EFFECT OF NON-UNIFORM TEMPERATURE GRADIENT ON MAGNETO-CONVECTION DRIVEN BY SURFACE-TENSION AND BUOYANCY

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Effect of Non-uniform Temperature Gradient on Magneto-Convection Driven by Surface-tension and Buoyancy

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Abstract

The effect of non-uniform temperature gradient and a uniform magnetic field on the onset of magneto-convection driven by surface tension and buoyancy force in a horizonaal layer of Boussinesq fluid is studied by means of linear stability analysis. The upper boundary is assumed to be free and adiabatic and the lower boundary is rigid and adiabatic. A Galerkin method is used to obtain the eigenvalues. A mechanism for suppressing or augmenting convection is discussed in detail. It is found that as the magnetic field increases the coupling between the two agencies causing instability becomes weaker even in the presence of non-uniform temperature gradient and a discontinuous change in cell size occurs at a certain value of the Rayleigh number as a result of sudden change over from convection dominated by one of the two agencies to that dominated by the other. The results obtained here are compared with the existing ones and found that even a single term Galerkin expansion gives reasonable results with minimum of mathematics.

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1. INTRODUCTION

The mechanism of controlling convection in a fluid, generated either by buoyancy force or by the variation in suface tension with temperature or by both, has recently assumed importance in material processing in space bacause of its application to the possibility of producing various new materials. The range of possibilities extends from producing large crystals of uniform properties to manufacturing materials with unique properties. The Lorentz force due to electromagnetic field, coriolis force dur to rotation and non-uniform temperature gradient due to the transient heating or cooling at the boundaries, which are ineffective at the terrestrial environment, become effective in the mirogravity environment. Such forces may be used to suppress or augment the convection (see Rudraiah and Chandna [1]).

The effect of non-uniform temperature gradient on the convection driven by surface-tension alone in the presence of rotation has been investigated by Rudraiah [2] and Friedrich and Rudraiah [3]. They have shown that suitable non-uniform temperature gradient arising due to sudden heating or cooling at the boundaries and the coriolis force suppress Marangoni convection. Recently Rudraiah, Rumachandramurthy and Chandna [4] have shown that the external constraint of magnetic field suppresses Marangoni convection driven by surface-tension. For material processing in the laboratory the convection induced by buoyancy force in addition to surface-tension is important. It may also be useful in the design of liquid containing systems, such as the coolant circulation systems and the heat pipes. Nield [5] has examined the effect of magnetic field on convection induced by buoyancy and surface-tension under the assumption of uniform temperature gradient. However, the non-uniform temperature gradient caused by rapid heating (see Sutton [6]) and a suitable strength of magnetic field may be important in inhibiting or augmenting convection in the applications cited above.

The object of this paper is to show that a suitable non-uniform temperature gradient and a transverse magnetic field suppress convection driven by buoyancy and surface-tension by considering infinitesimal perturbations. The non-uniform temperature gradient caused by transient heating or cooling at the boundaries is a function of position and time. In the present analysis, we introduce a simplification in the form of a quasi-static approximation (see Currie [7]) which consists of freezing the temperature distribution at a given instant of time. This hypothesis is sufficient as we are interested only in finding the conditions for the marginal stability. It is known that, depending upon the strength of the magnetic field (Chandrasekhar [8]) and the ratio of diffusivities (Rudraiah and Shivakumar [9]) both marginal and overstable convections are possible. In the present paper, however, we consider only the marginal state. In the absence of magnetic field, Lebon and Cloot [10] have considered the effect of non-uniform temperature gradient on convection using a quasi-variational technique with the help of Rayleigh-Ritz's method. This method requires elaborate numerical computations. Our aim here is to find the eigenvalues analytically by a method which simplifies the numerical computations considerably while retaining the essential feature of a non-uniform temperature gradient. For this purpose the Galerkin method (see Finlayson [11]) appears to be particularly well suited. Comparison of our results with those of Lebon and Cloot in the absence of Magnetic field and those of Nield [5] for uniform temperature gradient in the presence of magnetic field reveals that a single-term Galerkin expansion procedure used here gives reasonable results with minimum of mathematics.

2. FORMULATION OF THE PROBLEM

We consider the infinite horizontal layer of an electrically conducting Boussinesq fluid of depth d permeated by a uniform vertical magnetic field. It is bounded below by a rigid electrically and thermally perfect conducting wall and bounded above by a free surface. This free surface is adjacent to an electrically non-conducting medium and subject to a constant heat flux (ie. adiabatic). We assume a temperature drop ΔT acting between the upper and lower boundaries. The interface has a surface-tension σ which, following Pearson [12], can be assumed to vary linearly with temperature according to the formulae

$$\sigma = \sigma_0 - \sigma_1 \Delta T \ . \tag{1}$$

The applied uniform magnetic field H_0 is in the vertical direction. Using the cartesian coordinate system (x,y,z) with the origin in the lower boundary, Ox and Oy in it while Oz normal to it, the governing equations are

$$\nabla \cdot \vec{q} = \nabla \cdot \vec{H} = 0 \tag{2}$$

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} = -\frac{1}{\rho_0} \nabla\left(p + \frac{1}{2}\mu H^2\right) + \nu \nabla^2 \vec{q} - \frac{\rho}{\rho_0} g\hat{k} + \frac{\mu}{\rho_0} (\vec{H} \cdot \nabla)\vec{H} \tag{3}$$

$$\frac{\partial T}{\partial t} - (\vec{q} \cdot \nabla) T = k \nabla^2 T \tag{4}$$

$$\frac{\partial \vec{H}}{\partial t} = \nabla \times (\vec{q} \times \vec{H}) + \nu_m \nabla^2 \vec{H} \tag{5}$$

$$\rho = \rho_0 [1 - \alpha(T - T_0)] \tag{6}$$

where $\vec{q}$ is the velocity, ρ is the density, ρ_0 is the density at reference temperature T_0, p is the pressure, ν is the kinematic viscosity, $\vec{H}$ is the magnetic field, μ is the magnetic permeability, $\nu_m = \frac{1}{\mu\delta}$ is the magnetic viscosity, δ is the electrical conductivity, T is the temperature, g is the acceleration due to gravity, k is the thermal diffusivity and α is the coefficient of thermal expansion. In the quiescent state

$$\vec{q} = 0 \ , \quad \vec{H} = H_0\hat{k} \quad \text{and} \quad f(z) = -\frac{d}{\Delta T} \cdot \frac{dT_0}{dr}$$

where $\hat{k}$ is the unit vector in the z-direction and $f(z)$ is a non-dimensional temperature gradient satisfying

$$\int_0^1 f(z)\,dz = 1 \ .$$

Suppose that the initial state is slightly disturbed. The linearized equations of motion allow the solution for the perturbed field quantities in the form

(a function of z) . $\mathrm{Exp}\{i(\ell x+my) + \omega t\}$

where ℓ and m are the horizontal wave numbers, and ω is the growth rate. Using it in the linearized version of the equations (2)-(5) and eliminating the x and y components of the velocity and the induced magnetic field and making the resulting equations dimensionless by the introduction of d, $\frac{d^2}{\nu}$, $\frac{k}{d}$, H_0 and ΔT as the units of length, time, velocity, magnetic field and temperature scales respectively, we obtain

$$\omega(D^2-a^2)W = -Ra^2T + (D^2-a^2)W + \frac{\mu H_0^2 d^2}{\rho_0 \nu k}\frac{\partial}{\partial z}(D^2-a^2)H \qquad (7)$$

$$P_m \omega H = (D^2-a^2)H + \frac{k}{\nu_m} DW \tag{8}$$

$$P_r \omega T - (D^2-a^2)T = f(z)W \tag{9}$$

where $D \equiv \frac{d}{dz}$, $a^2 = \ell^2 + m^2$, $P_r = \frac{\nu}{k}$ is the Prandtl number, $P_m = \frac{\nu}{\nu_m}$ is the magnetic Prandtl number, $R = \frac{\alpha g \Delta T d^3}{\nu k}$ is the Rayleigh number, and $W(z)$, $H(z)$ and $T(z)$ being the amplitudes of the z-components of the velocity, magnetic field and of the temperature distribution respectively. The marginal state, $\omega = 0$, is assumed to be valid for particular values of P_r and a magnetic parameter. In that case equations (7)-(9) are considerably simplified. From these equations eliminating H , we obtain equations in W and T in the form

$$(D^2-a^2)^2 W = a^2 RT - QD^2 W \tag{10}$$

$$(D^2-a^2)T = -f(z)W \tag{11}$$

where $Q = \frac{\mu H_0^2 d^2}{\rho_0 \nu \nu_m}$, is the Chandrasekhar number.

In seeking the solution of these equations, we must specify certain boundary conditions on velocity and temperature. We note that the boundary conditions on the magnetic field do not affect the stability condition. We assume that the lower boundary is rigid and isothermal so that

$$W = DW = 0 \ , \quad T = 0 \quad \text{at} \quad z = 0 \ . \tag{12}$$

The upper boundary is assumed to be free and adiabatic, and Pearson [12] conditions are assumed to be valid here. This is true only when the deflection of the free surface by the transverse magnetic field is negligibly small. Under this assumption the required boundary conditions are

$$W = D^2W + a^2MT = DT = 0 \quad \text{at} \quad z = 1 \,, \tag{13}$$

where $M = \dfrac{\sigma_T \Delta T d}{\rho_0 \nu k}$ is the Marangoni number and $\sigma_T = \left(\dfrac{\partial \sigma}{\partial T}\right)_{T=T_0}$,

σ being the surface-tension.

3. CONDITION FOR THE ONSET OF CONVECTION

Multiplication of equation (10) by W , of equation (11) by T , integration of the resulting equations with respect to z from 0 to 1 , using the boundary conditions(12) and (13) and writing $W = AW_1$, $T = BT_1$ where A and B are constants and W_1 and T_1 are the trial functions, finally yeilds the following eigenvalue equation

$$M = -\frac{\left[<(D^2W)^2 + (2a^2+Q)(DW)^2 + a^4W^2> - \dfrac{a^2R<WT><f(z)WT>}{<(DT)^2 - a^2T^2>}\right]<(DT)^2 + a^2T^2>}{a^2DW(1) \;.\; T(1)<f(z)WT>} \tag{14}$$

where the angle bracket $<...>$ denotes the integration with respect to z from 0 to 1 .

We select the trail functions

$$W = z^2(1-z^2) \;, \qquad T = z(1-\frac{z}{2}) \tag{15}$$

which satisfy all the boundary conditions except the one given by

$$D^2W + a^2MT = 0 \quad \text{at} \quad z = 1 \,, \tag{16}$$

but the residual from this is included in a residual from the differential equations. Substituting equation (15) into equation (14), we get

$$M = \frac{\{8x^2 + 264x + (5292+132Q)\}(5+2x)}{<4725x<f(z)WT>} - R\frac{23}{420}$$

where $x = a^2$.

For any given $f(z)$, M attains its minimum value at $x_c = a_c^2$, x_c being the root of the cubic equation

$$x^3 + 17.75x^2 - (826.875 + 20.625Q) = 0 \,. \tag{17}$$

The variation of x_c with Q is computed for different vlaues of Q and the results are tabulated in the Table 1. From this it is clear that, the critical wave number increases with increasing Q and hence the effect of magnetic field is to contract the convection cells. When the layer of conducting fluid is heated from below, the non-uniform temperature gradient $f(z)$ is not only non-negative but also decreases monotonically. We are interested in the temperature profile which gives the maximum M_c subject to $f(z) \geq 0$. Fro this purpose, we consider different temperature profiles and obtain the condition for the onset of convection using the single term Galerkin expansion. To test the validity of this expansion we first apply this procedure for the uniform temperature gradient because in this case exact solutions are available.

3.1 Linear basic temperature distribution

For uniform temperature gradient $f(z) = 1$, let us denote the Marangoni number by M_1 so that equation (16) takes the form

$$M_1 = \frac{(8x^2+264x+5292+132Q)(5+2x)}{\frac{4725\times 23}{420}\,x} - R\frac{23}{420} \tag{18}$$

M_1 attains its minimum value denoted by $(M_c)_1$ at $x = x_c$. We note that in the absence of magnetic field $Q = 0$, equation (18) gives $(M_c)_1 = 78.44$ attained at $a_c = 2.4313$ and $R = 0$ which is close to the known exact value 79.6 attained at $a_c = 1.99$. Thus we conclude that the single term Galerkin expansion used here gives reasonable results. $(M_c)_1$ is computed for different values of Q and R and tabulated in the Table 1.

3.2 Piecewise linear profile for heating from below

For a piecewise basic linear temperature profile due to sudden heating from below, the temperature gradient [7] is

$$f(x) = \begin{cases} \frac{1}{\varepsilon} & \text{for } 0 \leq z < \varepsilon \\ 0 & \text{for } \varepsilon < z \leq 1 \end{cases} \tag{19}$$

where ε is a quasi-time-dependent thermal depth parameter ranging from 0 to 1. Then equation (16) becomes

$$M_2 = \frac{3360(8x^2+264x+5292+132Q)(5+2x)}{4725x(840\varepsilon^3-336\varepsilon^4-560\varepsilon^5+240\varepsilon^6)} - R\frac{23}{420}.$$

The corresponding critical Morangoni number $(M_c)_2$ is obtained from above when $x = x_c$ and $(840\varepsilon^3-336\varepsilon^4-560\varepsilon^5+240\varepsilon^6)$ is maximum. We see that as

ε increases from 0 to 1 $(M_c)_2$ decreases from ∞ to a minimum value, for example in case when $R = 0$, $Q = 0$ $(M_c)_2 = 75.9596$, attained at $\varepsilon = 0.9323$ and then increases to $(M_c)_1$ at $\varepsilon = 1$. $(M_c)_2$ is computed for different values of Q and R and tabulated in table 1.

3.3 Piecewise linear profile for cooling from above

For a liquid layer cooling suddenly from above, the non-uniform temperature gradient $f(z)$ is of the form

$$f(z) = \begin{cases} 0 & \text{for } 0 \leq z < 1 - \varepsilon \\ \frac{1}{\varepsilon} & \text{for } 1 - \varepsilon < z < 1 \end{cases} . \tag{20}$$

On substituting (20) into (16), we get

$$M_3 = \frac{(8x^2+264x+5292+132Q)(5+2x)}{\frac{4725}{3360} \times (1680\varepsilon-2800\varepsilon^2+840\varepsilon^3+1344\varepsilon^4-1120\varepsilon^5+240\varepsilon^6)} - \frac{23}{420} R .$$

The corresponding critical Marangoni number is obtained by taking $x = x_c$ and maximizing

$$(1680\varepsilon-2800\varepsilon^2 + 840\varepsilon^3+1344\varepsilon^4-1120\varepsilon^5+240\varepsilon^6) .$$

As ε increases from 0 to 1 , $(M_c)_3$ decreases from ∞ to a minimum value 47.7178 in the case $R = 0$, $Q = 0$ when $\varepsilon = 0.4275$ and then increases to $(M_c)_1$ at $\varepsilon = 1$. This is close to the known exact value 48 (see[10]).

Values of $(M_c)_3$ for different values of Q and R are tabulated in table 1.

3.4 Superposed fluid layer

For a superposition of two layers at different temperatures in which the basic temperature drops suddenly by an amount ΔT at $z = \varepsilon$ but is otherwise uniform, the temperature gradient is (see [3]):

$$f(z) = \delta(z-\varepsilon) \tag{21}$$

where δ is the Dirac delta function and ε is a time dependent thermal depth parameter ranging from 0 to 1. In this case equation (16) becomes

$$M_4 = \frac{2(8x^2+264x+5292+132Q)(5+2x)}{4725(2\varepsilon^3-\varepsilon^4-2\varepsilon^5+\varepsilon^6)} - R\frac{23}{420} .$$

Again the critical value is given by $x = x_c$ and when $(2\varepsilon^3-\varepsilon^4-2\varepsilon^5+\varepsilon^6)$ is maximum. It is given by

$$(M_c)_4 = 37.1935$$

in the case of $Q = 0$ and $R = 0$, attained at $\varepsilon = 0.7394$, which is close to the numerical value given in [10].

$(M_c)_4$ is computed for different values of Q and R and tabulated in table 1. As in sections 3.2 and 3.3, as ε increases from 0 to 1 $(M_c)_4$ first decreases from ∞ to a minimum value at $\varepsilon = 0.7394$ and then increases again. From the table 1 we see that the system is more unstable in the case of the superposed two-fluid layers model because the jump in temperature occurs nearer the free surface. We also notice that the increase in Q increases $(M_c)_4$ and hence the magnetic field inhibits the onset of Marangoni convection by balancing a part of the potential energy produced by the buoyancy force.

3.5 Inverted parabolic temperature profile

From an inverted parabolic temperature profile generated in a layer of conducting liquid through the Joule heating with an alternating current [13], the temperature gradient is given by

$$f(z) = 2(1-z) \tag{23}$$

Using it in equation (16), we obtain the critical Marangoni number $(M_c)_5 = 116.3981$ in the absence of magnetic field and buoyancy force attained at $a_c = 2.4313$. $(M_c)_5$ is computed for different values of Q and R and the results are tabulated in Table 1. We see that the inverted parabolic basic temperature profile is less destabilizing compared to the other profiles discussed earlier. We also observe that an external constraint such as magnetic field makes the system more stable. As such, this situation is more suitable for material processing in a microgravity environment simulated in the laboratory.

4. DISCUSSION AND CONCLUSIONS

The purpose of this paper has been to study the effect of magnetic field and non-uniform temperature gradient on the linear stability of a horizontal layer of a conducting liquid at rest with the object of knowing which temperature profile gives the maximum critical Marangoni number for different values of the Rayleigh number. The single term Galerkin procedure provides a quick method for achieving the above objective.

A comparison of the critical Marangoni numbers in table 1 shows that the system is more unstable in the case of superposed two fluid model because the jump of temperature occurs nearer the less restrictive free surface. It is also true in the case when the layer is cooled from above.

In all the cases we find that the critical Marangoni number and the wave number increase with the Chandrasekhar number Q having the asymptotic behaviour $a_c \to 2.74Q^{1/3}$; $M \to 1.02Q$ in the case of linear profile, $M \to 0.99Q$ in the case of piecewise linear profile for heating from below and $M \to 0.6Q$ in the case of delta function profile when $R \to 0$. This reveals that the magnetic field inhibits the onset of convection in all the cases.

The critical Marangoni number computed for different values of ε and Q for a fixed value of R are shown in figure 1. It is seen that as ε increases from 0 to 1, M_c decreases from infinity to a minimum value and then increases again.

The magnetic field and invested parabolic basic temperature profile increase M_c considerably making the system more stable than in other cases. Therefore, we conclude that a suitable strength of magnetic field and inverted parabolic basic temperature profile is favourable for material processing in the laboratory with simulated microgravity environment. In all the cases discussed above we find that the critical Marangoni number M_c decreases with the increase in Rayleigh number.

Our results, in the case of uniform temperature gradient are compared in table 2 with those of Nield [5] obtained through elaborate numerical procedure using Fourier series method. We note that the results obtained here agree well with those of Nield [5] for small values of

Q (<10.0) . This shows that a single term Galerkin procedure used here gives reasonable results with minimum of mathematics.

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Q	a_c	R	$(M_c)_1$	$(M_c)_2$	$(M_c)_3$	$(M_c)_4$	$(M_c)_5$	$(M_c)_6$
0	2.4313	0.0	78.4421	75.9596	47.7178	37.1935	116.3981	59.1531
		100.0	72.9658	70.4834	42.2416	31.7173	110.9219	53.6769
		300.0	62.0134	59.5310	31.2892	20.7650	99.9695	42.7245
		500.0	51.0611	48.5786	20.3368	9.8126	89.0171	31.7721
		679.18689	41.2485	38.7660	10.5242	0.0	79.2045	21.9595
		871.3670	30.7244	28.2419	0.0		68.6803	11.4354
		1080.1847	19.2891	16.8066			57.2451	0.0
		1387.0872	2.4834	0.0			40.4385	
		1432.4203	0.0				37.9561	
		2125.5266					0.0	
10^2	3.1894	0.0	211.8912	205.18545	128.8973	100.4689	314.4192	159.7868
		500.0	184.5102	177.8045	101.5163	73.0879	287.0382	132.4058
		1200.0	146.1769	139.4712	63.1830	34.7556	248.7049	94.0725
		1834.6503	114.4223	104.7165	39.6647	0.0	213.9501	59.3179
		2353.775	82.9940	76.2882	0.0		185.5220	30.8896
		2917.8464	52.1044	45.3986			154.6324	0.0
		3746.8652	6.7057	0.0			109.2337	
		3869.3182	0.0				102.5280	
		5741.5690						
10^4	7.3347	0.0	11021.996	10673.184	6704.8917	5226.1158	16355.22	8311.6696
		500.0	10994.616	10645.804	6677.5108	5198.7349	16327.87	8284.2887
		1800.0	10923.425	10574.613	6606.3203	5127.5444	16256.649	8213.0982
		57800.0	8036.759	7507.947	3539.6539	2060.878	13189.983	5146.4318
		95433.428	5795.881	5447.069	1478.7759	0.0	11129.105	3085.5538
		122437.16	4317.105	3968.293	0.0		9650.329	1606.7781
		151778.33	2710.327	2361.515			8043.551	0.0
		194901.65	348.812	0.0			5682.036	
		201271.25	0.0				5333.224	
		298660.57					0.0	

TABLE 1. Critical wave number a_c for different values of Chandrasekhar's number Q and critical Marangoni number $(M_c)_i$ $i = 1$ to 6 for different values of R and Q .

TABLE 2

Q	Present Analysis		Nield [5]	
	M_c	a_c	M_c	a_c
0	78.4421	2.4313	79.607	1.99
2.5	82.0570	2.4642	85.971	2.05
12.5	96.2639	2.5817	110.08	2.22
25.0	113.5765	2.7052	138.09	2.39
50.0	147.1794	2.9026	189.87	2.63

Table 2. Comparison of critical Marangoni numbers with those of Nield [5] for $f(z) = 1$.

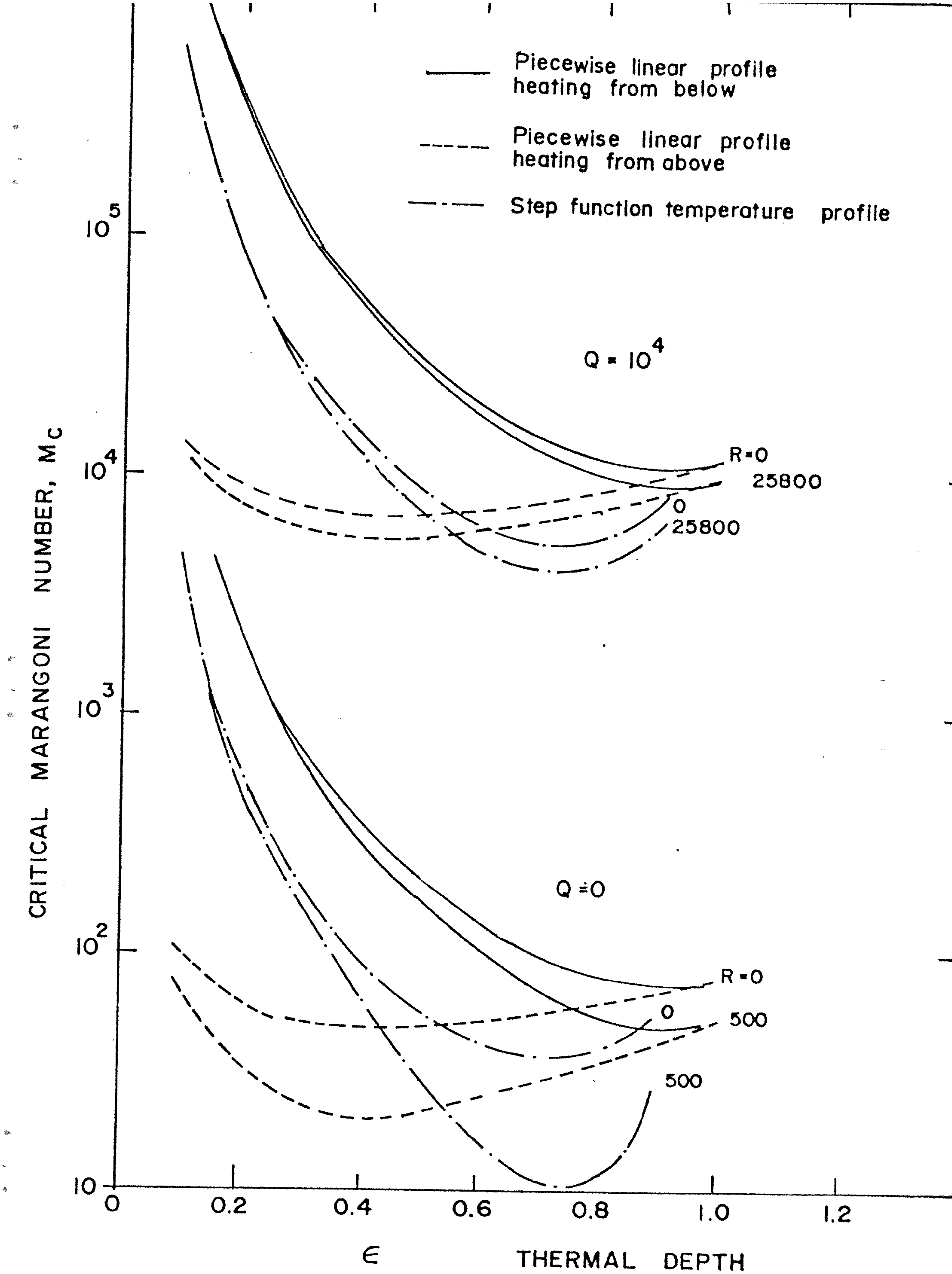

FIG. 1 CRITICAL MARANGONI NUMBER AS A FUNCTION OF THERMAL DEPTH PARAMETER ϵ FOR DIFFERENT VALUES OF Q AND R