HYDROMAGNETIC STABILITY OF COMPRESSIBLE FLUID

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The hydromagnetic stability of an adiabatic compressible perfectly conducting non-viscous fluid is investigated and it is shown that a necessary condition for instability is

$$J_H < \frac{1}{4P_+} + \frac{g^2}{C^2(W')^2}$$

where J_H is the Richardson number, P_+ the magnetic parameter, C the acoustic velocity, W' the basic velocity shear and g the gravity. This condition for instability is useful in studying the propagation of internal Alfvén-acoustic-gravity waves across a critical level.

INTRODUCTION

The theory of momentum transport by gravity waves in a conducting fluid in the presence of a magnetic field is an area of considerable interest in meteorology, oceanography, geophysics and astrophysics. Additional interest in this field stems from the attempts to simulate solar wind-geomagnetic interactions and geo-hydromagnetic secular variations. The momentum transport in a Boussinesq perfectly conducting fluid has been investigated by Rudraiah and Venkatachalappa (1972a, b, c) with velocity shear and by Acheson (1972) with magnetic shear. Rudraiah and Venkatachalappa have shown that the waves are attenuated by a factor $\exp - 2\pi(J - \frac{1}{4})$ with $J > \frac{1}{4}$, where J is the algebraic sum of the hydrodynamic and hydromagnetic Richardson numbers. Rudraiah (1964a, b, 1967, 1970) has shown that $J > \frac{1}{4}$ is the sufficient condition for stability. The above work (Rudraiah and Venkatachalappa 1972a, b, c) is mainly concerned with the situations in which the speed of fluid flow is much less than the speed of sound in the medium and the accelerations are slow compared with those associated with sound waves. Another important assumption made is the Boussinesq approximation, which amounts to the neglect of density variation except in the buoyancy term. This assumption that fluctuations in density occur principally as a result of thermal, rather than pressure variations, is a natural approximation in the case of a liquid, but is much more restrictive in the case of a

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compressible fluid. Conditions under which internal Alfvén-acoustic-gravity waves are important in geophysical and astrophysical problems are usually far removed from the idealization of a Boussinesq fluid in which the speed of the fluid flow is much less than that of sound in the medium. In meteorological case, variations in density and pressure within the toposphere can scarcely be regarded as small. Therefore, recently Rudraiah *et al.* (1976, 1977) and Rudraiah and Venkatachalappa (1978) have studied momentum transport by internal Alfvén-acoustic-gravity waves considering perfectly conducting isothermal compressible fluid in the absence of Boussinesq approximation. Acheson (1972) has shown, in the case of incompressible fluid, that there is a valve effect caused by the presence of non-zero components of rotation in a plane normal to the direction in which the medium varies. Recently Rudraiah and Venkatachalappa (1978) have established the valve effect in a compressible conducting fluid. In contrast to the incompressible results of Acheson (1972) they (Rudraiah and Venkatachalappa 1978) have shown that the valve effect in compressible flow no longer requires the presence of non-zero components of rotation in the plane normal to the direction in which the medium varies. Rudraiah and Venkatachalappa (1978) have also concluded that the compressibility increases the probability of a valve effect and so increases the capacity of a hydromagnetic wave to propagate across a field line, rather than be absorbed at some critical level which is of importance in geophysics. In particular, Rudraiah *et al.* (1976, 1977) have proved that the waves are attenuated at the critical layers by a factor $\exp(-2\mu_m\pi)$ where

$$\mu_m = \left[\frac{1}{(1+M^2)}\left(J' + \frac{l^2}{k^2} M^2 J\right) - \tfrac{1}{4}\right]$$

$$J' = \left(1 + \frac{l^2}{k^2}\right)\frac{(N')^2}{\left(\frac{dv}{dz}\right)^2}$$

is the modified Richardson number

$J = \dfrac{N^2}{\left(\frac{du}{dz}\right)^2}$ is the hydrodynamic Richardson number

$(N')^2 = \dfrac{g\beta - g^2}{c^2}$ is the modified Brunt-Väisälä frequency

$N^2 = g\beta$ is the Brunt-Väisälä frequency

$M = \dfrac{A}{C}$ is the magnetic Mach number.

For attenuation we require that

$$\frac{1}{1+M^2}\left[J' + \left(\frac{\rho^2}{k^2}\right) M^2 J\right] > \tfrac{1}{4} \text{ i.e. the fluid to be stable.}$$

Therefore the object of this investigation is to find a sufficient condition for instability of a perfectly conducting isothermal compressible fluid.

It may be mentioned that in the absence of isothermal stratified atmosphere the instability of a velocity discontinuity at a plane interface (usually vortex sheet) has been investigated for incompressible flow in the classical work of Helmholtz, Rayleigh and Kelvin (*see* Chandrasekhar 1961, section 100) and for compressible flow by Landau (1944), Hatanakar (1949), Pai (1954), Miles (1958) and Fejer and Miles (1963). In the investigation of instability Landau (1944), Hatanakar (1949) and Pai (1954) have ignored the existence of branch points for the eigenvalue equation and accepted the eigenvalues given by its two possible boundaries. Whereas, Miles (1958) has established the proper treatment of the branch points and the ultimate character of the motion by considering an initial value problem for the vortex sheet and has given asymptotic solution for large time. This two-dimensional problem has been extended to three dimensional disturbances by Fejer and Miles (1963) and they have shown that disturbances travelling at sufficiently oblique angles with respect to undisturbed flow are unstable. Recently Rudraiah (1975) and Rudraiah and Shanthakumar (1974) have investigated the stability of an isothermal compressible atmosphere flowing over an infinite liquid under the assumption of an inviscid fluid and found two separate and different types of instabilities. As the velocity of the isothermal atmosphere related to the liquid increases from zero, Rudraiah (1975) has shown that there first appears an instability of a relatively weak nature referred to as initial instability. This is followed at higher velocities, by a stronger type of instability called the gross instability. In that case they found that the effect of compressibility is to make the system more unstable. The above work on the instability of compressible flow was connected with superposed fluids. However, the instability of compressible stratified flow has not been given much attention. Warren (1968) had obtained a sufficient condition for stability of the compressible flow but this expression is weaker than the expected generalisation of the theory and is not the optimum condition. Chimons (1970) has improved Warren's condition using the procedure of Howard (1961). Recently Antia and Chitre (1978) have investigated the stability of an inviscid, perfectly conducting isothermal compressible fluid subject to a horizontal magnetic field in the presence of thermal dissipation and concluded that the magnetic field sets up the overstable motion and makes the system unstable. This situation is similar to the one discussed by Rudraiah and Vortmeyer (1978). In the absence of thermal convection and in the presence of stratified fluid under constant gravity the magnetic field makes the system more stable. Whereas, in the case of thermal convection in the presence of a magnetic field, the temperature difference sets up the potential energy which balances a part of the stabilizing effect of magnetic field and hence the system becomes unstable irrespective of the magnetic field strength. The stability analysis discussed by Antia and Chitre (1978) is not directly useful to discuss the condition for attenuation of Alfvén-acoutic waves.

Therefore, in this paper we extend Chimons work to conducting flows with the motive explained earlier.

The basic stability problem is formulated mathematically in section 2 [see eqn. (2.9) for the equilibrium configuration]. The shear flow in the y-direction is carried in this formulation with little extra expense in terms of algebraic manipulation. An elementary extension of Chimons analysis to conducting flow is presented in section 3 and a necessary condition for instability is obtained. Such a condition is useful in understanding the attenuation of internal Alfvén-acoustic gravity waves propagating across a critical level.

2. Mathematical Formulation

Let (x, y, z) form a right-handed set of co-ordinates with g, the gravitational acceleration, acting in the negative z direction and an applied magnetic field with components in x and y directions act in the directions normal to gravity. The fluid is assumed to be compressible, adiabatic and perfectly conducting. Under these approximations the basic equations of flow following Rudraiah and Venkatachalappa (1977) are

$$\frac{\partial \rho}{\partial t} + (\mathbf{q}\cdot\nabla)\rho + \rho\nabla\cdot\mathbf{q} = 0 \quad \text{...(2.1)}$$

$$\rho\left[\frac{\partial u}{\partial t} + (\mathbf{q}\cdot\nabla)u\right] = -\frac{\partial p}{\partial x} - \frac{\partial}{\partial x}\left(\frac{\mu H^2}{2}\right) + \mu(\mathbf{H}\cdot\nabla)H_x \quad \text{...(2.2)}$$

$$\rho\left[\frac{\partial v}{\partial t} + (\mathbf{q}\cdot\nabla)v\right] = -\frac{\partial p}{\partial y} - \frac{\partial}{\partial y}\left(\frac{\mu H^2}{2}\right) + \mu(\mathbf{H}\cdot\nabla)H_y \quad \text{...(2.3)}$$

$$\rho\left[\frac{\partial w}{\partial t} + (\mathbf{q}\cdot\nabla)w\right] = -\frac{\partial p}{\partial z} - \frac{\partial}{\partial z}\left(\frac{\mu H^2}{2}\right) + \mu(\mathbf{H}\cdot\nabla)H_z - \rho g \quad \text{...(2.4)}$$

$$\frac{\partial p}{\partial t} + (\mathbf{q}\cdot\nabla)p = c^2\left[\frac{\partial \rho}{\partial t} + (\mathbf{q}\cdot\nabla)\rho\right] \quad \text{...(2.5)}$$

$$\frac{\partial H_x}{\partial t} = (\mathbf{H}\cdot\nabla)u - (\mathbf{q}\cdot\nabla)H_x - H_x(\nabla\cdot\mathbf{q}) \quad \text{...(2.6)}$$

$$\frac{\partial H_y}{\partial t} = (\mathbf{H}\cdot\nabla)v - (\mathbf{q}\cdot\nabla)H_y - H_y(\nabla\cdot\mathbf{q}) \quad \text{...(2.7)}$$

$$\frac{\partial H_z}{\partial t} = (\mathbf{H}\cdot\nabla)w - (\mathbf{q}\cdot\nabla)H_z - H_z(\nabla\cdot\mathbf{q}) \quad \text{...(2.8)}$$

where $c^2(z) = \left(\frac{\partial p}{\partial \rho}\right)$ is the square of the acoustic velocity.

We consider the basic flow to be of the form

$$q = (U(z), V(z)\, 0),\ \mathbf{H} = (H_{x0}(z),\ H_{y0}(z),\ 0),\ \rho = \rho_0(z),\ p = p_0(z),$$

such that

$$\frac{dp_0}{dz} + \frac{d(\frac{1}{2}\mu H_0^2)}{dz} + \rho_0 g = 0. \quad ...(2.9)$$

On this steady flow we superpose small symmetrical disturbances of the form

$$u = U + u', \; v = V + v', \; w = w'$$

$$H_x = H_{x_0} + h'_x, \; H_y = H_{y_0} + h'_y, \; H_z = h'_z$$

$$p = p_0 + p', \; \rho = \rho_0 + \rho' \quad ...(2.10)$$

The usual process of linearisation and the assumption that each of the perturbed quantities vary in the form

$$f(x, y, z, t) = f(z) \exp \{i(\sigma t - \mathbf{k} . \mathbf{r})\}$$

where $\mathbf{k} . \mathbf{r} = k_x x + k_y y$, k_x and k_y being the wave numbers in x and y directions and $\sigma = \sigma_r + i\sigma_i$, leads to

$$i\rho\Omega + w\frac{d\rho_0}{dz} + \rho_0\left(\frac{dw}{dz} - i\mathbf{u} . \mathbf{k}\right) = 0 \quad ...(2.11)$$

$$\rho_0\left(iu\Omega + w\frac{dU}{dz}\right) = ik_x\pi + \mu\left(-\,i\mathbf{H} . \mathbf{k}h_x + h_z\frac{dHx_0}{dz}\right) \quad ...(2.12)$$

$$\rho_0\left(iv\Omega + w\frac{dV}{dz}\right) = ik_y\pi + \mu\left(-\,i\mathbf{H} . \mathbf{k}h_y + h_z\frac{dH_{y_0}}{dz}\right) \quad ...(2.13)$$

$$\rho_0 iw\Omega = -\frac{\partial\pi}{\partial z} + \mu(-\,i\mathbf{H} . \mathbf{k}h_z) - \rho g \quad ...(2.14)$$

$$i\Omega p + w\frac{dp_0}{dz} = c^2\left(i\Omega\rho + w\frac{d\rho_0}{dz}\right) \quad ...(2.15)$$

$$= -\,c^2\rho_0\left(\frac{dw}{dz} - i\mathbf{k} . \mathbf{u}\right) \quad ...(2.16)$$

$$ih_x\Omega + w\frac{dH_{x0}}{dz} = -\,i(\mathbf{k} . \mathbf{H})\,u + h_z\frac{dU}{dz} + H_{x_0}\left(i\mathbf{k} . \mathbf{u} - \frac{dw}{dz}\right) \quad ...(2.17)$$

$$ih_y\Omega + w\frac{dH_{y0}}{dz} = -\,i(\mathbf{k} . \mathbf{H})\,v + h_z\frac{dV}{zdz} + H_y\left(i\mathbf{k} . \mathbf{u} - \frac{dw}{dz}\right) \quad ...(2.18)$$

$$ih_z\Omega = -\,i(\mathbf{k} . \mathbf{H})\,w \quad ...(2.19)$$

$$-\,i\mathbf{k} . \mathbf{H} + \frac{dh_z}{dz} = 0 \quad ...(2.20)$$

where

$$\pi = p + \frac{\mu H^2}{2}$$

$$\Omega = \sigma - \mathbf{U} \cdot \mathbf{k} = \sigma - (Uk_x + Vk_y)$$

$$\mathbf{H} \cdot \mathbf{k} = H_{x_0} k_x + H_{y_0} k_y$$

$$\mathbf{h} \cdot \mathbf{k} = h_x k_x + h_y k_y$$

$$\mathbf{u} \cdot \mathbf{k} = uk_x + vk_y$$

and for simplicity the primes are omitted in the above set of equations.

After eliminating all the perturbed quantities except ω and π and simplifying, the above set of equations can be reduced to :

$$i\pi = \frac{\rho_0 \Omega^2 (1 - S^2) \dfrac{d}{dz}\left(\dfrac{w}{\Omega}\right) - \dfrac{\rho_0 g \Omega^2}{c^2}\left(\dfrac{w}{\Omega}\right)}{\left(k^2 - \dfrac{\Omega^2}{c^2}\right)} \qquad \ldots(2.21)$$

and

$$\frac{\partial \pi}{\partial z} + \frac{\pi g}{c^2} = \frac{\rho_0 w}{i\Omega}\,[\Omega^2(1 - S^2) - n^2] \qquad \ldots(2.22)$$

where

$$S^2 = \frac{A_0^2}{\Omega^2} = \frac{\mu \Omega_m^2}{\rho_0 \Omega^2},\ \Omega_m^2 = (\mathbf{H} \cdot \mathbf{k})^2,\ n^2 = -\frac{g^2}{c^2} + N^2,\ N^2 = g\beta \qquad \ldots(2.23)$$

and $\qquad k^2 = k_x^2 + k_y^2 \cdot$

Note that w appears as $w/i\Omega$ in (2.21) and (2.22) and hence setting

$$\phi(z) = \frac{w}{i\Omega},\ \psi(z) = \pi(z) \qquad \ldots(2.24)$$

eqns. (2.21) and (2.22) become

$$\phi' - p_0 \phi = A(z)\,\psi \qquad \ldots(2.25)$$

$$\psi' + q_0 \psi = B(z)\,\phi \qquad \ldots(2.26)$$

where

$$p_0 = \frac{g}{c^2}(1 - S^2)$$

$$q_0 = \frac{g}{c^2}$$

$$\left.\begin{aligned} A(z) &= \frac{\left(k^2 - \dfrac{\Omega^2}{c^2}\right)}{\rho_0\,\Omega^2(1 - S^2)} \\ B(z) &= \rho_0\,[\Omega^2(1 - S^2) - n^2]. \end{aligned}\right\} \qquad \text{...(2.27)}$$

Note that $S^2 \neq 1$ in (2.25), otherwise either ϕ and ψ or their coefficients in (2.25) are zero, or else ϕ' becomes infinite. Multiplying (2.25) by $\exp\,(-\int p_0\,dz)$ and (2.26) by $\exp\,(\int q_0\,dz)$ and simplifying we obtain

$$q' = A\psi \exp\,(-\textstyle\int p_0\,dz) \qquad \text{...(2.28)}$$

$$\frac{d}{dz}\left(\frac{q'r}{\rho_0 A}\right) = \frac{Bqr}{\rho_0} \qquad \text{...(2.29)}$$

where

$$r = \rho_0 \exp\,(\textstyle\int (p_0 + q_0)\,dz)$$

Equation (2.29) using (2.27) becomes

$$\left[\frac{r(1 - S^2)\,\Omega^2 q'}{k^2 - \dfrac{\Omega^2}{c^2}}\right]' + r(1 - S^2)\left[\frac{n^2}{1 - S^2} - \Omega^2\right] q = 0 \qquad \text{...(2.30)}$$

where the primes denote differentiation with respect to z and

$$q = \frac{w(z)}{i\Omega} \exp\left[-\int \frac{g}{c^2(1 - S^2)}\,dz\right]$$

For statically stable fluids n^2 and r are both real and positive. Since both the velocity and magnetic field equations can be combined into one single eqn. (2.30) the boundary conditions will depend only on the velocity w. We accept two forms of boundary conditions on w. Either the domain z terminates at a rigid parallel wall on which q must be zero or outside of some finite region of z the unperturbed fluid assumes constant values of C, U, V, H_{x_0} and H_{y_0}. In the latter case the outer going radiation condition viz. the boundary conditions of the third kind are applied and it is found that $r^{1/2}q$ goes to zero exponentially as $|\,z\,| \to \infty$.

3. The Stability Conditions

As Ω has non-zero imaginary part we uniquely define a branch $\Omega^{1/2}$ and make the substitution

$$q = \Omega^{-1/2}\,\phi \qquad \text{...(3.1)}$$

in (2.30) to obtain

$$\left[\frac{r(1-S^2)\,\Omega\phi'}{k^2\left(1-\dfrac{\Omega^2}{(k^2c^2)}\right)}\right]' - \frac{\Omega'^2 r(1-S^2)\,\phi}{4k^2\Omega\left(1-\dfrac{\Omega^2}{(k^2c^2)}\right)}$$

$$-\tfrac{1}{2}\left[\frac{r(1-S^2)\,\Omega'}{k^2\left(1-\dfrac{\Omega^2}{(k^2c^2)}\right)}\right]'\phi + \frac{r(1-S^2)\left(\dfrac{n^2}{1-S^2}-\Omega^2\right)}{\Omega}\phi = 0. \quad \ldots(3.2)$$

Multiplying (3.2) by $\bar{\phi}$ and integrating over the domain of z gives with a minor rearrangement of terms

$$\int\left[\left\{\frac{r(1-S^2)\,\Omega\bar{\phi}\phi'}{k^2\left(1-\dfrac{\Omega^2}{(k^2c^2)}\right)}\right\}' - \frac{r(1-S^2)\Omega\,|\,\phi'\,|^2}{k^2\left(1-\dfrac{\Omega^2}{(k^2c^2)}\right)}\right.$$

$$-\frac{(\Omega')^2\,r(1-S^2)\,|\,\phi\,|^2}{4k^2\Omega\left(1-\dfrac{\Omega^2}{(k^2c^2)}\right)} - \tfrac{1}{2}\left\{\frac{r(1-S^2)\,\Omega'\,|\,\phi\,|^2}{k^2\left(1-\dfrac{\Omega^2}{(k^2c^2)}\right)}\right\}'$$

$$\left.+\tfrac{1}{2}\,\frac{r(1-S^2)\,\Omega'(\phi\bar{\phi})'}{k^2\left(1-\dfrac{\Omega^2}{(k^2c^2)}\right)} + \frac{r(1-S^2)\left(\dfrac{n^2}{1-S^2}-\Omega^2\right)|\,\phi\,|^2}{\Omega}\right]dz = 0. \quad \ldots(3.3)$$

The first and the fourth terms integrate directly and are seen from boundary conditions to yield zero identically. The remaining terms are

$$\int\left[(T_r+iT_i)\left\{\frac{\Omega\,|\,\phi'\,|^2}{k^2\alpha} + \frac{\left(\dfrac{\Omega'}{2}\right)^2|\,\phi\,|^2}{\Omega k^2\alpha} - \frac{\dfrac{\Omega'}{2}(\phi\bar{\phi})'}{k^2\alpha} - \left(\frac{n^2}{B\Omega}-\Omega\right)|\,\phi\,|^2\right\}\right]dz=0 \quad \ldots(3.4)$$

where

$$B = 1 - S^2$$

$$\alpha = 1 - \frac{\Omega^2}{(k^2c^2)}$$

$$T_r = \rho_0\,e^{x}(B_r\cos y + B_i\sin y)$$

$$T_i = \rho_0\,e^{x}(B_i\cos y - B_r\sin y)$$

$$B_r = 1 - \frac{A_0^2\,(\Omega_r^2-\Omega_i^2)}{|\,\Omega\,|^4},\quad B_i = \frac{2A_0^2\,\Omega_r\Omega_i}{|\,\Omega\,|^4}$$

$$x = \int^{z} \frac{g}{c^2} \left\{1 + \frac{|\Omega|^4 - A^2(\Omega_r^2 - \Omega_i^2)}{|\Omega^2 - A^2|^2}\right\} dz$$

$$y = \int^{z} \frac{g}{c^2} \frac{2\Omega_r\Omega_i}{|\Omega^2 - A^2|^2} dz.$$

The imaginary part of (3.4) is

$$\int \Bigg\{ T_r\Omega_i \Bigg[\left\{ \frac{\alpha\,|\phi'|^2 + \left(\frac{\Omega'}{2kc}\right)^2 \alpha\,|\phi|^2 - \frac{\Omega'\Omega_r}{k^2c^2}(\phi\bar{\phi})'}{k^2\,|\alpha|^2} \right\}$$

$$+ \left\{P_+n^2 - \left(\frac{\Omega'}{2k}\right)^2\right\} \frac{|\phi|^2}{|\Omega|^2} + |\phi|^2 \Bigg]$$

$$+ T_i\Omega_r \Bigg[\left\{ \frac{R\,|\phi'|^2 + \left(\frac{\Omega'}{2kc}\right)^2 R\,|\phi|^2}{k^2\,|\alpha|^2} - \frac{\Omega'}{2\Omega_r}\left(1 - \frac{\Omega_r^2 - \Omega_i^2}{k^2c^2}\right)(\phi\bar{\phi})' \right\}$$

$$- \left\{P_-n^2 - \left(\frac{\Omega'}{2k}\right)^2\right\} \frac{|\phi|^2}{|\Omega|^2} + |\phi|^2 \Bigg]\Bigg\} dz = 0. \qquad \ldots(3.5)$$

where

$$\alpha = 1 + \frac{|\Omega|^2}{(k^2c^2)}$$

$$R = 1 - \frac{|\Omega|^2}{(k^2c^2)}$$

$$P_+ = \frac{1 + A_0^2/|\Omega|^2}{|B|^2}, \quad P_- = \frac{1 - A_0^2/|\Omega|^2}{|B|^2}.$$

The coefficient of $T_r\Omega_i$ in (3.5) is similar to that of Chimons except that his n^2 has been replaced here by P_+n^2. Equation (3.5) may be written in the form

$$\Omega_i = \frac{\int T_i\Omega(z)\, Y(z)\, dz}{\int T_r(z)\, X(z)\, dz}$$

where

$$Y = \left\{P_-n^2 - \left(\frac{\Omega'}{2k}\right)^2\right\} \frac{|\phi|^2}{|\Omega|^2} - |\phi|^2 - \frac{1}{k^2\,|\alpha|^2}$$

$$\times \left\{R\,|\phi'|^2 + \left(\frac{\Omega'}{2kc}\right)^2 R\,|\phi|^2 - \frac{\Omega'}{2\Omega_r}\left(1 - \frac{\Omega_r^2 - \Omega_i^2}{k^2c^2}\right)(\phi\bar{\phi})'\right\}$$

$$X = \frac{\alpha \mid \phi \mid^2 + \left(\frac{\Omega'}{2kc}\right)^2 \alpha \mid \phi \mid^2 - \frac{\Omega'\Omega_r(\phi\bar{\phi})'}{k^2c^2}}{k^2 \mid \alpha \mid^2}$$

$$+ \left\{P_+n^2 - \left(\frac{\Omega'}{2k}\right)^2\right\} \frac{\mid \phi \mid^2}{\mid \Omega \mid^2} + \mid \phi \mid^2.$$

The first term in X is always positive as shown by Chimons. Therefore, if

$$n^2 \geqslant \frac{1}{P_+}\left(\frac{\Omega'}{2k}\right)^2, \; X > 0 \qquad \text{...(3.6)}$$

and if $T_r \neq 0$, in addition to (3.6) the motion is unstable ($\Omega_i \neq 0$) if $T_i\Omega_r\, Y \neq 0$ and is stable ($\Omega_i = 0$) if it is equal to zero.

However, we note that when the frequency σ_r of the perturbations balances the sum $k_xU + k_yV$, i.e. $\Omega_r = 0$, which is one of the critical points; from (3.5) it follows that the motion is stable if (3.6) is satisfied. Hence a necessary condition for instability when $\Omega_r = 0$ is

$$n^2 < \frac{1}{P_+}\left(\frac{\Omega'}{2k}\right)^2 \qquad \text{...(3.7)}$$

for Ω_i to be not equal to zero. That is

$$N^2 - \frac{g^2}{C^2} < \frac{(W')^2}{4P_+} \qquad \text{...(3.8)}$$

where

$$W' = \frac{(k_xU' + k_yV')}{\sqrt{k_x^2 + k_y^2}} = \frac{k_xU' + k_yV')}{k}$$

is the 'shear' of the basic velocity $W(z) = [k_xU(z) + k_yV(z)]/k$

In other words

$$J_H < \frac{1}{4P_+} + \frac{g^2}{C^2(W')^2} \qquad \text{...(3.9)}$$

For an incompressible fluid $C \to \infty$ and for a non-conducting flow $P_+ = 1$, condition (3.9) reduces to $J_H < \frac{1}{4}$ which is the Miles-Howard condition for instability.

Thus in this paper we have been able to obtain a necessary condition [condition (3.7) or (3.9)] for the flow to be unstable ($\Omega_i \neq 0$) under the restriction $\Omega_r = 0$. This restriction amounts to saying that the phase velocity of the perturbations c_r given by $c_r = \sigma_r/k$ is equal to the basic velocity $W(z)$ defined in (3.8) above. In other words the flow may be unstable for the band of disturbances for which $W(z) - c_r = 0$, the condition (3.4) is satisfied. From the work of Rudraiah and Venkatachalappa (1976, 1977) we know that $W(z) - c_r = 0$ is one of the critical levels at which the propagation of Alfvén-acoustic waves has been investigated. Thus, we conclude that the condition (3.9) is useful in the study of propagation of internal Alfvén-acoustic-gravity waves and in particular in the study of attenuation of waves across the critical layers.

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