CONVECTION OVER A NATURALLY PERMEABLE BED

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ABSTRACT

The temperature distribution for a poiseuille flow over a naturally permeable bed is presented for the two cases, conducting and insulating upper plate. It is found that, the temperature distribution increases linearly in the case of insulating wall, whereas, it is parabolic in nature, in case of conducting wall. It is also found that, the Nusselt number increases steadily up to the value of $\sigma = 2$ and it is uniform for $\sigma > 2$; where $\sigma = h/\sqrt{k}$. h is the distance between the upper plate and the permeable bed and k is the permeability of the bed.

1. INTRODUCTION

THE poiseuille flow of a Newtonian fluid over a porous surface and the corresponding slip boundary condition at the permeable wall has been investigated, both theoretically and experimentally, by Bevers and Joseph (1967). Their results show that the rectilinear flow of a viscous fluid over the surface of a permeable material induces a boundary layer region, within the material. The effect of this boundary layer is to alter the nature of the tangential flow near the nominal boundary. The slip boundary condition proposed by them is in agreement with their experimental data. They have shown that the discharge is greatly enhanced over the value it would have if the bed were impermeable, indicating the presence of a boundary layer in the bed. Throughout their analysis, they have considered only the velocity of the flow but not the temperature distribution in the bed. Since there exists a boundary layer, i.e., viscous dissipation in the bed, we should take into account, however small, the variation of temperature induced by the viscous dissipation. This is one of the convection problems over a permeable surface.

In general, the problem of convection consists of the transport of heat by a moving fluid in which variation of temperature, and hence of density, produces a distributed buoyancy force that itself modifies the flow pattern. This interaction of density and velocity fields is the essential feature of convection. Thus to find the convection in poiseuille flow both the velocity and temperature fields must be determined together, throughout the whole region of flow. Solutions of this kind are mathematically complicated and in such circumstances simple problems like forced and free convection are usually studied theoretically in detail.

In forced convection problem, the density of the flow is regarded entirely unaffected by the heat transfer, which therefore acts only to mark part of fluid passing near certain boundaries. These marked elements are swept away in the flow produced by externally applied forces (namely the applied pressure in the poiseuille flow) and are distributed throughout the region by convection and diffusion, without affecting the field of flow in any way. This amounts to finding the temperature distribution due to heated or cooled boundaries in a specified velocity field. However, in the problem of free convection the motion of fluid is caused solely by the buoyancy forces due to the action of some field of force on the unevenly unheated fluid.

The aim of the present paper is to consider only the problem of forced convection taking the velocity field as given by Bevers and Joseph and assuming the Muskat model. The corresponding problem of free convection will be presented elsewhere.

2. Basic Equations and Solutions of the Problem

We consider the steady rectilinear flow of a viscous fluid through a two-dimensional parallel channel formed by an impermeable upper wall $y = h$ and a permeable lower bed $y = 0$. The plane $y = 0$ defines a nominal surface for the permeable material. A uniform pressure gradient is maintained in the longitudinal direction both in the channel and in the impermeable material. The flow through the body of the permeable material is given by Darcy's law

$$Q = -\frac{k}{\mu} \times \frac{dp}{dx} \tag{1}$$

where k is the permeability of the material, Q is the volume of flow rate per unit cross-sectional area, and μ is the viscosity.

The free flow between the permeable surface and the rigid upper boundary having the horizontal component of velocity u is governed by the Navier-Stokes equation. For this problem Bevers and Joseph (1967) has given the velocity distribution in the form:

$$u = u_B\left(1 + \frac{a}{\sqrt{k}} y\right) + \frac{1}{2\mu}(y^2 + 2ay\sqrt{k})\frac{dp}{dx} \tag{2}$$

where the slip velocity is given by

$$u_B = -\frac{k}{2\mu}\left(\frac{\sigma^2 + 2a\sigma}{1 + a\sigma}\right) \times \frac{dp}{dx} \tag{3}$$

with

$$\sigma = \frac{h}{\sqrt{k}}$$

The temperature distribution is given by the energy equation

$$\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{K}\left(\frac{du}{dy}\right)^2 = 0 \tag{4}$$

where μ is the viscosity and K is the thermal diffusivity of fluid. The boundary condition on the temperature distribution at the permeable surface is

$$\frac{dT}{dy} = \beta(T_B - T_0) \quad \text{at} \quad y = 0. \tag{5}$$

The boundary condition on the temperature at the impermeable surface will depend on the nature of the material of the plate. If the upper plate is conducting the boundary condition is

$$T = T_0 \quad \text{at} \quad y = h \tag{6}$$

If the upper plate is insulating, the boundary condition, since no heat can pass through it, is

$$\frac{dT}{dy} = 0 \quad \text{at} \quad y = h. \tag{7}$$

Making equations (2) to (7) dimensionless, using the quantities

$$V = \frac{u}{u_0}, \ V_B = \frac{u_B}{u_0}, \ \theta = \frac{T}{T_0},$$

$$\eta = \frac{y}{h}, \ \gamma = \frac{x}{h} \tag{8}$$

where u_0 is the maximum velocity in the channel, T_0 is the room temperature, we obtain

$$V = -\frac{R}{2} \times \frac{dp_0}{dr}(1 - \eta^2) + a\sigma\left(V_B + \frac{R}{\sigma^2} \times \frac{dp_0}{dr}\right)(\eta - 1) \tag{9}$$

$$V_B = -\frac{R}{2\sigma} \times \frac{(\sigma + 2a)}{(1 + a\sigma)} \times \frac{dp_0}{dr} \tag{10}$$

$$\frac{d^2\theta}{d\eta^2} = -\xi\left(\frac{dV}{d\eta}\right)^2 \tag{11}$$

and the corresponding boundary conditions are:

(*i*) When the upper plate is conducting

$$\theta = 1 \quad \text{at} \quad \eta = 1$$

$$\frac{d\theta}{d\eta} = a\sigma(\theta_B - 1) \quad \text{at} \quad \eta = 0 \tag{12}$$

(*ii*) When the upper plate is insulating

$$\frac{d\theta}{d\eta} = 0 \quad \text{at} \quad \eta = 1$$

$$\frac{d\theta}{d\eta} = a\sigma(\theta_B - 1) \quad \text{at} \quad \eta = 0 \tag{13}$$

where

$$R = \frac{u_0 h}{\nu}$$

is the Reynolds number.

$$\xi = P_r(E_c)$$

$$P_r = \frac{\mu g c_p}{K}$$

is the Prandtl number

$$E_c = \frac{u_0^2}{g c_p T_0}$$

is the Eckert number

When the upper plate is conducting, the temperature distribution is given by

$$\theta = 1 - \xi \left[\frac{\alpha\sigma\{2b^2 + (a + 2b)^2\}(1 - \eta)}{12(1 + \alpha\sigma)} - \frac{b^2}{2}(1-\eta^2) - \frac{ab}{3}(1-\eta^3) - \frac{a^2}{12}(1-\eta^4) \right] \quad (14)$$

where

$$b = \frac{a\alpha(2 - \sigma^2)}{2\sigma(1 + \alpha\sigma)}, \quad a = R\frac{dp_0}{dr}. \quad (15)$$

In this case the Nusselt number N, defined by

$$N = \frac{-\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=1}}{\theta_B - 1} \quad (16)$$

is given by

$$N = \frac{(2a + 3b)^2(1 + \alpha\sigma) + 3b^2(1 - \alpha\sigma) - \alpha\sigma(a^2 + 4ab)}{2b^2 + (a + 2b)^2}. \quad (17)$$

Also, when the upper plate is insulating, the temperature distribution is given by

$$\theta = 1 + \frac{\xi}{3\alpha\sigma}[b(a + 2b) + (a + b)^2] + \frac{\xi}{3}[b(a+2b) + (a+b)^2]\eta - \xi\left[\frac{b^2}{2} + \frac{ab}{3}\eta + \frac{a^2}{12}\eta^2\right]\eta^2. \quad (18)$$

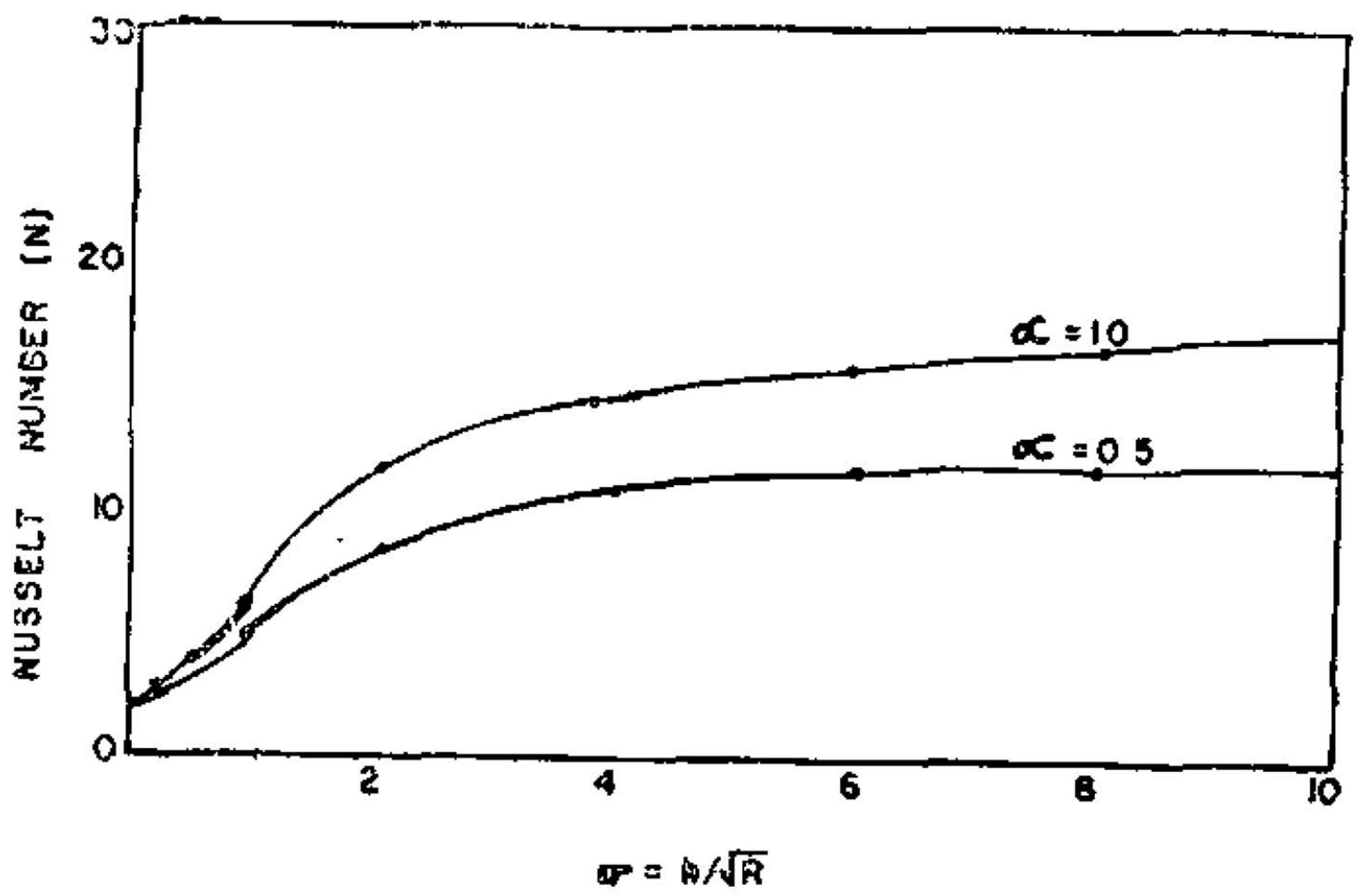

FIG. 1 Nusselt number *vs.* permeability.

The Nusselt number given by equation (17) is numerically evaluated for different values of σ, a and α and is shown in Fig. 1, for $a = 1$. We observe that, when α increases, the Nusselt number increases steadily up to the value of $\sigma = 2$ and for $\sigma > 2$, the Nusselt number is uniform. Typical behaviour is observed for other values of a.

3. Discussion

Temperature distribution given by equations (14) and (18) are numerically evaluated and it is observed that in the case of insulating upper wall the variation of temperature is linear whereas in the case of conducting upper wall, the variation of temperature is parabolic in nature.

1. Beavers, S. G. and Joseph, D. D., *J. Fluid Mech.* 1967, **30**, Part I, 197.