LIQUID METAL MAGNETOHYDRODYNAMIC POWER GENERATOR

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THE theoretical aspect of magnetohydrodynamic power generator using a rectangular channel of uniform cross-section is investigated in this article. The experimental arrangements and the results obtained will be presented elsewhere. The physical problem consists of the flow of a conducting, incompressible, heterogeneous and non-viscous fluid bounded by a rectangular channel made of electrodes and insulating walls in the presence of a transverse magnetic field. The purpose of using an heterogeneous conducting fluid is to achieve increased power output.

The required equations following Rudraiah (1964), using

$$u_x = u\left(\frac{\rho}{\rho_0}\right)^{\frac{1}{2}} \tag{1}$$

$$u_y = v\left(\frac{\rho}{\rho_0}\right)^{\frac{1}{2}} \tag{2}$$

and using small perturbation

$$u_x = u' + U$$
$$u_y = v' \tag{3}$$

where u and v are the x and y components of velocity, ρ is the variable density, ρ_0 is some reference density, and U is the free upstream velocity, become

$$\nabla^2 \Phi = N\left[\frac{\partial \Phi}{\partial \xi} + \left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}} \frac{\partial \Psi}{\partial \xi}\right] \tag{4}$$

$$\nabla^2 \Psi = -N\left[\frac{\partial \Phi}{\partial \xi} + \left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}} \frac{\partial \Psi}{\partial \xi}\right] \tag{5}$$

with the boundary conditions

$$\Phi = \pm \Phi_w \qquad \zeta = \pm \frac{\pi}{2h} \; (\xi > 0) \tag{6}$$

$$\frac{\partial \Phi}{\partial \xi} = 0 \qquad \zeta = \pm \frac{\pi}{2h} \; (\xi < 0) \tag{7}$$

$$\Psi = \pm \frac{\pi}{2h} \qquad \zeta = \pm \frac{\pi}{2h} \; (-\infty < x < \infty) \tag{8}$$

where

$$\phi = UBh\Phi, \quad \psi' = Uh\Psi, \quad x = h\xi$$

and $y = h\xi$, ϕ is the electric potential and ψ' is the stream function,

$$N = \frac{\sigma B^2 h}{\rho_0 U}$$

is the interaction parameter, which we assume to be small.

To solve equation (4) we use,

$$\Phi = \Phi_0 + N\Phi_1 + \cdots \tag{9}$$
$$\Psi = \Psi_0 + N\Psi_1 + \cdots \tag{10}$$
$$\rho = \rho_0 + N\rho_1 + \cdots \tag{11}$$

We note that Φ_0 is sufficient (Sutton and Carlson, 1961) to calculate the power output. Thus, equation (4) is solved using the technique of conformal mapping, where we use the transformation

$$e^z = \sin w \tag{12}$$

$$\eta_h = \xi + \frac{\pi}{2h} \tag{13}$$

$$z = \xi + i\eta_h \tag{14}$$

$$w = \xi' + i\,\eta_h', \quad \eta_h = \frac{\eta}{h} \tag{15}$$

The required solution for the potential is

$$\Phi_0 = 2\Phi_w \frac{\xi' h}{\pi} \tag{16}$$

or in terms of dimensional quantities

$$\phi_0 = 2\phi_w \frac{x'}{\pi} \tag{17}$$

where

$$\xi' = \frac{x'}{h} \quad \eta'_h = \frac{\eta'}{h}.$$

The expression for the current, using Ohm's law, will be

$$J_0 = \frac{2\sigma}{\pi} \phi_w \eta' - \left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}} \sigma \, UBx. \qquad (18)$$

For large η' along $x = \pi/2$, equation (12) becomes

$$e^{x} = \tfrac{1}{2} e^{\eta'} \qquad (19)$$

and hence

$$\eta' = x + \log 2. \qquad (20)$$

If the channel length is L, the total current to the electrodes per unit length in the direction of the magnetic field is

$$J_L = \sigma \left[\frac{2\phi_w}{h} - \left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}} UB\right] L + \frac{4}{\pi} \sigma \phi_w \log 2. \qquad (21)$$

The efficiency of the power generator is given by

$$\epsilon_g = \frac{\text{Power output}}{\text{Flow work}}$$

$$= \frac{2\phi_w \left[L \left\{\frac{2\phi_w}{h} - \left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}} UB\right\} + \frac{4}{\pi} \log 2\right]}{UBLh \left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}} \left[\frac{2\phi_w}{h} - UB \left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}}\right]}. \qquad (22)$$

If $L \to \infty$, the end losses become negligible and the efficiency becomes

$$\epsilon_g = \frac{2\phi_w}{UBh} \left(\frac{\rho}{\rho_0}\right)^{\frac{1}{2}}. \qquad (23)$$

We conclude that using an heterogeneous conducting fluid, the total current per unit length in the direction of the magnetic field, the power output and efficiency are increased.

1. Sutton, G. W. and Carlson, A. W., "End effects in inviscid flow in a magnetohydrodinamic channel," *J. Fluia. Mech.*, 1961, **2**, 121.
2. Rudraiah, N., "Magnetohydrodynamic stability of Heterogeneous dissipative conducting liquids," *Appl. Sci. Res. Sec. B*, 1964, **2**, 180.