

# Star clusters in the Magellanic Clouds and their mass functions

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*Study of star clusters in a galaxy throws light not only on the processes of star formation and evolution in the galaxy but also on the structure and evolution of the galaxy. In this respect, star clusters of the Magellanic Clouds are of extreme importance because they differ from those of our galaxy in many important respects, e.g. they occupy regions of the age and metallicity domain which are not populated in our galaxy. Therefore they extend the range of comparison between stellar evolutionary theory and observational data. As the evolutionary history of the Magellanic Clouds has been very different from that of our galaxy, study of the mass function of those young star clusters which are rich in stars, and span a wide mass range, can provide the answer to the astrophysically important question of the universality of the shape of the initial mass function in time and space.*

A star cluster is a group of dynamically associated stars, presumably created from the same material at

about the same time and, consequently, located at the same distance. All the cluster members move together through the star fields of its galaxy and will maintain their identity for some time which is known as the cluster lifetime. A colour photograph of the southern

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galactic globular star cluster NGC 104 is shown in Figure 1. It is a naked eye object. The cluster lies at a distance of  $\sim 13,000$  light years and is amongst the younger ( $\sim 10^{10}$  years) galactic globular clusters. Its stars have a fairly high ( $\sim 10$  times) content of heavy elements in comparison to that of older globular clusters. It is believed that this difference is due to early supernova explosions that enriched the interstellar medium with heavier elements, when the Milky Way galaxy was still very young. The oldest clusters would thereby have fewer metals than the younger ones. Study of star clusters is important not only because insight is gained into their formation and evolution but also, since it throws light on other related problems like the processes of star formation and evolution, the structure and evolution of galaxy, etc. Studies of star clusters therefore occupy a central place in the modern astrophysical research.

Trumpler<sup>1</sup> in 1925 emphasized the special value of Hertzsprung-Russel (HR) diagram of star clusters in comparison to that of field stars. Position of a star in the HR diagram depends mainly upon its mass, age

(evolutionary stage) and chemical composition. Presence of rotation, peculiarity and binarity, etc. in the stars changes the position slightly in the HR diagram. As all members in a star cluster are nearly at the same distance, their colour-magnitude diagram (CMD) is equivalent to their HR diagram. Because of common origin of the cluster members, their HR diagram reflects the evolutionary dispersion of stars of different masses, after a lapse of time typified by the age of the star cluster. Consequently, observational HR diagrams of star clusters are a natural test bed for the theories of stellar evolution.

The star clusters found in the Magellanic Clouds (MCs), two largest satellite galaxies of the Milky Way, are different in many ways from those of our galaxy (see refs. 2, 3). Figures 2 and 3 show the colour pictures of the Large Magellanic Cloud (LMC) and the Small Magellanic Cloud (SMC) respectively where a large number of star clusters are clearly visible in the form of



Figure 1. Colour photograph of the galactic globular cluster NGC 104 or 47 Tucanae reproduced from the ESO slide No. 10. It is the second brightest globular cluster in the sky and located in the constellation of Tucana, close to the Small Magellanic Cloud.

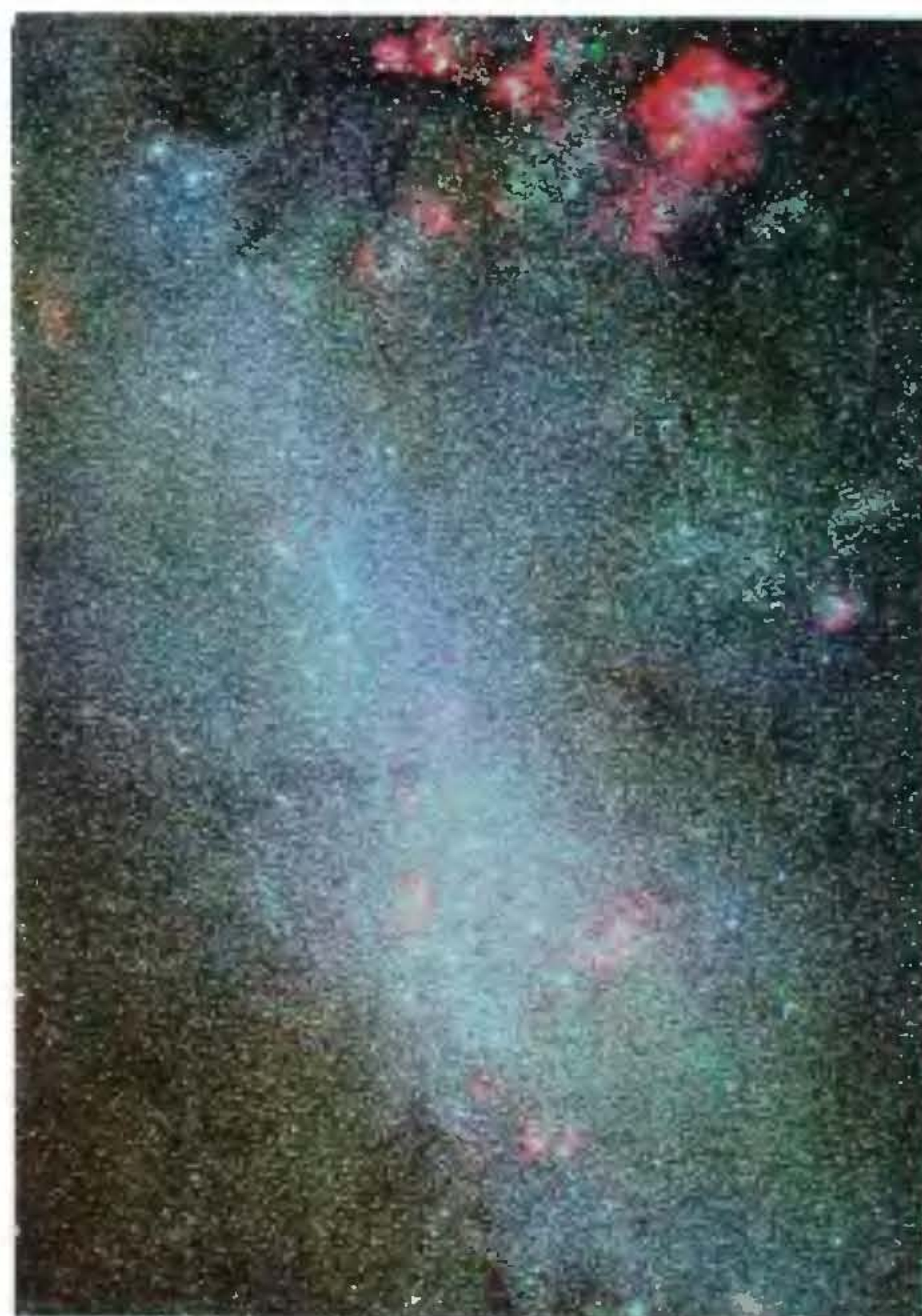


Figure 2. The Large Magellanic Cloud colour photograph reproduced from the ESO slide No. 12. It is our nearest neighbour galaxy in the Universe. Its distance is  $\sim 180,000$  light-years which is only a little more than the diameter of our galaxy, the Milky Way. The LMC is the prototype of irregular galaxies; that is galaxies which can neither be classified as elliptical nor spiral.





Figure 3. The colour picture of the Small Magellanic Cloud reproduced from the ESO slide No. 15. It is our second nearest neighbour galaxy in space, at a distance of  $\sim 250,000$  light-years. The LMC and SMC are quite close in space, the distance is about 90,000 light-years. A band of neutral hydrogen, the so-called Magellanic Stream, connects the LMC and SMC and can be detected with radio telescopes. The SMC is also an irregular galaxy, but it is less massive than the LMC.

diffuse objects. When we sort the star clusters of our galaxy and those of the MCs according to their integrated absolute magnitude (see Figure 4), it is noticed that there exists a clear-cut dichotomy between massive globular clusters and much less massive open star clusters. No such distinction is observed for star clusters in the MCs. In fact, MCs contain a class of populous star clusters that have no galactic counterpart. They exhibit parts of the age and metallicity domain different from those in our galaxy. Furthermore, studies of the populous clusters in the MCs allow one to study the evolution of massive ( $\geq 4M_{\odot}$ ) stars in rapid evolutionary phases, which are unlikely to be observed in sparser galactic open clusters. Consequently, the star clusters in MCs extend the range of comparison between stellar evolutionary theory and observational data. A few other important differences

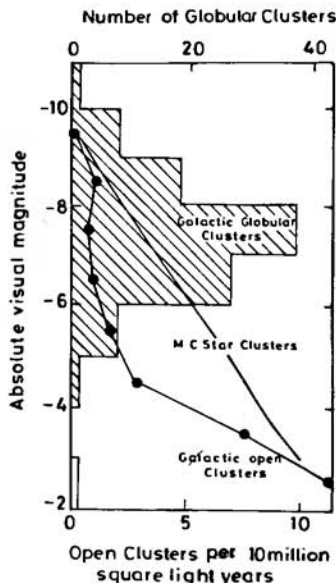


Figure 4. The luminosity function of star clusters of the Milky Way and Magellanic Clouds. Data for the galactic open clusters are taken from Sagar *et al.*<sup>4</sup>, while those for galactic globulars and Magellanic Clouds are taken from van den Bergh<sup>5</sup>. The Milky Way's open and globular clusters form two distinct groups. The Magellanic Clouds' young and intermediate-age star clusters fall roughly along a straight line.

between the star clusters of our galaxy and those of MCs are given below.

#### Sizes of star clusters

For characterizing the size of star clusters, the radius,  $r_h$ , which contains half of the cluster light is more appropriate than tidal radius or core radius because the value of  $r_h$  is relatively less affected by the galactic tidal forces as well as dynamical evolutionary effects of the cluster. A comparison of these values indicates that the star clusters in MCs are, on an average, 3-4 times larger than their galactic counterparts<sup>3</sup>. The most obvious explanation for this observation may be that clusters with larger radii are preferentially formed in regions of low density as the overall density of the clouds is lower than that of the galaxy. This picture seems to be consistent with the observation that the globular cluster NGC 1409, in the low-density Fornax dwarf, is even larger than that of NGC 121 in the SMC.

### Shape of the clusters

Galactic globular clusters are predominantly spherical (see Figure 1), whereas the star clusters in MCs are highly flattened. There are no Galactic counterparts to such highly flattened objects as NGC 121 in the SMC and NGC 1978 in the LMC. Star clusters in the SMC are, on an average, even more flattened than those in the LMC<sup>6</sup>. The reason for these differences in the mean ellipticity of star clusters in different stellar systems is not well understood. If one assumes that flattening of a star cluster can be due mainly to rotation, then the difference between the mean ellipticity of Galactic and MC star clusters suggests that there may be a systematic difference between the angular momentum with which star clusters are formed in the Galaxy and the MCs.

### Age distribution of MC star clusters

Figure 5 shows age distribution of MC star clusters derived from the data given by Sagar and Pandey<sup>7</sup>. This indicates some interesting differences in the evolutionary histories of the two clouds. The figure suggests that the LMC was relatively quiescent between 0.1 and 0.5 Gyr ago and between 3 and 10 Gyr ago. No obvious bursts of cluster formation or long periods of quiescence are visible in the age distribution of SMC star clusters. The absence of a particularly large age gap (~ 3 to 10 Gyr) in the LMC cannot be due to selection effects because star clusters with ages 3–10 Gyr have been found in the SMC. The peak-and-valley nature of the LMC plot suggests that discrete episodes of cluster

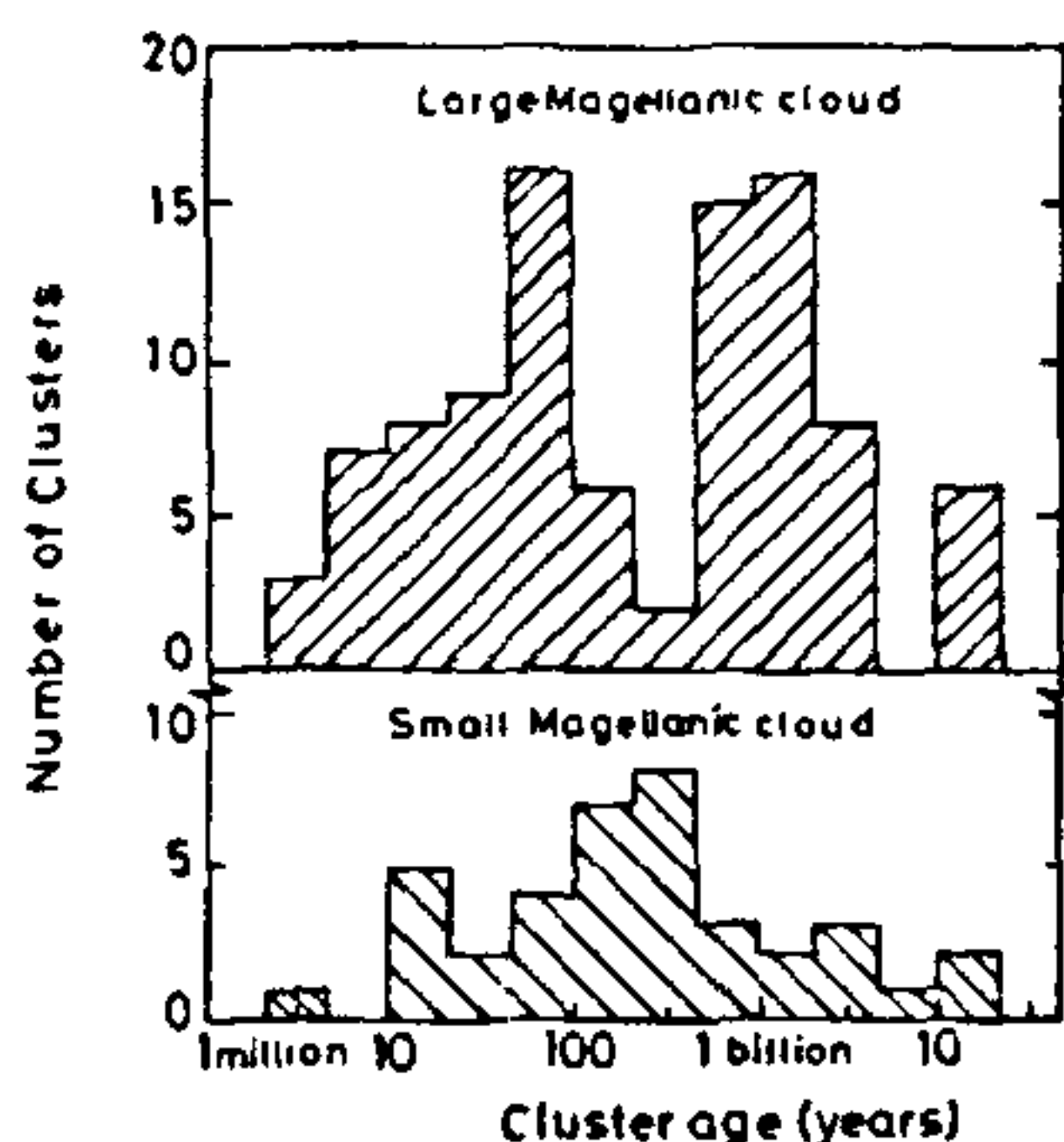


Figure 5. Age distribution for star clusters in the LMC and SMC from data compiled by Sagar and Pandey<sup>7</sup>. The peaks for the LMC suggest this galaxy experienced episodic bursts of star formation, whereas the comparatively smoother distribution for the SMC indicates a fairly constant rate of cluster production

formation occurred. This is in obvious contrast to the gradual rate of cluster formation in the SMC, which mimics the situation in our own galaxy. Assuming that bursts of cluster formation should occur simultaneously in both members of an interacting pair, we can conclude that the LMC bursts were not triggered by close encounters with either the Milky Way or the SMC.

### Star clusters in different galaxies

The above discussions as well as studies of star clusters in other external galaxies suggest that the frequency of star clusters differs from galaxy to galaxy. This may indicate that formation of star clusters in a galaxy is affected by the environment in which they form. However, the relation between environment and formation of star clusters in a galaxy is far from understood and knowledge of such relation is of extreme importance for our understanding of galaxy formation and evolution. In this respect, the star clusters of Magellanic Clouds are valuable because the MCs are amongst a few external galaxies where the CMDs of star clusters based on the observations of individual stars can still be constructed. With the advent of the Hubble space telescope and the European Southern Observatory 3.6 meter new technology telescope where imaging with sub arcsecond (0.2–0.3 arcsec) resolution is possible, this situation may improve slightly. However, they still cannot provide observations of individual stars in the star cluster of galaxies located farther than ~ 800 kpc. Consequently, the MC star clusters are, indeed, link between the star clusters of our galaxy and those in more distant galaxies where only integrated light observations can be at our disposal.

### Colour-magnitude diagrams

One important aspect of the MC star cluster research is to obtain their accurate photometric CMDs. They are valuable tools for testing stellar evolution theories upto very advanced phases. They also provide basic information for understanding the star formation and chemical evolutionary processes in these neighbouring galaxies. An inspection of the recently published catalogue on the CMD studies of star clusters in MCs (see refs. 2, 3, 7 and references therein) reveals that most studies are based on photographic and electronographic observations which are, generally, not only restricted to bright stars ( $V \sim 17$ –18 mag) but are also limited in accuracy. They may have systematic errors as well<sup>8</sup>. A detailed comparison of theoretical stellar evolutionary models with the narrow and well-defined stellar sequences in the CMDs of MC star clusters was thus questionable upto now. The advent of new large



telescopes in the south and modern CCD astronomical detectors has stimulated studies on MC star clusters. In obtaining the accurate CMD of MC star clusters, recent CCD observations by Sagar *et al.* and Mateo<sup>9</sup>, among others, have shown their superiority over conventional photographic and other types of photometry. The linearity, dynamic range and sensitivity of the CCD have dramatically reduced the internal photometric scatter in the CMDs of the star clusters. The software technique used for the reduction of CCD data shares at least equal responsibility with the detectors in producing the vastly improved CMDs of star clusters in MCs. The key ingredient in the new software is replacement of fixed-aperture photometric techniques with the point spread function of stellar images. Consequently, accurate stellar photometry becomes a reality even in crowded stellar regions.

### Initial mass function

Another important topic which can be tackled by investigating the  $V$ ,  $(B-V)$  CMDs of young (age  $< 10^8$  yr) MC star clusters is the shape of the initial mass function (IMF) of stars that are born in a star-forming region. The IMF ( $\equiv \xi(M)$ ) is described as the frequency distribution of stellar masses on the hydrogen burning main sequence (MS) of an ensemble of stars at the time of their birth. It is generally defined as  $dN/dM (= \xi(M)) \propto M^{-(1+x)}$ , where  $dN$  is the number of stars in the mass interval  $dM$  at mass  $M$  and the Salpeter<sup>10</sup> value of  $x$  is 1.35. Detailed knowledge of the IMF in different environments is crucial for studies that attempt to describe the spectral, photometric, and chemical evolution of integrated stellar systems ranging in complexity from star clusters to galaxies because mass is one of the primary parameters which dictate the evolution of stars. Knowledge of IMF is also important for constraining star formation theories and for understanding the early evolution of star clusters because it is a fossil record of the very complex process of star formation and provides an important link between the easily observable population of luminous stars in a stellar system and the fainter, but dynamically more important low-mass stars.

A fundamental question in the theories of stellar and galactic evolution is whether the shape of the IMF is universal in time and space or whether it depends on parameters like metallicity, age, environment, etc. The answers are still unknown. At first glance, this seems surprising in view of the simplicity of the concept of IMF. After considering the steps involved in the observational determination of an IMF<sup>11</sup>, one becomes aware of the large variety of difficulties and error sources involved in it. In order to emphasize this, some basic definitions are given below.

The stellar luminosity function, LF ( $\equiv \vartheta(M_i)$ ) represents the number of stars per unit magnitude interval in a given volume derived from observations in passband  $i$ . The observed present day mass function, MF ( $\equiv N(M)$ ) denotes the number of stars per unit mass in a given volume.

The LF and MF are related by

$$N(M, \dots) = \vartheta(M_i) dM(z, \tau) / dM_i, \quad (1)$$

where  $z$  refers to metallicity;  $\tau = \text{age}/\text{total MS lifetime}$  and  $dM/dM_i$  is the MS mass-absolute magnitude relation for a passband  $i$  which can be derived using theoretical stellar evolutionary and atmospheric models.

The relation between MF and IMF is given by

$$N(M) = \int_{\max[0, T-t(M)]}^T \xi(M, z, t \dots) \psi(t) dt, \quad (2)$$

where  $\psi(t)$  is the star formation rate (number of stars formed per unit time interval in a given volume);  $T$  is the age of the galaxy being considered and  $t(M)$  is the MS lifetime of a star with mass  $M$ . Therefore, derivation of IMF ( $\equiv \xi(M, z, t \dots)$ ) from the observed LF ( $\equiv \vartheta(M_i)$ ) involves many steps. Consequently, determining the IMF using field stars is complicated because the population of field stars is a mixture of stars with different ages, distances, and metallicities. Untangling these effects and correcting for the many biases inherent in the problem is an arduous task. In such work, the universality of the IMF (one of the questions) is assumed instead of being investigated and the shape of the IMF can only be estimated if further assumptions regarding the past star formation rate are made.

The behaviour of the IMF with time or metallicity or environment can be studied in a more direct way if one observes many star clusters with different age, abundances etc., because star clusters contain an (almost) coeval set of stars at the same distance with the same metallicity. Therefore,  $\psi(t)$  in equation (2) can be replaced with  $\psi_{cl} \delta(t - t_0)$ ; where  $\psi_{cl}$  is the scale factor describing the number of stars formed in the cluster;  $t_0$  is the cluster age and  $\delta$  is the Dirac delta function. In such situation equation (2) becomes

$$N(M, z, t \dots) = \xi(M, z, t \dots) \psi_{cl}. \quad (3)$$

Therefore, in the simplest imaginable situation, the MF of a star cluster is equivalent to its IMF and it can in principle be determined from the LF using theoretical stellar evolutionary models. In reality the situation is different, because the stellar MF of any star cluster changes with time due to stellar as well as dynamical evolutionary effects. Such changes can, however, be



ignored if one considers clusters younger than their dynamical evolution time. The young populous clusters in the MCs are therefore the ideal laboratories for not only investigating the IMF in stellar systems but also for extending the study of IMFs to other galaxies. They are only slightly less massive than the globular clusters in the Milky Way, but in several other respects they resemble the galactic open star clusters. MC star clusters are all at the same and relatively well known distances. They are rich in stars, and span a wide mass range. Thus, they overcome both the limitation of small number statistics imposed by young star clusters of our galaxy and the problem of narrow mass range present in galactic globular clusters. Also, unlike for galactic open clusters, where corrections for reddening are not always simple because it could be large as well as variable<sup>12</sup>, for MC star clusters the treatment of reddening is normally not a problem because it is relatively small. Many of the MC star clusters are so young that dynamical processes will not have had enough time to alter their IMF. The evolutionary history of the MCs has been very different from that of our galaxy. Consequently, work on young MC star clusters is of importance for providing the answer to the question of the universality of the IMF in time and space.

However, in such distant stellar systems, we encounter another type of serious problem, namely crowding of stellar images which makes the IMF determination somewhat complicated. At the distance of the LMC, 1 arcsec corresponds to 0.25 pc. Most of the LMC star clusters are therefore immensely crowded from the observational point of view. In spite of this, in 1988, we obtained BV CCD data for 5 young LMC star clusters (NGC 1711, 2004, 2100, 2164 and 2214) and attempted to estimate the IMF of MC star clusters. They are located in different parts of the LMC (see Figure 6). While CCD data reductions were still in progress, we came across the two main contributions on this subject namely by Elson *et al.*<sup>13</sup> and Mateo<sup>9</sup>. Elson *et al.* derive the IMFs for 6 young (30–50 Myr) LMC star clusters by star counts on photographic plates. They find flat IMFs with  $x$  ranging from  $-0.2$  to  $0.8$  in the mass range  $1.5-6.0 M_{\odot}$ . On the other hand, Mateo using CCD BV photometric data, finds a very different range in IMF slopes from 1.5 to 3.6 for another 6 MC star clusters spanning a range in age from 10 Myr to 2.5 Gyr; in metallicity from about solar  $1/20$ th solar and in the mass from  $0.9-10.5 M_{\odot}$ . Thus the two results differ significantly. This gave us an additional motivation for the study of the IMF of MC star clusters in greater detail.

#### Observational data

The BV CCD data were obtained using the 1.54 m

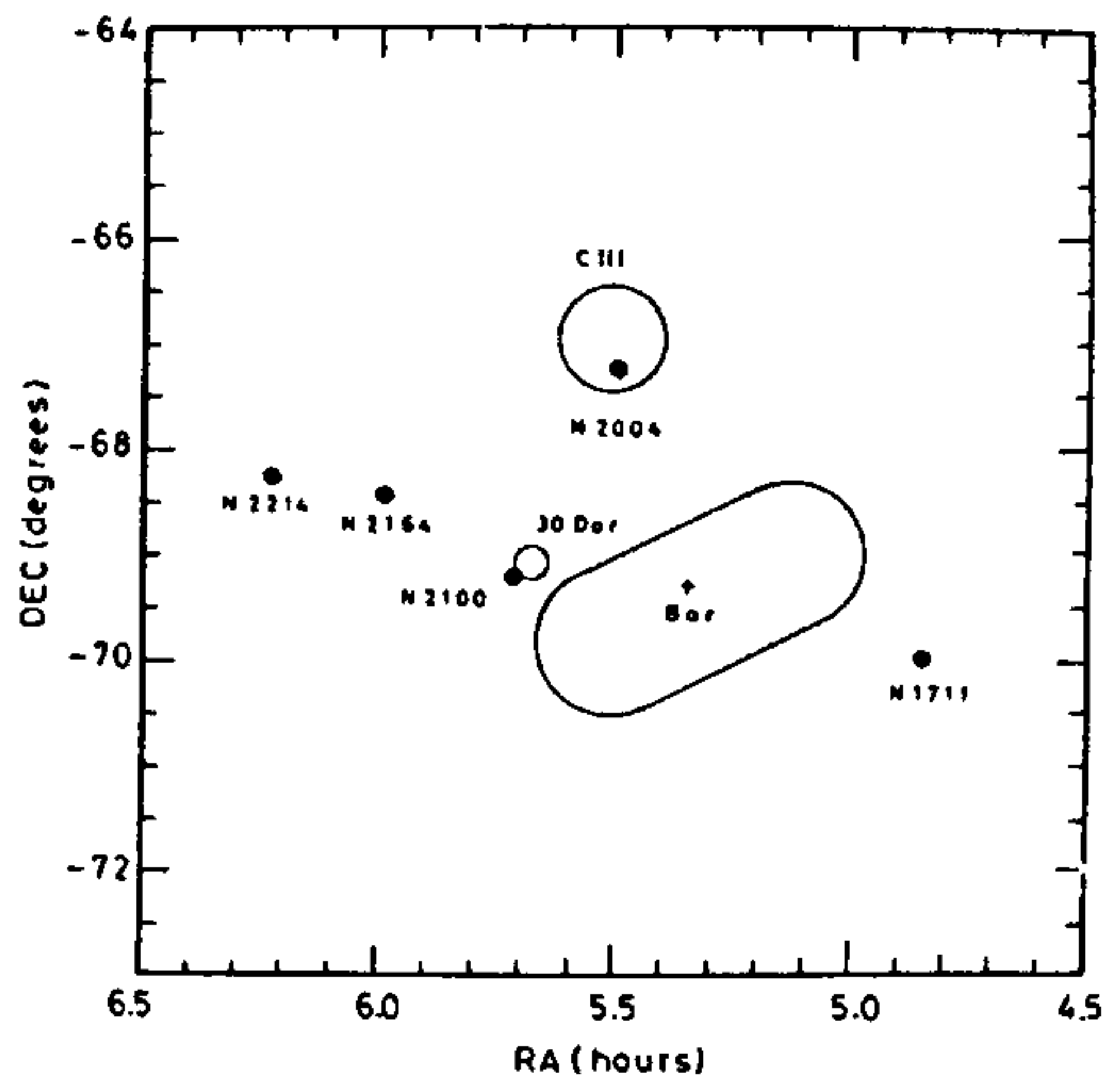


Figure 6. Positions on the sky of the clusters in our sample. The Bar, 30 Doradus, Constellation III and the centre of the LMC are also identified.

Danish telescope at the European Southern Observatory, La Silla, Chile. A detailed description of the observations and data reductions is given by Sagar *et al.*<sup>8</sup>. In addition to cluster region, we also imaged a field region located  $\sim 4$  arcmin away from the cluster frame which was used to correct the cluster luminosity function (LF) for field star contamination. Angular diameters of the programme clusters are less than 3 arcmin. This indicates that the probability of the presence of a significant number of cluster members in the field regions is very low, while the probability that the distribution of field stars in the cluster region can be represented very well by the stars of the imaged field region is very high.

Based on the present CCD observations the CMDs for NGC 2004 and its neighbouring field regions are shown in Figure 7. In the CMD of the cluster, a well defined stellar sequence from the main sequence to the red supergiant phase is clearly visible. There is a concentration of stars at  $V \sim 19.5$  and  $(B-V) \sim 0.9$  mag in the CMDs of both cluster and field regions. These stars are intermediate-age core helium burning stars of the LMC field forming a clump in the CMD.

#### Data incompleteness

The main factor which limits the precise determination of LFs from observations is data incompleteness. A quantitative evaluation of completeness of the photometric data with respect to brightness and position on a

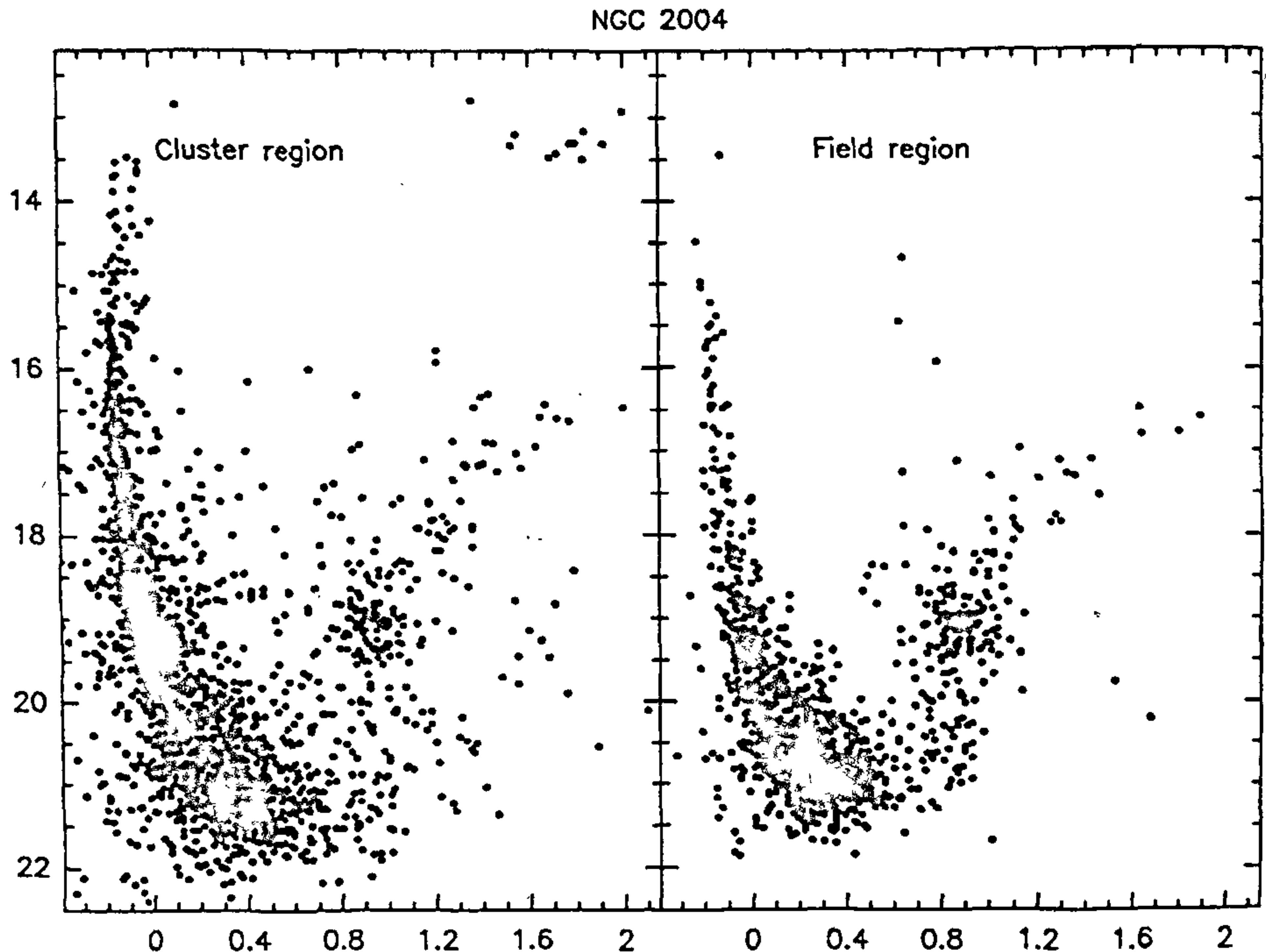


Figure 7. The  $V, (B-V)$  diagrams for the LMC star cluster NGC 2004 and its neighbouring field region. A well defined stellar sequence is clearly visible in the cluster region. The red supergiants are missing in the field region, indicating that a population as young as the cluster is not present in the imaged field region.

given image frame is required because it converts an observed LF into a true LF. Quantification of data completeness was not straightforward about a decade ago. Thanks to the latest developments in software techniques which provide a way to quantify the completeness of the stellar data. The method is to perform experiments with artificial stars, which are inserted randomly in the original image with known magnitude and positions. This new image frame is then reduced in the same way as the original image frame. The ratio of recovered to inserted stars gives directly the completeness factor, CF. Since in the above procedure the added artificial stars should not change the crowding characteristics of the original image frame, only a limited number ( $\sim 10\text{--}15\%$  of the number of originally detected stars) can be added at one time. Thus, to have a good number statistics for the estimation of CF, one must repeat the above process on a given image frame many times, which means

considerable computer CPU time for data reductions. In fact we used more than 2 months of CPU time on a micro VAX 3500. The luminosity distribution of the artificial stars has been chosen such that more stars are added in the domain of high incompleteness, e.g., the fainter ones or in the central region of the cluster, because only a small fraction of them will be recovered during photometric reduction. Details of the artificial added star experiments were given by Sagar and Richtler<sup>14</sup>.

The procedure outlined above will give CF for stellar data derived from one photometric pass band, e.g., in our case in  $B$  and  $V$ . What is the value of CF for the LF derived from  $V, (B-V)$  CMD? The answer to this question is by no means simple. Mateo<sup>9</sup>, for instance, assumes that the completeness factor,  $CF(V_i, (B-V)_i)$ , at a point  $(V_i, (B-V)_i)$  in the  $V, (B-V)$  CMD is the product of the CF values of the  $V$  and  $B$  frames at magnitude  $V_i$  and  $B_i$  respectively. This would mean that



stars on  $V$  and  $B$  frames are independent. But the geometrical configuration of stars in both  $V$  and  $B$  frames of a stellar region is exactly the same, only the magnitude distribution is slightly modified due to the range of colours of the stars. Therefore,  $B$  and  $V$  frames of a region are by no means independent and the assumption of a multiplicative nature for  $CF(V_i, (B-V)_i)$ , as Mateo used, cannot be justified. Actually, the completeness at a given magnitude in  $V$ ,  $(B-V)$  diagram will be mainly controlled by that  $CF$  value of those of the two wavelength bands where completeness is less<sup>14</sup>. Thus, the effective value of  $CF(V_i, (B-V)_i)$  at point  $(V_i, (B-V)_i)$  cannot be larger than the smaller value of the pair  $(CF(V_i), CF(B_i))$ , so this is the minimum correction. It is difficult to estimate the actual correction. As it is not too different from the minimum correction, we have corrected our data for incompleteness using this factor.

Mass function

Despite difficulties mentioned above, true cluster LF can be derived from the observed one by accounting properly for the data incompleteness and for field star contamination. In order to transform the true LF of a cluster into its MF, we require not only the cluster reddening, distance, metallicity and age but also the appropriate theoretical stellar evolutionary models. Cluster age and MF slope derived for solar metallicity using Maeder's<sup>15</sup> model are given in Table 1.

As the ages of the clusters are less than their dynamical evolution times ( $\sim 10^9$  yr), the slope of the present-day MFs can be considered as the slope of IMF. The star clusters are located in different parts of the LMC (see Figure 6) and the different MF slopes agree within errors. From this we conclude that IMF of the young LMC star clusters is not too different from the solar neighbourhood IMF of Salpeter with  $x = 1.35$ .

In terms of stellar mass range, the young galactic open star clusters provide the most analogous stellar systems to compare with the young LMC star clusters. The MF slopes derived from accurate observations using reliable cluster membership criteria are given in Table 2.

The slope given by Sagar *et al.* is based on homogeneous photoelectric UBV data and reliable

Table 1. Mass function slopes for five young LMC star clusters

Cluster	Age (Myr)	Mass range ( $M_{\odot}$ )	MF Slope (x)
NGC 1711	32	2.6 - 8.1	$1.3 \pm 0.3$
NGC 2004	16	2.6 - 11.9	$1.0 \pm 0.1$
NGC 2100	16	3.7 - 12.4	$0.8 \pm 0.3$
NGC 2164	63	1.9 - 5.7	$1.1 \pm 0.2$
NGC 2214	63	2.1 - 6.5	$1.1 \pm 0.3$

Table 2. Mass function slopes for young galactic open star clusters

Source	N	Mass range ( $M_{\odot}$ )	MF slope (x)
Piskunov <sup>16</sup>	61	1 - 25	1.3
Sagar <i>et al.</i> <sup>17</sup>	5	1 - 80	1.4
Kjeldsen and Frandsen <sup>18</sup>	13	1 - 6	1.3

(proper motion) cluster membership while that given by Kjeldsen and Frandsen is derived from CCD observations. The open star clusters studied by these authors are located in different regions of the Galactic disk while the star clusters listed in Table 1 are situated in different parts of the LMC (see Figure 6). An agreement between the MFs of these various stellar systems indicates that star formation in star clusters can be described with some universal IMF at least in our Galaxy and in the LMC. Considering the fact that the MCs have experienced a quite different evolutionary history, one may think that the similarity of the slopes of the IMF in the young LMC and Galactic star clusters supports the idea of a universal IMF.

Given the fact that only a few of the large number of MC star clusters have been used either for the study of the IMF or for testing the theoretical stellar evolutionary models, it is clear that we are at the very beginning in utilizing the vast potential offered by them.

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