Nonlinear electrical conductivity response of shaly-sand reservoir

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A brief review of electrical conductivity modelling of the porous medium is presented here to establish that, merely for the sake of applicability, the sound physico-mathematical model of Maxwell has been replaced by Archie's empirical one. The extension of clean sand models to shaly-sand models is discussed emphasizing the inadequacy of the former to represent true physical situations. Since the experimental data show nonlinear dependence of rock conductivity on pore fluid conductivity, three nonlinear model equations of Glover, Mixing and Bussian are studied for a wide range of parameters and the results are analysed in the light of physical expectations. It is concluded that only the Bussian model is able to simulate the behaviour of effective conductivity of saturated shaly-sand reservoir over the complete range of water and matrix conductivity and porosity values.

Keywords: Effective electrical conductivity, nonlinear response, shaly-sand reservoir.

THE electrical conductivity of reservoir rock is used to estimate porosity, clay content, permeability and water saturation as well as oil saturation. These estimations can be efficiently performed if a proper theoretical model is developed for computation of electrical response of heterogeneous porous medium comprising solid and fluid phases having different conductivities, volume fraction and connectivities.

The theoretical approach for calculating effective electrical conductivity (σ_0) of a porous medium of porosity ϕ, saturated with fluid of conductivity σ_w, started with Maxwell's[1] classical work, where he considered the electric current flow in a two-phase medium comprising uniform spheres immersed in a continuum. The final relation given below is obtained by solving the relevant Laplace's equation (Appendix eq. (A-14)),

$$\sigma_0 = \sigma_w \frac{2\phi}{3-\phi}. \tag{1}$$

Wagner[2] extended this formulation to very dilute solutions and came up with a linear form given by

$$\sigma_0 = 0.5\sigma_w(3\phi - 1). \tag{2}$$

Rayleigh, and later Fricke[3], extended the treatment to dispersed particles of regular but non-spherical shapes. Meredith and Tobias[4] treated the case of composite spheres with an inner core and an outer shell of different radius and conductivity. Using the concept of tortuosity, Slawinski[5] presented a treatment of the bulk conductivity of an electrolyte containing non-conducting spheres, as

$$\sigma_0 = a\sigma_w\phi. \tag{3}$$

Here, a is tortuosity of the reservoir. However, the most satisfactory expression for the conductivity of a water-bearing rock is attributed to Archie[6], who empirically established, both for unconsolidated and consolidated porous media, that the bulk conductivity also slightly depends on permeability of the sample and accounted for it through the well-known relation having ϕ exponent m, called the cementation factor, as given below

$$\sigma_0 = a\sigma_w\phi^m. \tag{4}$$

Archie's equation has been extremely useful in the computation of water and hydrocarbon saturations in brine-bearing non-shaly oil reservoirs.

In the case of consolidated medium, Slawinski's model has no theoretical basis and Archie's cementation factor is no longer independent of porosity; it becomes a function of other non-specified factors. This is the prime reason of its restricted use in shaly-sand log evaluation. Archie's equation occasionally results in misleading conclusions about aquifers in case of groundwater wells and also in case of oil wells in freshwater environments[7–12]. Hence there has been an explicit need for the development of a more appropriate model.

The complex distribution of solid and fluid phases having different conductivities, volume fractions and connectivities makes the modelling of heterogeneous porous medium a non-trivial problem. In shaly-sand reservoir, due to the clay particles possessing a net negative charge compensated by an excess number of cations close to the clay surfaces, surface conduction leads to a net increase in the effective fluid conductivity. Patnode and Wylie[13] empirically established that in case of shaly formation, the electric currents are also carried through the rock by a medium other than the electrolyte. They proposed that the conducting

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solids and the saturating fluid are electrically parallel so that the effective conductivity, σ_0 can be expressed as

$$\sigma_0 = \sigma_w \phi^m + \sigma_c. \quad (5)$$

Here, σ_c is the solid conductivity associated with clay and is directly proportional to the volume of clay. Winsauer and McCardell[14] attributed the clay conductivity contribution to mobile positively charged cations, or counterions, at the clay–brine interface called the double layer and termed it as 'excess conductivity', σ_s. They expressed effective conductivity as

$$\sigma_0 = \phi^m (\sigma_w + \sigma_s). \quad (6)$$

Since then, a large number of semi-empirical models based on statistical treatments of experimental observations have been proposed. These models are either shale fraction models or cation-exchange models derived using the concept of parallel conductor. In oil industry, for log analysis the two widely used models are those of Waxman and Smits[15] (WS) and Clavier *et al.*[16] (the dual water model).

It is important to emphasize here that the experimentally measured bulk conductivity of saturated shaly-sand formation shows nonlinear behaviour[17]. However, there are only a limited number of nonlinear equations capable of describing this characteristic of experimental data. The objective of this article is to critically evaluate the three relevant nonlinear equations.

The nonlinear models can be grouped into two classes. The first group of models is based on rigorous electrochemical principles[18–21] and these models contain some macroscopic geometric parameters (such as formation factor, electrical tortuosities and specific surface) interrelated with physico-chemical terms (as counter-ion charge density, effective ion mobility and Hittorf transport numbers). In the second group, the relations are formulated using unique macroscopic parameters such as formation factor, clay content and equivalent grain conductivity for the solid matrix. As the required physico-chemical parameters are not easily obtainable, it is difficult to use the equations of the first group in geophysical well log data interpretation. The equations of the second group are physically consistent with the general behaviour of experimental data and these have the advantage of circumventing the need of explicit evaluation of microscopic parameters related to electrical conduction through charged double layers. As the required parameters can be derived from the available set of log measurements, these equations are suitable for geophysical well log data interpretation. In this context, we will consider three equations that are shown to be robust.

The first equation is due to Glover *et al.*[22]. It is a modified Archie's equation that can be used with two conducting phases of any conductivity and volume fraction. It retains the ability to model variable conductivities within the phases that result from variations in their distribution. The modified model has two exponents (m and p) that describe the connectivity of each of the two phases. The exponents are related through an equation that also depends on the volume fractions of the two phases. It is claimed that the new model describes the experimental electrical behaviour of the system extremely well, improving greatly on the conventional Archie model for the two conducting phases. The response is given as

$$\sigma_0 = \sigma_w \phi^m + \sigma_s (1-\phi)^p, \qquad p = \frac{\log(1-\phi^m)}{\log(1-\phi)}. \quad (7)$$

The second and third equations studied in great detail by Lima *et al.*[17] are based on general mixture and effective medium theories respectively. In the general mixing rule, the electrical conductivity of a fully water-saturated rock is represented as a binary mixture of solid grains immersed in a continuous electrolyte and it can be expressed in terms of electrical conductivities, relative amount and topological distribution of the constituents. This representation is exact and derivable from the theory of functional equations under appropriate boundary conditions[23,24]. The equation can be written as

$$\sigma_0 = [\sigma_w^{1/m} \phi + (1-\phi) \sigma_s^{1/m}]^m. \quad (8)$$

The effective medium model is based on Bruggeman–Hanai[25,26] theory, which is an extension of the Maxwell–Wagner solution for dilute suspension in a constructive step process. Bussian[27] used the Bruggeman–Hanai equation – a simple formula for gauging the dielectric properties of random mixtures – in its low-frequency limit to determine the conductivity of the saturated shaly-sand reservoir. Lima and Sharma[28,29] modified the Bussian equation to develop a new scheme to describe the conductivity behaviour of clay gels, shales and shaly-sands under saline as well as freshwater saturations. The effective model avoids the assumption of parallel conductor, however, one must choose sand–shale geometry. The model is flexible enough to incorporate cation exchange data. The Bussian equation is written as

$$\sigma_0 = \sigma_w \phi^m \left[\frac{1-\sigma_s/\sigma_w}{1-\sigma_s/\sigma_0} \right]^m. \quad (9)$$

Substituting $(\sigma_s/\sigma_w) = \alpha$ and $(\sigma_0/\sigma_w)^{1/m} = x$, we can write eq. (9) as

$$x^m - \phi(1-\alpha)x^{m-1} - \alpha = 0. \quad (10)$$

Lima *et al.*[17] observed that, as the required parameters can be derived from the available set of log data, both Mixing and Bussian equations satisfactorily describe experimental core data and are suitable for well log analysis. It is instructive to consider a few interpreted parameters obtained by Lima *et al.*[17] using the data of Waxman and Smits[15] (WS) (Table 1). It is evident that the values of these parameters for different models vary over a wide range and therefore there exists a need for proper choice of a relevant model.

Since all the three nonlinear conductivity responses of saturated shaly-sand reservoir, Glover[22], Mixing[23] and Bussian[27], satisfy the experimental bulk conductivity variation with conductivity of water, here we evaluate these equations on the basis of exhaustive simulations over a wide range of parameters. We have used the 'bisection method'[30] of finding roots of a nonlinear equation. The values of parameters m, ϕ, σ_w and σ_s used in simulation are: m = 1.5, 2.0, 2.5, 3.0, 3.5 and 4.0, ϕ = 0.041, 0.074, 0.111, 0.131, 0.153, 0.198, 0.231, 0.289, 0.362, and 0.439, σ_w = 0.001, 0.002, 0.005, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, 200, 500, 1000 S/m and σ_s = 0.025, 0.22, and 5.5 S/m.

All the computed values are compared with the corresponding data computed using Archie's equation. It is pertinent to mention here that for all simulated results, the error function (the absolute value of the left hand side function in eq. (10)) for Bussian equation is significantly lower than that for Glover and Mixing equations. One such representative comparative plot is given in Figure 1. On inspection of the generated synthetic data, a general observation can be made that at low porosities all the three nonlinear equations, Glover (G), Mixing (M) and Bussian (B), lead to approximately the same values and that these values deviate from that of Archie (A), starting at $\sigma_w < 10$ S/m towards higher values of σ_0. Further, these values asymptotically approach the corresponding σ_s at lower values of σ_w. However, as porosity increases from low to high values, the computed values of bulk conductivity using Bussian equation deviate from the other two towards A for σ_w below 0.1 S/m. The other two equations almost become independent of σ_w and always approach asymptotically to σ_s. In order to support our observations, we present here only a limited number of results in Figure 2 *a–d*. These are for the two values 0.041 and 0.439 representing respectively, low and high porosities. The two m values chosen are 1.5 and 2.5, while the only σ_s considered is 0.22.

Table 1. Parameters determined using linear and nonlinear equations

Data source	Model	ϕ (%)	m	σ_s (S/m)
WS (C-22)	Linear	22.9	2.39	0.063
	Bussian	6.55	1.31	0.083
	Mixing	6.55	1.35	0.074
WS (C-26)	Linear	22.9	2.64	0.120
	Bussian	6.05	1.31	0.144
	Mixing	6.06	1.37	0.136

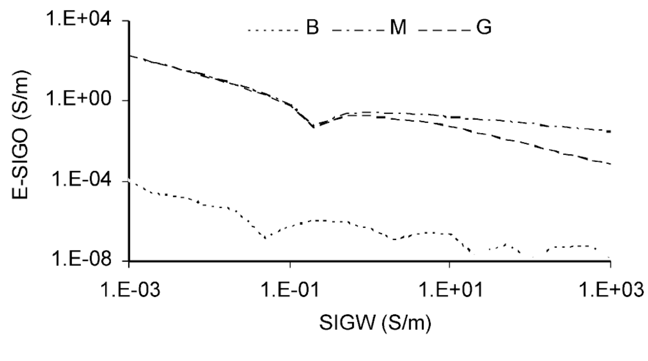

Figure 1. Computational errors in σ_0 (E-SIGO) for Bussian (B), Mixing (M) and Glover (G) models.

In order to understand the causes behind the significant differences in the behaviour of the three nonlinear equations for low σ_w, we obtained the low σ_w range asymptotic expressions of the Glover, Mixing and Bussian equations respectively, as

$$\sigma_0 \cong \sigma_w \phi^m + \sigma_s (1-\phi^m), \quad (11)$$

$$\sigma_0 \cong \sigma_s (1-\phi)^m, \quad (12)$$

$$\sigma_0 \cong \sigma_w \phi^{m/(1-m)}. \quad (13)$$

A glance at these equations reveals that for a given value of ϕ and for low σ_w, σ_0 is linearly proportional to σ_s in the case of Glover and Mixing equations, while it is linearly proportional to σ_w for the Bussian equation. The difference in the intercepts of Glover and Mixing equations increases as porosity increases.

Another test undertaken was to compare the above three equations for m = 0, a case simulating no cementation or equivalently 100% porosity. In this case, Glover, Mixing and Bussian equations give σ_0 equal to $(\sigma_s + \sigma_w)$, $\max(\sigma_s, \sigma_w)$ and σ_w respectively. Thus, Bussian equation simulates physics of the situation more realistically. It may be added here that for m = 1, all the three equations lead to σ_0 equal to $(\sigma_s(1-\phi) + \sigma_w\phi)$.

Next we tested the accuracy of the Bussian equation by comparing the numerical solution and the exact solution for m = 2, which leads to a more practical situation. In this case, eq. (9) reduces to a quadratic equation having two roots as

$$\sigma_0 = \sigma_s + \frac{[(\sigma_w - \sigma_s)\phi]^2}{2\sigma_w} \left[1 \pm \left(1 + \frac{4\sigma_s\sigma_w}{[(\sigma_w - \sigma_s)\phi]^2} \right)^{1/2} \right]. \quad (14)$$

Equation (14) represents a parabolic structure having vortex at $\sigma_w = \sigma_s$. For $\sigma_s < \sigma_w$, expanding the term under square

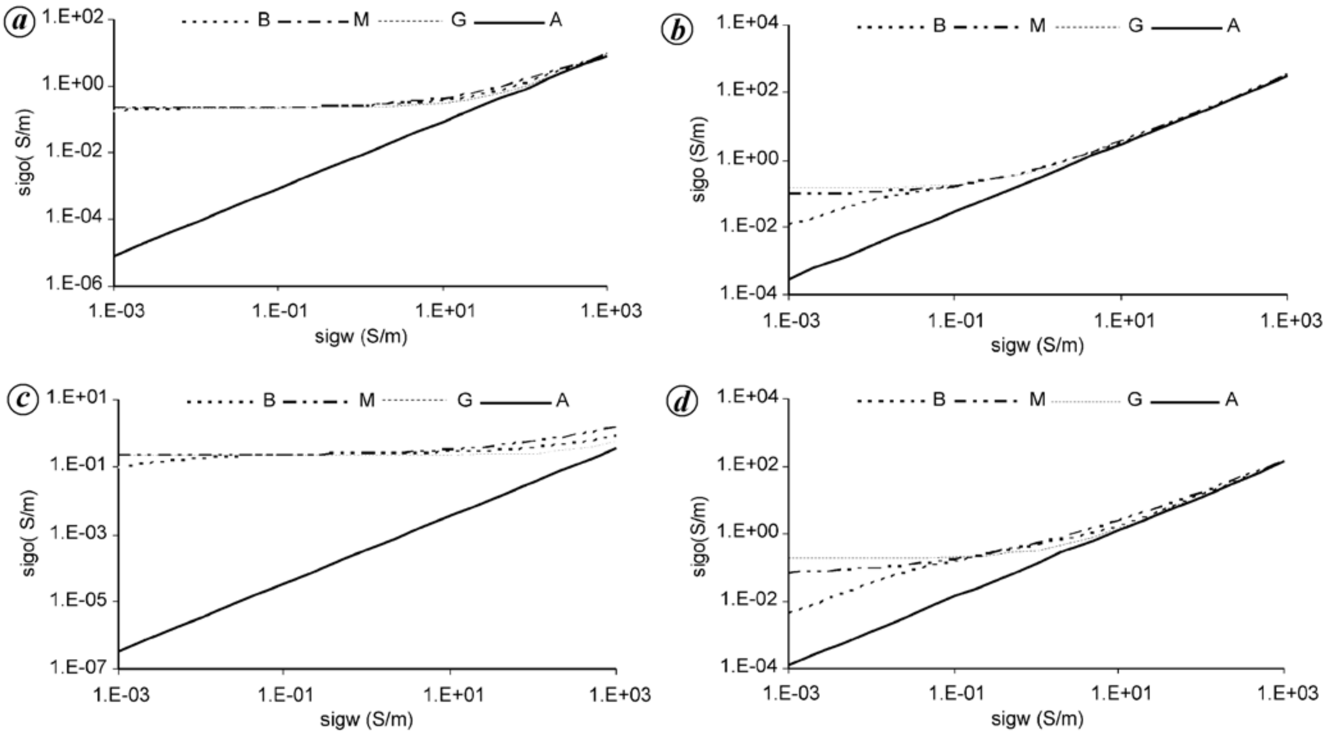

Figure 2. Effective conductivity (sigo) vs water conductivity (sigw) obtained for the four models, Glover (G), Mixing (M), Bussian (B) and Archie (A) for $m = 1.5$, $\phi = 0.041$ and $\sigma_s = 0.22$ (***a***), $m = 1.5$, $\phi = 0.439$ and $\sigma_s = 0.22$ (***b***) $m = 2.5$, $\phi = 0.041$ and $\sigma_s = 0.22$ (***c***) and $m = 2.5$, $\phi = 0.439$ and $\sigma_s = 0.22$ (***d***).

root and retaining only the linear order terms, the first root (with +ve sign) of eq. (14) is simplified to

$$\sigma_0 = 2\sigma_s + \frac{[(\sigma_w - \sigma_s)\phi]^2}{\sigma_w}. \tag{15}$$

This can be further simplified by rejecting the term with second power of small σ_s as

$$\sigma_0 = \phi^2[\sigma_w + 2(\phi^{-2} - 1)\sigma_s]. \tag{16}$$

The asymptotic solution for $\sigma_w \to \infty$ can be written as

$$\sigma_0 = \phi^2\sigma_w + \sigma_s. \tag{17}$$

It may be emphasized that eq. (16), the Revil *et al.*[20] equation as the high salinity asymptote of their general equation, and the linearized solution of equation of Bussian[27] obtained by Lima and Sharma[28,29] (for $m = 2$) are identical. Further, eq. (17) is the same as the one given by Patnode and Wyllie[13] that was further modified by Winsaur and McCardell[14]. Thus all the existing linear models can be simulated using Bussian equations in the respective ranges of parameters.

In conclusion, we make the following observations: (i) the empirical linear models based on parallel conductor concept are unable to simulate effective conductivity of a shaly-sand formation saturated with water of low conductivity; (ii) the existing nonlinear equation of Glover *et al.*[22] and the Mixing[23] equation studied by Lima *et al.*[17] are able to simulate the effective conductivity curves for the entire range of water conductivity only for low porosity and these fail to simulate the real behaviour as the porosity increases, (iii) the Bussian equation simulates the effective conductivity curves for all ranges of porosity and water conductivity, and (iv) the Bussian equation which is more consistent with the physics, reduces to the existing linear models and thus is consistent with these models. Hence, for nonlinear interpretation of well log data Bussian equation is the most appropriate one.

Appendix 1. Derivation of Maxwell's equation of effective conductivity.

The law of conservation of charges ($\nabla \cdot J = 0$; J being current density) and the Ohm's law ($J = \sigma E$, $E = -\nabla U$, σ is the electrical conductivity of the medium and U the electrical potential developed) results in Laplace's equation ($\nabla^2 U$) for the potential due to electrical conduction in a conductor. For its solution, let us assume that a sphere having radius r_i and conductivity σ_i is immersed in an electrolyte of conductivity σ_w and is brought under the influence of a uniform electric field E, which has lines of force parallel to the x-axis. Assuming zero charge density, the system satisfies Laplace's equation, whose general solution in spherical polar coordinates (r, θ, φ) can be obtained through the method of separation of variables as

$$U = \sum_{n=0}^{\infty} (K_n r^n + \chi_n r^{-n-1}) P_n(u), \tag{A1}$$

where K_n and χ_n are arbitrary constants to be determined using relevant boundary conditions, and $P_n(u)$ are Legendre polynomials given by

$$P_n(u) = \frac{1}{2^n n} \frac{d^n}{du^n} (u^2 - 1)^n, \; u = \cos\theta. \tag{A2}$$

As $r \to 0$, U remains finite and as $r \to \infty$, $U = -Er\cos\theta$. The potential inside and outside the sphere can be written as

$$U_i = \sum_{n=0}^{\infty} K_n r^n P_n(\cos\theta); \; r < r_i, \tag{A3}$$

and

$$U_w = \sum_{n=0}^{\infty} \chi_n r^{-n-1} P_n(\cos\theta); \;\; r > r_i. \tag{A4}$$

Applying the boundary conditions of continuity of potential and radial current density at $r = r_i$ given as

$$U_i = U_w \quad \text{and} \quad \sigma_i \frac{\partial U_i}{\partial r} = \sigma_w \frac{\partial U_w}{\partial r} \quad \text{at } r = r_i, \tag{A5}$$

to eqs (A3) and (A5), we obtain $K_n = \chi_n = 0$. For $n = 1$, we get

$$K_1 = -\frac{3E\sigma_w}{\sigma_i (2\sigma_w + \sigma_i)} \quad \text{and} \quad \chi_1 = -\frac{r_i^3 E(\sigma_w - \sigma_i)}{(2\sigma_w + \sigma_i)}. \tag{A6}$$

Now, eqs (A3) and (A4) can successively be written as

$$U_i = -\frac{3E\sigma_w r\cos\theta}{\sigma_i (2\sigma_w + \sigma_i)}, \tag{A7}$$

and

$$U_w = -\frac{r_i^3 E(\sigma_w - \sigma_i)\cos\theta}{(2\sigma_w + \sigma_i) r^2} - Er\cos\theta. \tag{A8}$$

Let us assume now that there are N equal spheres of radius r_i arranged in such a fashion so as to form a larger sphere of radius r_N, and that electric interference among the smaller spheres is negligible. Thus the potential outside the system resulting from the presence of a bunch of spheres is given by

$$U_w = -\frac{N r_i^3 E(\sigma_w - \sigma_i)\cos\theta}{(2\sigma_w + \sigma_i) r^2} - Er\cos\theta. \tag{A9}$$

Similarly, if σ_0 represents the bulk conductivity of the sphere of radius r_N, the potential outside the system can be written as

$$U_w = -\frac{r_N^3 E(\sigma_w - \sigma_0)\cos\theta}{(2\sigma_w + \sigma_0) r^2} - Er\cos\theta. \tag{A10}$$

Equating eqs (A9) and (A10) we get

$$\sigma_0 = \sigma_w \frac{r_N^3 (2\sigma_w + \sigma_i) - 2N r_i^3 (\sigma_w - \sigma_i)}{N r_i^3 (\sigma_w - \sigma_i) + r_N^3 (2\sigma_w + \sigma_i)}. \tag{A11}$$

In the light of the definition of fractional porosity

$$N r_i^3 = r_N^3 (1 - \phi). \tag{A12}$$

Equation (A11) can be modified as

$$\sigma_0 = \sigma_w \frac{3\sigma_i + 2\phi(\sigma_w - \sigma_i)}{3\sigma_w - \phi(\sigma_w - \sigma_i)}. \tag{A13}$$

In case of a sphere of insulating material ($\sigma_i = 0$), we can write Maxwell's equation as

$$\sigma_0 = \sigma_w \frac{2\phi}{(3 - \phi)}. \tag{A14}$$

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