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Coherence vs. decoherence in (some) problems of condensed matter physics

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Abstract. We present an 'overview' of coherence-to-decoherence transition in certain selected problems of condensed matter physics. Our treatment is based on a subsystem-plus-environment approach. All the examples chosen in this paper have one thing in common – the environmental degrees of freedom are taken to be bosonic and their spectral density of excitations is assumed to be 'ohmic'. The examples are drawn from a variety of phenomena in condensed matter physics involving, for instance, quantum diffusion of hydrogen in metals, Landau diamagnetism and *c*-axis transport in high T_c superconductors.

Keywords. Coherence; decoherence; high T_c superconductivity.

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1. Introduction

Environment-induced decoherence is an important issue in the topical subjects of mesoscopic phenomena and quantum information processes. We shall discuss this issue in the context of certain problems in condensed matter physics which we have worked on recently. At the outset, it is necessary to indicate what we mean by coherence and decoherence. For this, the following classical paradigm, borrowed from a recent article by Imry [1] suffices. Consider a beam of particle wave with momentum $\underline{k}_0$ incident from the left on two rigid, elastic, point scatterers denoted by crosses and located at x_1 and x_2 (figure 1). The beam is scattered with a wave vector $\underline{k}$ resulting in a momentum transfer $\underline{K} = \underline{k}_0 - \underline{k}$. The scattering intensity, assuming that the two amplitudes of scattering are the same, can then be written as

$$S_K = |A_K \left(e^{i\underline{K}.\underline{x}_1} + e^{i\underline{K}.\underline{x}_2} \right)|^2, \tag{1}$$

which has two parts:

$$S_K = S_K^{\mathrm{I}} + S_K^{\mathrm{II}}, \tag{2}$$

where

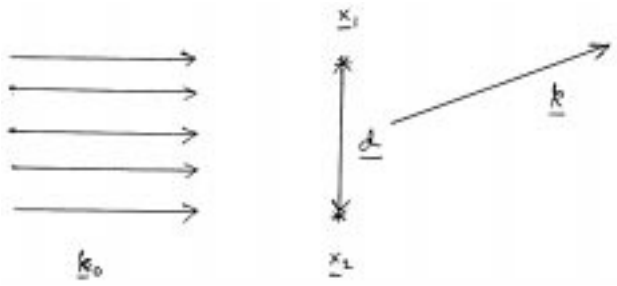

Figure 1. Elastic scattering from two rigid point scatterers.

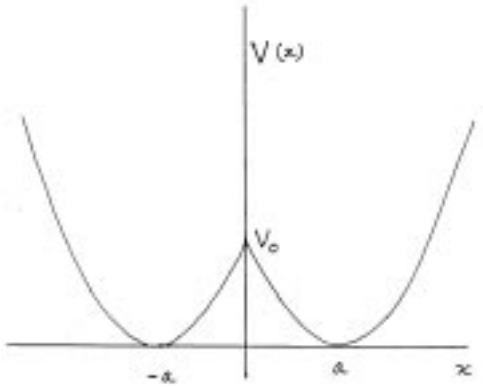

Figure 2. Symmetric double-well with minima at $\pm a$. The barrier height V_0 equals $1/2m\omega^2 a^2$, ω being the small oscillation frequency in each well.

$$S_K^{\mathrm{I}} = 2|A_K|^2 \tag{3}$$

and

$$S_K^{\mathrm{II}} = 2|A_K|^2 \cos(\underline{K}.\underline{d}), \tag{4}$$

$\underline{d}$ being $\underline{x}_1 - \underline{x}_2$. Note that the first contribution to the intensity, S_K^{I} would have resulted had we simply added the two probabilities (each being $|A_K|^2$ in this case) – this contribution may then be regarded as 'classical'. The second contribution: S_K^{II} directly relates to the wave aspect of the incident and the scattered beams and depends on what is called the 'phase shift', $(\underline{K}.\underline{d})$. It is this contribution which is responsible for interference in wave optics and quantum mechanics, and has its origin in *coherence*. Decoherence (or dephasing) then is the process of diminishing of the interference of the two waves. Decoherence occurs due to interaction of the interfering entity with the degrees of freedom of the environment, e.g., lattice vibrations, electromagnetic fields, other entities not measured in the experiment, and so on.

One of the simplest schemes for studying coherence vs. decoherence in quantum mechanics can be formulated in terms of a one-dimensional symmetric double-well model, illustrated schematically in figure 2, and defined by the potential

$$V(x) = \frac{1}{2}k(|x| - a)^2. \tag{5}$$

The two lowest eigenfunctions and the energy eigenvalues, as a function of a dimensionless distance (which is proportional to a), are shown in figures 3 and 4 [2]. From figure 3 it is clear that the symmetric $(\psi_0 + \psi_1)$ and the antisymmetric $(\psi_0 - \psi_1)$ combinations of the two lowest eigenfunctions yield two functions which are localized and peaked about the right minimum and the left minimum of the double-well, respectively. These may then be considered to be the 'physical' states corresponding to wave packets localized in

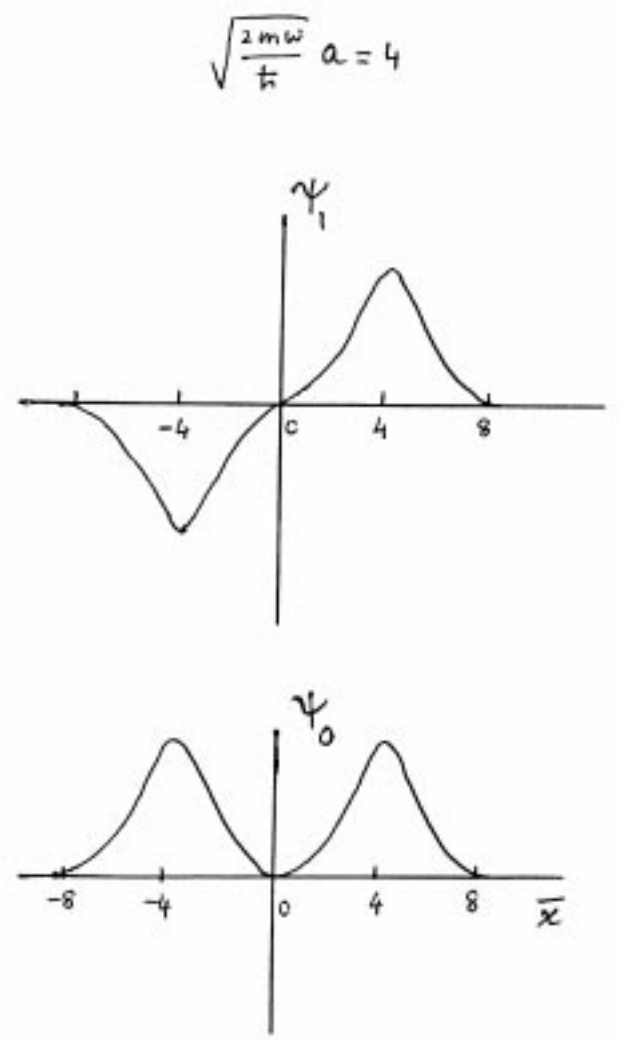

Figure 3. The ground state wave function ψ_0 and the first excited state wave function ψ_1, plotted against a dimensionless distance $\bar{x} = \sqrt{2m\omega/\hbar}x$, for $\sqrt{2m\omega/\hbar}a = 4$.

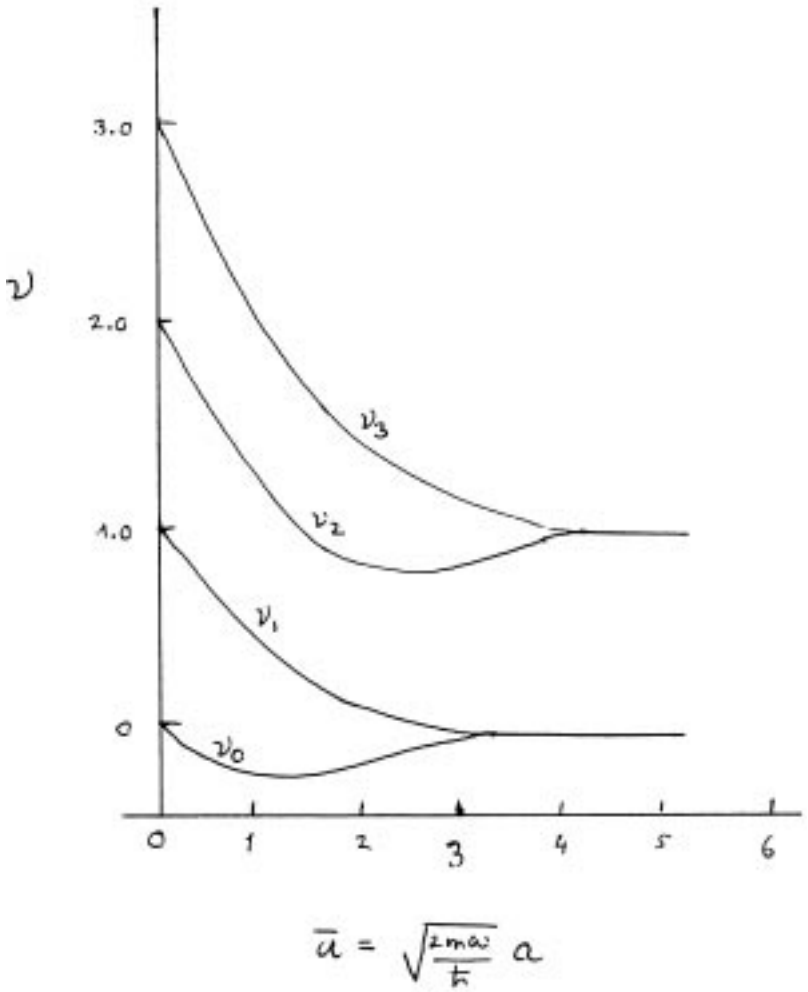

Figure 4. The quantum number $\nu = (E/\hbar\omega) - (1/2)$, E being the energy eigenvalue, is plotted against $\bar{a} = \sqrt{2m\omega/\hbar}a$. For $\bar{a} = 0$, we have the special case of a single harmonic oscillator with the origin as the minimum whereas for $\bar{a} \to \infty$ we have two disjoint harmonic oscillators with minima at $\pm\infty$. Thus for $\bar{a}$ 'large', the eigenvalues collapse pairwise and become doubly degenerate.

the right and the left wells. Furthermore, these are the only states to reckon with if the temperature is low enough such that the Boltzmann populations of the levels other than the first two energy levels can be ignored. The Hilbert space of the system is then shrunk into a two-state one for which the physical states can be represented by the pseudospin eigenstates of the Pauli operator σ_Z, $|+\rangle$ corresponds to the wave packet being localized in the right well, whereas $|-\rangle$ corresponds to the wave packet being localized in the left well. The system tunnels between the two states with a frequency which is the difference between the lowest energy eigenvalues and is given in the WKB approximation by

$$\Delta_0 = \sqrt{\frac{8\omega V_0}{\hbar\pi}} \exp\left(-\frac{2V_0}{\hbar\omega}\right). \tag{6}$$

We have earlier discussed the issue of coherence in a static picture, through eq. (4). Equally well, we can think of a time-dependent description of coherence in which the mean position of the wave packet is an oscillatory function of time:

$$\langle x(t)\rangle = X_0 \cos(\Delta_0 t). \tag{7}$$

On the other hand, when decoherence sets in, the mean position is expected to be exponentially damped:

$$\langle x(t)\rangle = X_0 \exp(-t/\tau), \tag{8}$$

wherein, however, the time constant τ may depend on the tunneling frequency Δ_0, the temperature T and the strength K of interaction with other degrees of freedom, as we shall analyze later.

The plan of this 'overview' is then as follows: In §2 we will discuss a physical realization of the two-state model in terms of the tunneling of an interstitial hydrogen in niobium in the presence of other trapping impurities, e.g. oxygen. Decoherence in this system is occasioned by the coupling of the tunneling hydrogen (or proton, for that matter) with metallic electrons. Coherence-to-decoherence transition, as the temperature is increased, will be illustrated in terms of the structure factor of neutron scattering from the tunneling centres. The model employed in §2 for describing the heat bath which induces 'quantum dissipation', and thereby decoherence, is a generic one that serves to exemplify similar decoherence phenomena in other very different systems. Thus, in §3, we will consider the quantum theory of diamagnetism and argue that the transition from the Landau to the Bohr–Van Leeuwen regime can indeed be viewed as a coherence to decoherence transition. The problem discussed in §2 relates to a subsystem (of the tunneling centre) which has a finite Hilbert space whereas the one in §3 has a subsystem (of the circulating charge in a magnetic field) that is characterized by an infinite-dimensional Hilbert space. An effective combination of the two paradigms in which the subsystem is a composite one, with both finite and infinite-dimensional character, is introduced in §4, which deals with the issue of *c*-axis transport in layered superconductors. Again, the observed suppression of *c*-axis tunneling is viewed in the light of a coherence-to-decoherence transition. Finally, in §5, we address the question: 'Can there be dissipationless decoherence?', and conclude the paper with a few remarks.

2. Quantum diffusion of H in $Nb(OH)_{0.002}$

The niobium metal has a bcc structure and becomes superconducting at low temperatures. However, we shall consider only those experiments in which superconductivity is annuled by an applied magnetic field and hence we will have to deal only with the normal state electrons. Furthermore, our focus of interest will be the dilute system of hydrogen interstitials which occupy tetrahedral sites, indicated by $a, b, \ldots, e$, and which can be trapped by an interstitial oxygen that can occupy an octahedral site (see figure 5). From a large body of experimental data involving X-ray and channeling measurements, it has been ascertained

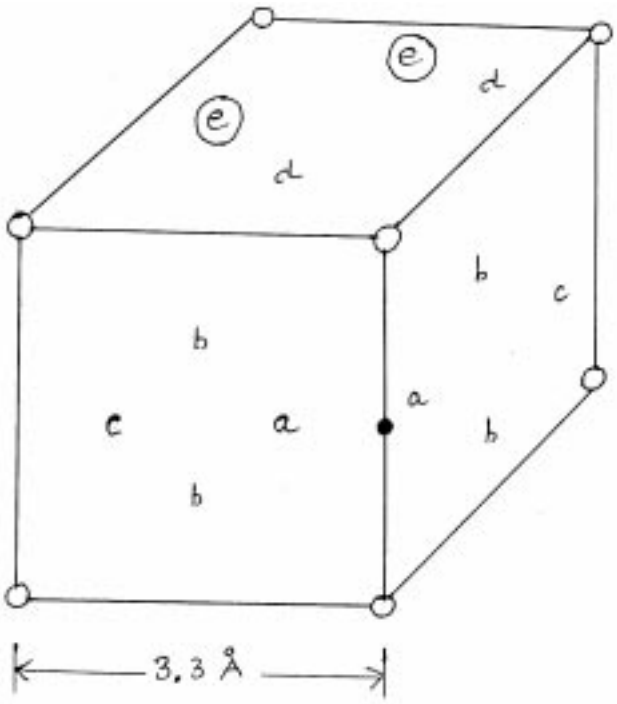

Figure 5. The bcc structure of Nb. The big open circles are Nb sites, the solid circle is the octahedral site of immobile oxygen whereas a, b, c, d, e are possible symmetry-allowed tetrahedral sites of the hydrogen. Because of the presence of oxygen, the hydrogen occupies one of the two sites marked as 'encircled e'.

that the hydrogen, which can be considered one at a time in view of the specified dilution, can occupy one of the two sites marked as encircled e in figure 5 [3]. We then have our text book example of a double-well in which the hydrogen, being a light quantum particle, tunnels back and forth, but unlike the one mentioned in §1, this double-well is asymmetric because of long-range elastic interactions between OH-pairs (figure 6). Of course, the asymmetry is random at different hydrogen sites and has to be averaged over in the final analysis.

At this stage it is appropriate to ask what are the relevant energy scales in the problem. It turns out that the tunneling energy Δ_0, the asymmetry energy $\hbar\varepsilon$ and the thermal energy $K_{\mathrm{B}}T$ (at temperatures of experimental interest) are all of the same order: a few meV. On the other hand, the harmonic oscillator like energy separation (in each well), the Debye energy (associated with phonons), and the Fermi energy are all at least two orders of magnitude higher. Therefore, the two-state abstraction of the double-well, in which the subsystem-Hamiltonian can be described in terms of Pauli operators alone, as discussed in §1, is indeed a valid model for the tunneling centre.

As has been mentioned already, our treatment will be focused on neutron scattering measurements. It is well-known that neutrons have a large scattering cross section with hydrogen atoms resulting in a structure factor which is the Fourier transform of a Van Hove correlation function:

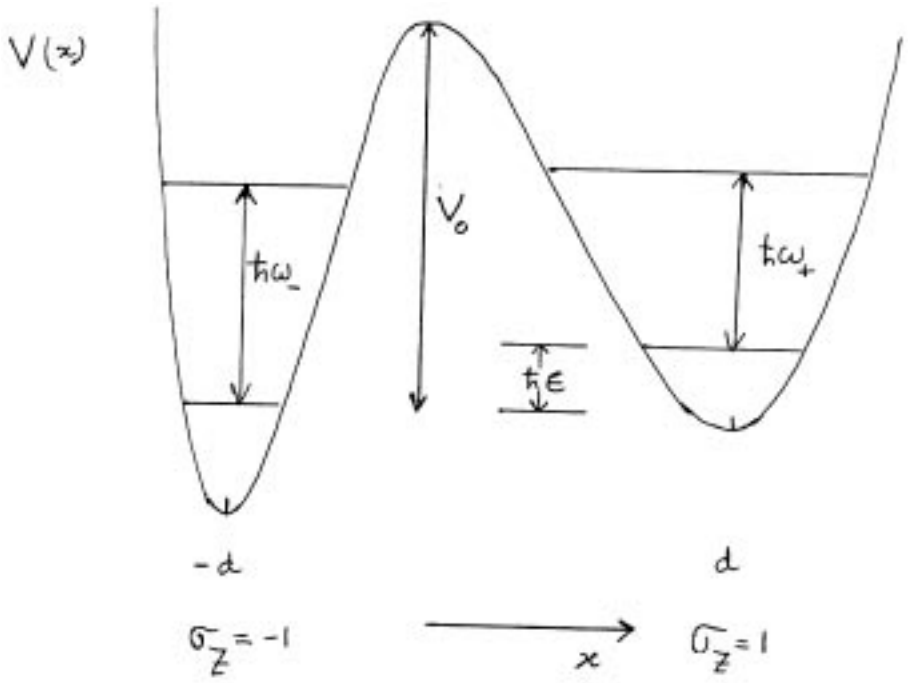

Figure 6. The two-level abstraction of the asymmetric double-well in which the hydrogen moves in the presence of the trapping impurity, oxygen. The energy associated with small oscillation frequencies $\omega_{\mp}$ is much larger than the asymmetry energy $\hbar\varepsilon$, the tunneling energy $\hbar\Delta_0$ and the thermal energy $K_B T$. In this limit the left and the right minima of the double well are mapped into the pseudospin states for $\sigma_Z = -1$ and $\sigma_Z = 1$, respectively.

$$S(\underline{K}, \omega) = \int_{-\infty}^{\infty} \mathrm{d}t \mathrm{e}^{iwt} \langle \mathrm{e}^{-i\underline{K}.\underline{r}(0)} \mathrm{e}^{i\underline{K}.\underline{r}(t)} \rangle. \tag{9}$$

In eq. (9), $\underline{K}$ is the wave vector transfer in the scattering process (as in figure 1), ω is the corresponding frequency transfer, and $\underline{r}$ at times 0 and t are the positions of the scatterer, in this case the hydrogen. The point to note is that $\underline{r}(t)$, which is obtained from the Heisenberg evolution of $\underline{r}(0)$, is a quantum operator that *does not* commute with $\underline{r}(0)$. Finally, the angular brackets $\langle \ldots \rangle$ imply a quantum statistical average over the entire system of interest, maintained at a fixed temperature T.

What is then the system of interest? The answer is, it is the subsystem comprising of the tunneling centre plus the environment with which it is coupled. Now, as the hydrogen (which is normally stripped off its electron and therefore behaves as a proton) moves, it is dragged by the cloud of conduction electrons of niobium. Therefore, the environment is a heat bath of conduction electrons that causes 'quantum friction'. Since the thermal energy is much smaller than the Fermi energy the relevant excitations are those of electron-hole pairs of the Fermi surface and can be represented by bosons. Motivated by these considerations the system of interest may be described in terms of what Leggett *et al* term as the spin-boson Hamiltonian [4]:

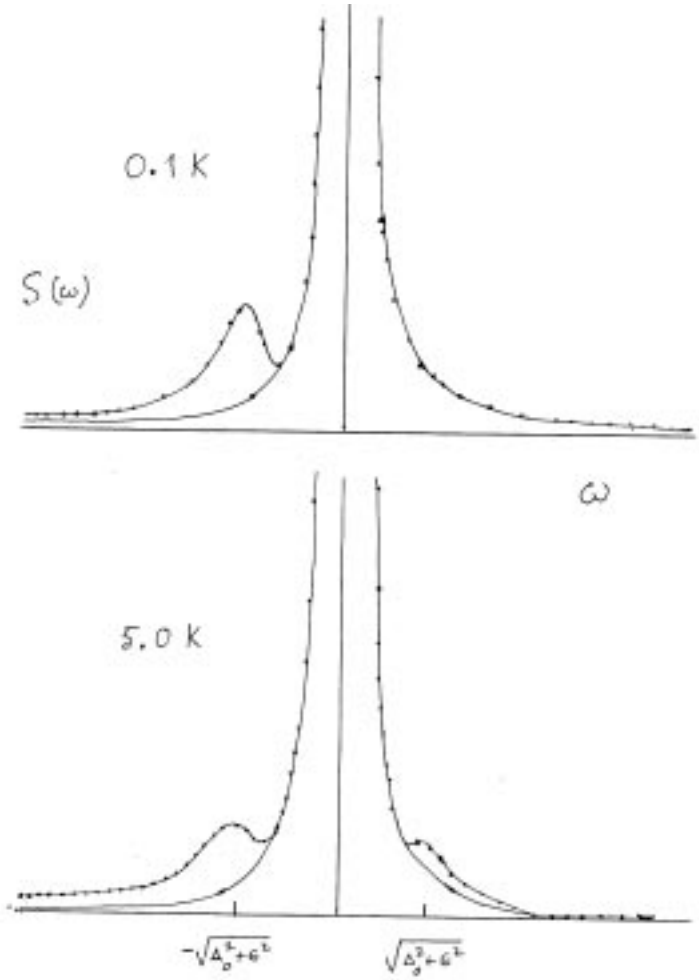

Figure 7. The inelastic neutron scattering structure factor $S(\omega)$, for fixed $\underline{K}$, as a function of ω, for two different temperatures 0.1 K and 5 K, when the sample of $Nb(OH)_{0.002}$ is put in an external magnetic field of 0.7 tesla.

$$\mathscr{H} = \mathscr{H}_s + \sum_k G_k(b_k + b_k^+)\sigma_Z + \sum_k \hbar\omega_k b_k^+ b_k. \tag{10}$$

In eq. (10), $\mathscr{H}_s$ is the subsystem Hamiltonian given by

$$\mathscr{H}_s = \frac{\hbar}{2}(-\Delta_0\sigma_X + \varepsilon\sigma_Z). \tag{11}$$

As the details of the calculations are already available in the literature [5,6], we will simply discuss the main results in relation to the measured neutron scattering data [7]. What are shown in figures 7a and 7b are the structure factor as a function of the neutron energy

gain ω (in meV) for fixed $\underline{K}$, when the sample is placed in a magnetic field of 0.7 tesla. The data in figure 7a, taken at 0.1 K, clearly indicate an 'inelastic' peak located at energy $-\hbar\sqrt{\Delta_0^2+\varepsilon^2}$, which is the consequence of the *coherent* tunneling motion of the hydrogen, picked up by the scattered neutron. The other inelastic peak on the right shoulder, located at energy $\hbar\sqrt{\Delta_0^2+\varepsilon^2}$ is suppressed because of the detailed balance factor, but shows its presence at a somewhat higher temperature of 5 K (figure 7b). These two figures 7a and 7b are then the signatures of coherence, as far as neutron scattering is concerned. Now, as the temperature increases, there is more phase space available for the tunneling centre to interact with the excitations of the heat bath. The coherent clock-like motion of the hydrogen is then impeded and decoherence sets in. As a result, neutron scattering changes over to the 'quasi-elastic' regime from the inelastic one, yielding a central line (figure 8) [7]. The latter, for a fixed $\underline{K}$, can be fitted to a sum of Lorentzians:

$$S(\omega) = \sum_j \frac{a_j \Gamma_j}{\omega^2 + \Gamma_j^2}, \tag{12}$$

where a_j's are constant coefficients and Γ_j's are widths related to the jump rate of the hydrogen. We should emphasize, however, that these jumps are associated with what may be called the *quantum* diffusion of the hydrogen.

One of the intriguing results is the plot of the jump rate of the hydrogen as a function of temperature, derived from figure 8 and shown in figure 9. It is seen that in the temperature range of about 10–60 K the jump rate *decreases* with temperature, following a power law: T^{2K-1}, where K is a measure of the strength of the coupling with the bath. This power law is a direct consequence of the fact that the heat bath in the present instance is composed of fermions and that the electron-hole excitations can be characterized by a spectral density which is ohmic [8]. The upturn in the curve at temperatures beyond 60 K is the result of phonon-assisted quantum diffusion of the hydrogen which, naturally, would be accompanied by non-ohmic dissipation [9].

3. Landau diamagnetism in a dissipative environment

We turn our attention in this section to an enigmatic problem in solid state physics that has prompted Peierls to dubb it as 'one of the surprises of theoretical physics' [10]. The vexed issue is that of orbital diamagnetism of an ideal gas of electrons in a box under the influence of an applied magnetic field. It is well known through the work of Bohr and Van Leeuwen that if one calculates the diamagnetic moment using the tool of *classical* statistical mechanics the answer one gets is zero! The textbook proof of what is called the Bohr–Van Leeuwen theorem is somewhat facile in that the presence of the magnetic field is shown to be 'gauged away' from the partition function of the system – hence its logarithmic derivative which yields the average moment is identically zero [11]. Only later it was clarified by Van Vleck, amongst others, that the reason for the vanishing of the orbital moment is a more subtle one – the contribution from the circulating electrons in the bulk is exactly canceled out by the contribution coming from the so-called 'skipping orbits' due to the electrons bumping onto the walls of the enclosure and getting reflected back. Thus, the 'surprise' element is that the boundary of the enclosure which holds the

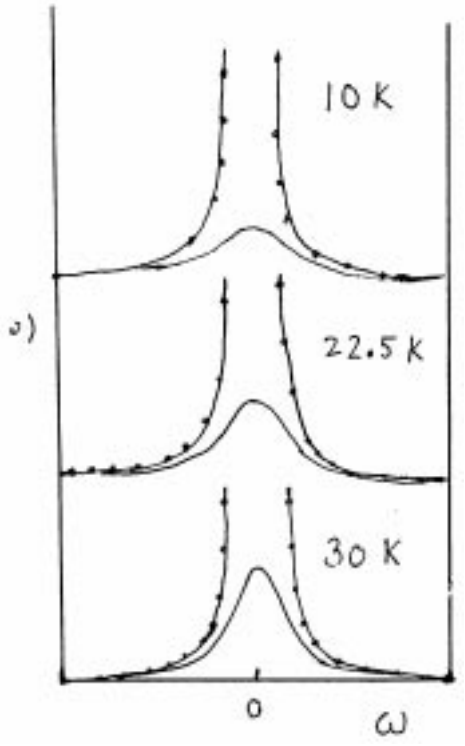

Figure 8. The quasielastic scattering at temperatures higher than those depicted in figure 7.

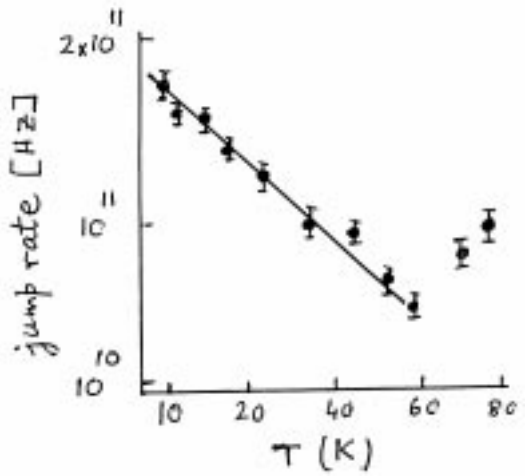

Figure 9. The jump rate of hydrogen in the temperature range of 10–60 K, deduced from the width of figure 8.

electrons comes into essential reckoning, even in the thermodynamic limit. It was Landau who showed that diamagnetism could be explained only by invoking the rules of quantum mechanics. Because the position and the momentum of the electron do not commute the bulk contribution is different from the surface one, yielding a nonvanishing diamagnetic moment.

Diamagnetism has its origin in coherent circular motion of the electron in a plane normal to the magnetic field such that the position vector follows an equation similar to eq. (7), Δ_0 being a measure of the cyclotron frequency. We may then ask the question: what would happen to the Landau diamagnetism if the coherent precessional motion of the position vector of the electron is disturbed due to interaction with other degrees of freedom, e.g., defects, other electrons, phonons, etc? This question can again be addressed within a system-plus-bath approach, already introduced in §2, in which the total Hamiltonian for the system at hand can be written as

$$\mathscr{H} = \mathscr{H}_s + \sum_j \left[\frac{1}{2m_j} \underline{p}_j^2 + \frac{1}{2} m_j \omega_j^2 (\underline{q}_j - \underline{r})^2 \right], \tag{13}$$

where the subsystem Hamiltonian $\mathscr{H}_s$ is given by

$$\mathscr{H}_s = \frac{1}{2m} \left(\underline{p} - \frac{e\underline{A}}{c} \right)^2. \tag{14}$$

In eqs (13) and (14), m is the mass of the electron of charge e, $\underline{p}$ its canonical momentum, $\underline{r}$ its position vector and $\underline{A}$ is the vector potential associated with the magnetic field $\underline{B}$.

The strategy is to integrate the bath variables $\underline{p}_j(t)$ and $\underline{q}_j(t)$ in terms of their initial values, say at time $t = 0$, employ quantum statistical expressions for their fluctuations at a fixed temperature T and thereby derive a *quantum* Langevin equation for the subsystem variables $\underline{r}$ and $\underline{p}$ [12]. In this description one has to consider the case in which the number of bath oscillators is infinitely large and their spectrum is continuous. Introducing then the spectral density as

$$J(\omega) = \frac{\hbar}{2} \sum_j m_j \omega_j^3 \delta(\omega - \omega_j), \tag{15}$$

ohmic dissipation, considered already in §2, can be modeled as

$$J(\omega) = \gamma\omega, \tag{16}$$

γ being a 'friction' coefficient, proportional to the parameter K of the previous section.

From the underlying Langevin equation the time-dependent magnetic moment can be computed using the formula:

$$M_Z(t) = \frac{|e|}{4c} \mathrm{Im} \langle (\dot{\xi}\xi^\dagger + \xi^\dagger \dot{\xi}) \rangle, \tag{17}$$

where ξ is the complex variable $x + iy$, x and y being the planar coordinates of the electron circulating around the $\underline{B}$ field along the Z-axis, dot denotes time derivative, dagger implies hermitian adjoint and the angular brackets indicate a statistical average over the 'noise' in

the Langevin equation, itself a quantum operator. The results are given in [13] but here we simply quote the asymptotic ($t \to \infty$) form, in the case of ohmic dissipation, though the non-ohmic case can be easily tackled [14]. We find

$$M = \lim_{t\to\infty} M_Z(t) = -\frac{2K_B T}{B} w_c^2 \sum_{j=1}^{\infty} \frac{1}{(\nu_j + \gamma)^2 + \omega_c^2}, \tag{18}$$

where w_c ($= eB/mc$) is the cyclotron frequency and ν_j is defined as

$$\nu_j = \frac{2\pi}{\hbar} K_B T\, j. \tag{19}$$

As is to be expected, when the coupling between the subsystem and the environment is switched off the friction coefficient γ goes to zero and we recover the Landau expression:

$$\lim_{\gamma\to 0} M = \frac{e\hbar}{2mc}\left[\frac{2K_B T}{\hbar\omega_c} - \coth\left(\frac{\hbar\omega_c}{2K_B T}\right)\right]. \tag{20}$$

The result given in eq. (18) is a remarkable one in that an equilibrium property like the diamagnetic moment has an explicit dependence on a rate constant such as the friction coefficient γ. This suggests in view of the exactness of the result that the solution presented above, as it were, is the outcome of a full many-body treatment of an electron in a magnetic field, in interaction with bosonic excitations. The result in eq. (18) is also reminiscent of another familiar expression in solid state physics, viz., the Drude formula for the electrical conductivity which also, in the steady-state, depends on the damping parameter γ. Indeed this observation has motivated us to re-express eq. (18) as a 'Landau–Drude' formula [15]:

$$-\frac{mc}{e\hbar} M = \frac{1}{\nu} \sum_{j=1}^{\infty} \frac{1}{1 + (\bar{\mu}_j + \bar{r})^2}, \tag{21}$$

where

$$\nu = \frac{\hbar w_c}{2K_B T}, \qquad \bar{\mu}_j = \frac{\pi j}{\nu}, \tag{22}$$

and $\bar{r}$ is a scaled resistance which is the ratio of the Drude resistivity r to the Hall resistivity R:

$$\bar{r} = \frac{r}{R}, \quad r = \frac{m\gamma}{ne^2}, \quad R = \frac{B}{nec}, \tag{23}$$

n being the number density of electrons. In figure 10, we show a plot of the left-hand side of eq. (21) vs. $\bar{r}$, for two different values of ν. As resistance is the most basic manifestation of dissipation it is evident that larger values of $\bar{r}$ lead to decoherence. Further, larger the cyclotron frequency ν, the more perceptible is the persistence of coherence. Figure 10 is then a nice illustration of how the system transits from the coherent 'Landau regime' to the decoherent 'Van Leeuwen regime', at least asymptotically.

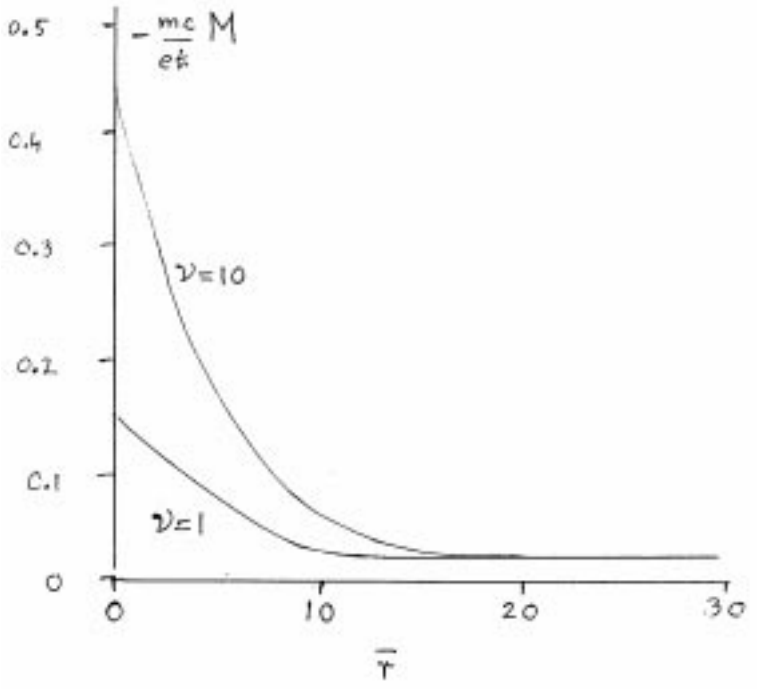

Figure 10. The negative of the dimensionless magnetization $-(mc/e\hbar)M$ is plotted against the scaled resistance $\bar{r}$, for two different values of the scaled cyclotron frequency ν. For $\bar{r}=0$, we recover the Landau limit.

4. 'Zeno blocking' of *c*-axis transport in YBCO

The high-T_c superconductors like $YBa_2Cu_3O_{6+x}$, in short YBCO, are characterized by anisotropy, in the direction normal to the CuO planes which are stacked up as layers. One of the manifestations of this anisotropy is the measured value of the resistivity $\rho(T)$ as a function of temperature. Both the in-plane resistivity ρ_{ab} and the *c*-axis resistivity ρ_c (*c* being normal to the *ab*-plane of CuO), over a significantly large temperature range and concentration values *x*, have the following temperature dependence in the *normal* phase [16]:

$$\rho = \frac{a}{T} + BT. \tag{24}$$

What is significant as far as the linear temperature-dependence is concerned is the value of *B*:

$$B_{ab} = 1.4 \times 10^{-6}, \quad B_c = 3 \times 10^{-5}. \tag{25}$$

Thus, transport along the *c*-axis is an order of magnitude 'slower' though the mechanism of transport, as evidenced by the identical temperature-dependence, must be the same for both the intra-layer and the inter-layer electrons.

Following the discussion in §2, the two planes involved in the tunneling process are mapped into the two states of a pesudo-spin Pauli matrix T_Z, whereas the tunneling between the two states is thought to be mediated by the operator T_X which is completely off-diagonal in the representation of T_Z. The normal state electron is a free one but undergoes 'quantum Brownian motion' due to interaction with bosonic degrees of freedom of the bath, as envisaged in §2 and §3. The full Hamiltonian may be written as

$$\mathscr{H} = \frac{\hat{p}^2}{2m} + \hat{x}\left[\left(\frac{1}{2}+T_Z\right)\sum_q g_q(a_q+a_q^+) + \left(\frac{1}{2}-T_Z\right)\sum_q G_q(b_q+b_q^+)\right] \\ -\hbar\delta T_X + \sum_q \hbar\left(\omega_q a_q^+ a_q + \Omega_q b_q^+ b_q\right). \tag{26}$$

Again, we skip the detailed derivations which have been published recently [17]. Instead the principal result is sketched in figure 11 in which we plot the 'stay-put' probability $P(t)\left(=\frac{1}{2}+\langle T^Z(t)\rangle\right)$ vs. a scaled time δt, defined by the tunneling frequency δ. Choosing the initial condition: $P(0)=1$, $P(t)$ is a measure of the 'leakage' of the electron from a given plane. The oscillations shown in figure 11 for moderately low values of the friction coefficient γ are signatures of quantum coherence. The transition to decoherence occurs for values of $\gamma = 0.5$ and above. Since resistive transport is in fact an incoherent process we propose that the exponential relaxation of $P(t)$ can be fitted to

$$P(t) = \exp\left(-\hat{\delta}^2\tau t\right), \tag{27}$$

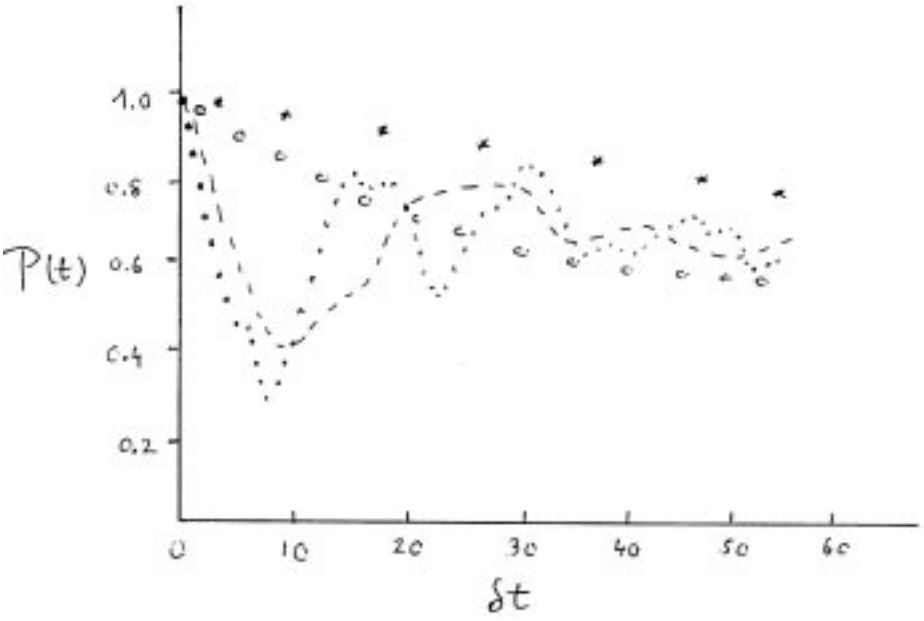

Figure 11. The stay-put probability $P(t)$ vs. the scaled time δt for four different values of the damping parameter γ; $\gamma = 0.1$: dots, $\gamma = 0.25$: dashes, $\gamma = 0.5$: circles and $\gamma = 0.75$: stars.

where $\tilde{\delta}$ is a renormalized tunneling frequency and τ is the intra-planar inelastic scattering time proportional to γ^{-1}. If d is the inter-planar distance, eq. (27) allows us to define a mobility

$$\mu = d\tilde{\delta}^2\tau, \tag{28}$$

which is then inversely proportional to ρ_c. On the other hand, it is the same τ which determines the in-plane resistivity through the Drude formula:

$$\rho_{ab} = \frac{m^*_{ab}}{ne^2\tau}, \tag{29}$$

m^*_{ab} being the effective mass and n is the number density of electrons. Thus, our proposal is that it is the parameter τ which dictates the temperature dependence of both ρ_c and ρ_{ab}.

5. Dissipationless decoherence

In all the examples discussed in §§ 2–4, the interaction Hamiltonian is chosen to be off-diagonal in the representation in which the subsystem and the heat bath Hamiltonians are simultaneously diagonal. The point is, it is the interaction Hamiltonian which causes transitions between the energy levels of both the subsystem and the heat bath. As a result there is continuous energy transfer between the subsystem and the heat bath though the latter, being a very large system, has its thermal equilibrium undisturbed. In the language of scattering theory we are then led to consider only inelastic scattering processes mediated by the coupling between the subsystem and the heat bath. As a variant to this approach we now ask: what if the interaction Hamiltonian continues to be off-diagonal in the heat bath representation but is chosen to be diagonal in the representation of the subsystem? A concrete example can be constructed in which the interaction Hamiltonian is taken to be proportional to the subsystem Hamiltonian $\mathscr{H}_{\rm s}$ itself! Thus we write

$$\mathscr{H} = \mathscr{H}_{\rm s} + \mathscr{H}_{\rm s}\sum_k g_k\left(b_k + b_k^+\right) + \sum_k \hbar\omega_k b_k^+ b_k. \tag{30}$$

Defining a 'reduced' density operator $\rho_{\rm s}$ for the subsystem from the total density operator ρ as

$$\rho_{\rm s} = Tr_{\rm b}(\rho), \tag{31}$$

where $\mathrm{Tr}_{\rm b}(\ldots)$ denotes trace over the bath degrees of freedom, we can derive a master equation [18]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{\rm s} = -i\left[\mathscr{H}_{\rm s}, \rho_{\rm s}\right] + i\eta(t)\left[\mathscr{H}_{\rm s}^2, \rho_{\rm s}\right] - \gamma(t)\left(\mathscr{H}_{\rm s}^2\rho_{\rm s} - 2\mathscr{H}_{\rm s}\rho_{\rm s}\mathscr{H}_{\rm s} + \rho_{\rm s}\mathscr{H}_{\rm s}^2\right). \tag{32}$$

In eq. (32), $\eta(t)$ and $\gamma(t)$ are damping parameters arising from the heat bath.

As the subsystem Hamiltonian $\mathscr{H}_{\rm s}$ commutes with the total Hamiltonian $\mathscr{H}$ there is no energy dissipation in the subsystem. But, is there any decoherence? Clearly, this question can be meaningfully posed only if the initial state of the subsystem is prepared in

a *non*-eigenstate. In this restrictive sense, coherence-to-decoherence transition has been discussed in a couple of model examples of eq. (30) [18]. The discussion also corroborates an observation made earlier that the crossover from coherent to incoherent dynamics in damped quantum systems depends in general on the specific dynamical quantity under consideration and the initial preparation [19].

The process of decoherence that we have analyzed here is also known as 'dephasing', especially in the context of the contemporarily interesting topic of mesoscopic physics [20]. Whether dephasing can occur when the temperature $T \to 0$ – which is possible to be studied within the presently discussed models because of the quantum nature of the heat bath – has generated a great deal of controversy in recent times. It is generally believed that purely elastic scattering does not cause decoherence at zero temperature. Yet, in the model governed by eq. (30), there are decoherence effects in certain dynamical quantities perhaps because of the presence of energy non-conserving terms involving heat bath variables, even though the subsystem remains dissipationless! The destruction of quantum coherence by environmental influence is therefore still a hotly debated issue and it is hoped that the present overview will be an useful contribution to this debate.

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