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Modelling of crustal deformation in volcanic areas

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Abstract : Mogi (1958) used a centre of dilatation in an elastic half-space to model the ground deformation in volcanic areas. We consider four additional axially-symmetric source models, namely, a single force, a dipole, a tensile dislocation and a compensated linear vector dipole (CLVD) and compare the corresponding surface deformation with the surface deformation due to a centre of dilatation. It is found that the vertical displacements due to a dipole, a tensile dislocation and a CLVD decay much faster as compared to the vertical displacement due to a centre dilatation. Moreover, while a centre of dilatation predicts uplift at all epicentral distances, a dispole of a CLVD predicts subsidence after a certain epicentral distance.

(Kewwords : ground deformation modelling/volcanic areas)

Introduction : Mogi (1958) used a centre of dilatation in an elastic half-space to interpret the ground deformation produced in volcanic areas. This model is often called Mogi's model after him and has been used very extensively since then. Dieterich and Decker (1975) used a finite element method with different shapes for the magma chamber to model surface deformation associated with volcanism. Bianchi *et al.* (1987) also used a finite element method to calculate the surface deformation, assuming axial symmetry about the vertical axis through the caldera centre. They concluded that Mogi's model requires an extremely large value for the pressure increase to explain the large and localized surface displacements observed at Campi Flegrei, Italy for the 1970-72 and 1982-1984 events. McTigue (1987) obtained approximate analytical expressions for the displacements and stresses due to a pressurized spherical cavity in an elastic half-space. In this method, known as the method of images, the boundary conditions are statisfied alternatively at the cavity surface and at the plane boundary. McTigue concluded that the effect of the finite radius of the cavity on the surface uplift is quite weak.

Bonafede *et al.* (1986) obtained an analytical solution for the displacement field due to a centre of dilatation in a viscoelastic half-space. The results obtained are applied to the volcanic area of Campi Flegrei. It is shown that the consideration of viscoelasticity allows the same deformation to take place with much lower pressure values in the magma chamber than are required by purely elastic models. Davis (1983) advocated the use of a tensile dislocation in a half-space for interpreting naturally induced hydrofracture of the earth's crust such as the magmatic intrusion into sills and dikes in volcanic zones, including the

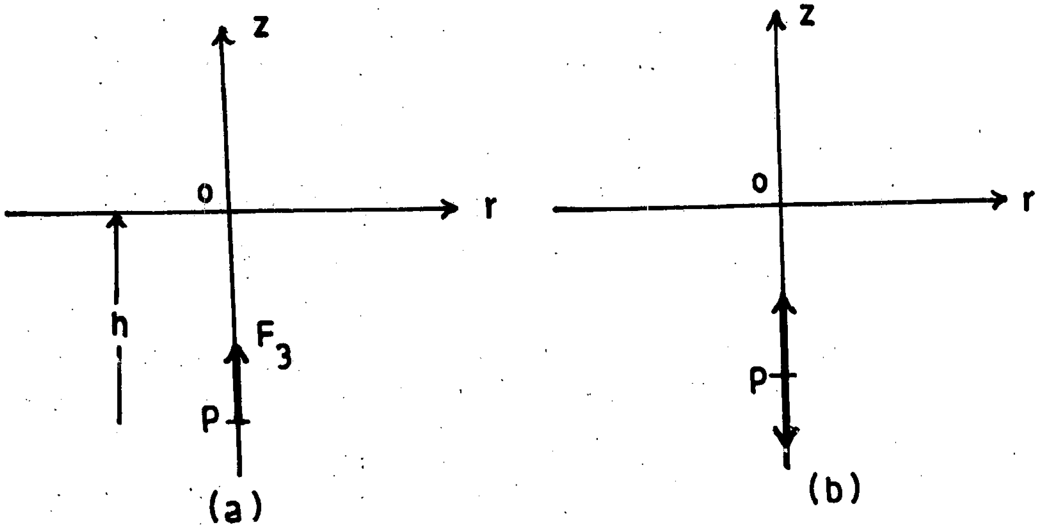

Figure 1– (a) Vertical force of magnitude F_3 placed at the point P of a uniform half-space at a depth, h.

(b) Vertical dipole.

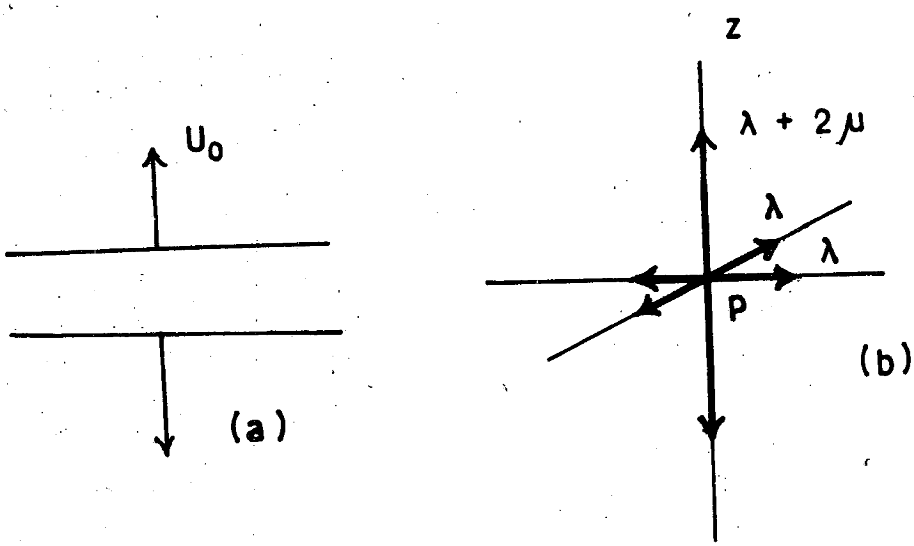

Figure 2– (a) A tensile dislocation of magnitude U_0 on a horizontal plane.

(b) Equivalent body force.

mid-ocean ridges, volcanic rift zones and continental rifts. Okada and Yamamoto (1991) constructed a dislocation model for the 1989 seismo-volcanic activity off Ito, central Japan, based on crustal movement data and seismic data. This model consists of two tensile faults corresponding to magma intrusions in May and July 1989, and a right-lateral reverse fault representing the largest seismic event (M 5.5) of July 9, 1989.

We consider four additional axially symmetric sources, namely, a vertical force, a vertical dipole, a tensile dislocation on a horizontal fault and a compensated linear vector dipole (CLVD) in an elastic half-space to model the ground deformation in volcanic areas. The displacement and strain fields due to these four sources are compared with the corresponding fields due to a centre of dilatation.

There is a fundamental difference between the single force source model and the remaining four source models considered in this paper. While the single force is a unipolar source, the remaining four source are dipolar and the corresponding equivalent force systems have zero total force and zero total torque. Takei and Kumazawa (1994) have given a theoretical justification for the physical existence of sigle-force and torque components from indigenous earthquake sources. According to them, the most promising indigenous events for detecting and identifying single forces and torques are those beneath volcanic regions.

Theory : The problem of the static deformation due to a point source buried in an elastic half-space has been discussed extensively in literature (Mindlin 1936, Mindlin and Cheng 1950, Maruyama 1964, Okada 1985, Singh and Singh 1989). The expressions for the surface displacements for various sources given below have been obtained from Singh and Singh (1989). We assume that the half- space occupies the region $z = 0$ the plane $z = 0$, is stress-free and the point source is located at a depth h below the free surface. We use a cylindrical co-oridnate system (r, e, z) with the z-axis vertically upwards.

Vertical Force : For a vertical point force of magnitude F_3 (Fig. 1a), the radial and vertical components of the displacements and strain at the surface of the half-space ($z = 0$) are given by :

$$u_r = \frac{F_3 r}{4\pi\mu R}\left(\frac{h}{R^2} + \frac{1-2\sigma}{R+h}\right), \tag{1}$$

$$u_z = \frac{F_3}{4\pi\mu R}\left(2(1-\sigma) + \frac{h^2}{R^2}\right), \tag{2}$$

$$e_{rr} = \frac{F_3}{4\pi\mu R}\left[\frac{h}{R^2}\left(1-\frac{3r^2}{R^2}\right) + (1-2\sigma)\left(\frac{h}{R^2}-\frac{1}{R+h}\right)\right], \tag{3}$$

$$e_{zz} = \frac{\sigma F_3 h}{4\pi\mu(1-\sigma)R^3}\left(2\sigma-\frac{3h^2}{R^2}\right), \tag{4}$$

where μ is the rigidity, σ the Poisson's ratio and

$$R^2 = r^2 + h^2. \tag{5}$$

Vertical Dipole : For a vertical dipole of moment F_{33} (Fig. 1b), the surface deformation is given by :

$$u_r = \frac{F_{33}\, r}{4\pi\,\mu R^3}\left(\frac{3h^2}{R^2} - 2\sigma\right), \tag{6}$$

$$u_z = \left(\frac{h}{r}\right) u_r, \tag{7}$$

$$e_{rr} = \frac{F_{33}}{4\pi\,\mu R^3}\left[3h^2\left(\frac{1}{R^2} - \frac{\sigma r^2}{R^4}\right) - 2\sigma\left(1 - \frac{3r^2}{R^2}\right)\right], \tag{8}$$

$$e_{zz} = -\frac{\sigma F_{33}}{4\pi\,\mu(1-\sigma)R^3}\times\left[2\sigma - \frac{3(3+2\sigma)h^2}{R^2} + \frac{15h^4}{R^4}\right]. \tag{9}$$

Centre of Dilatation : A centre of dilatation is equivalent to three equal mutually orthogonal dipoles. If the moment of each of these dipoles is M_0, the surface deformation is given by

$$u_r = \frac{(1-2\sigma)\, M_0}{2\pi\,\mu}\left(\frac{r}{R^3}\right), \tag{10}$$

$$u_z = \left(\frac{h}{r}\right) u_r, \tag{11}$$

$$e_{rr} = \frac{(1-2\sigma)\, M_0}{2\pi\,\mu R^3}\left(1 - \frac{3r^2}{R^2}\right), \tag{12}$$

$$e_{zz} = \frac{\sigma(1-2\sigma)\, M_0}{2\pi\,\mu(1-\sigma)R^3}\left(1 - \frac{3h^2}{R^2}\right). \tag{13}$$

Tensile Dislocation : A tensile dislocation U_0 (Fig. 2a) on a horizontal planer element of area dS is equivalent to a vertical dipole of moment $(\lambda + 2\mu)U_0 dS$ plus two mutually orthogonal horizontal dipoles of moment $\lambda U_0 dS$ each (Fig. 2b, Ben-Menahem and Singh 1981). The corresponding surface deformation is found to be

$$u_r = \frac{3U_0 dS\, h^2 r}{2\pi R^5}, \tag{14}$$

$$u_z = \left(\frac{h}{r}\right) u_r, \tag{15}$$

$$e_{rr} = \frac{3U_0 dS\, h^2}{2\pi R^5}\left(1 - 5\frac{r^2}{R^2}\right), \tag{16}$$

$$e_{zz} = \frac{3U_0 dS\, h^2}{2\pi R^5}\left(3 - 5\frac{h^2}{R^2}\right). \tag{17}$$

Compensated Linear Vector Dipole : A compensated linear vector dipole (Knopoff and Randall 1970) consists of three mutually orthogonal dipoles with moments in the ratio (− 1, −1, 2). If the principal dipole of moment 2M is vertical (Fig. 3), the surface deformation is given by

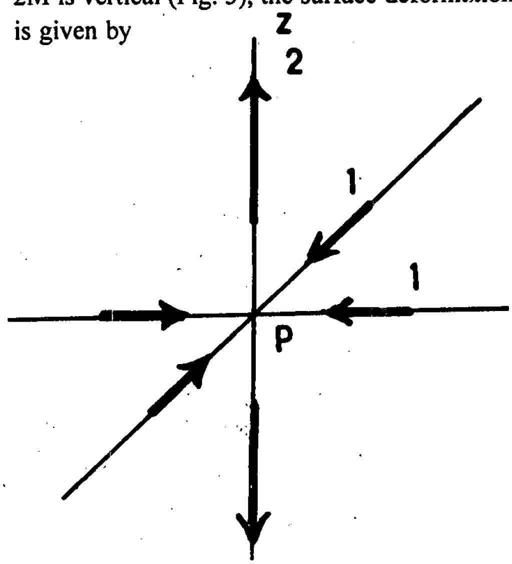

Figure 3– Body forces equivalent to a compensated linear vector dipole.

$$u_r = \frac{Mr}{4\pi\mu R^3}\left(\frac{9h^2}{R^2} - 2(1+\sigma)\right), \quad (18)$$

$$u_z = \left(\frac{h}{r}\right)u_r, \quad (19)$$

$$e_{rr} = -\frac{M}{4\pi\mu R^3}\left[2(1+\sigma)\left(1-\frac{3r^2}{R^2}\right) \times \frac{9h^2}{R^2}\left(1-\frac{5r^2}{R^2}\right)\right], \quad (20)$$

$$e_{zz} = -\frac{\sigma M}{4\pi\mu(1-\sigma)R^3} \times \left(2(1+\sigma) - 3(11+2\sigma)\frac{h^2}{R^2} + \frac{45h^4}{R^4}\right). \quad (21)$$

Numerical Results : We define dimensionless epicentral distance D, dimensionless radial displacement U and dimensionless vertical displacement (uplift) W by the relations

$$D = \frac{r}{h},\ U = \frac{A}{h}u_r,\ W = \frac{A}{h}u_z,$$

$$E_{11} = A\,e_{rr},\ E_{33} = A\,e_{zz}, \quad (22)$$

where A is a dimensionless constant.

For each source, we choose A in such a manner that $W = 1$ at $r = 0$.

Vertical Force : From equations (1)-(5), we find

$$U = \frac{D}{(3-2\sigma)(1+D^2)^{1/2}} \times \left[\frac{1}{1+D^2} + \frac{1-2\sigma}{1+(1+D^2)^{1/2}}\right], \quad (23)$$

$$W = \frac{1}{(3-2\sigma)(1+D^2)^{1/2}} \times \left[2(1-\sigma) + \frac{1}{1+D^2}\right], \quad (24)$$

$$E_{11} = \frac{1}{(3-2\sigma)(1+D^2)^{1/2}} \times \left[\frac{1-2D^2}{(1+D^2)^2} + (1-2\sigma) \times \left\{\frac{1}{1+D^2} - \frac{1}{1+(1+D^2)^{1/2}}\right\}\right], \quad (25)$$

$$E_{33} = \frac{\sigma}{(3-2\sigma)(1-\sigma)(1+D^2)^{3/2}} \times \left[2\sigma - \frac{3}{1+D^2}\right], \quad (26)$$

$$A = \frac{4\pi\mu h^2}{(3-2\sigma)F_3}. \quad (27)$$

Vertical Dipole : From equations (6)-(9), we have

$$U = \frac{D}{(3-2\sigma)(1+D^2)^{3/2}}\left(\frac{3}{1+D^2} - 2\sigma\right), \quad (28)$$

$$W = U/D, \quad (29)$$

$$E_{11} = \frac{1}{(3-2\sigma)(1+D^2)^{5/2}} \times \left[\frac{3(1-4D^2)}{1+D^2} - 2\sigma\left(1-2D^2\right)\right], \quad (30)$$

$$E_{33} = -\frac{\sigma}{(3-2\sigma)(1-\sigma)(1+D^2)^{3/2}} \times$$

$$\left[2\sigma - \frac{3(3+\sigma)}{1+D^2} + \frac{15}{(1+D^2)^2}\right], \tag{31}$$

$$A = \frac{4\pi\mu h^3}{(3-2\sigma)F_{33}}. \tag{32}$$

Centre of Dilatation : From equations (10)-(13), we find

$$U = \frac{D}{(1+D^2)^{3/2}}, \tag{33}$$

$$W = U/D, \tag{34}$$

$$E_{11} = \frac{1-2D^2}{(1+D^2)^{5/2}}, \tag{35}$$

$$E_{33} = \frac{\sigma(D^2-2)}{(1-\sigma)(1+D^2)^{5/2}}, \tag{36}$$

$$A = \frac{2\pi\mu h^3}{(1-2\sigma)M_0}. \tag{37}$$

Tensile Dislocation : From equations (14)-(17), we get

$$U = \frac{D}{(1+D^2)^{5/2}}, \tag{38}$$

$$W = U/D, \tag{39}$$

$$E_{11} = \frac{1-4D^2}{(1+D^2)^{7/2}}, \tag{40}$$

$$E_{33} = \frac{\sigma(3D^2-2)}{(1-\sigma)(1+D^2)^{7/2}}, \tag{41}$$

$$A = \frac{2\pi h^3}{3U_0\, dS}. \tag{42}$$

Compensated Linear Vector Dipole : From equations (18)-(21), we have

$$U = \frac{D}{(7-2\sigma)(1+D^2)^{3/2}}\left[\frac{9}{1+D^2} - 2(1+\sigma)\right] \tag{43}$$

$$W = U/D, \tag{44}$$

$$E_{11} = -\frac{1}{(7-2\sigma)(1+D^2)^{3/2}} \times$$

$$\left[\frac{2(1+\sigma)(1-2D^2)}{1+D^2} - \frac{9(1-4D^2)}{(1+D^2)^2}\right] \tag{45}$$

$$E_{33} = \frac{\sigma}{(7-2\sigma)(1-\sigma)(1+D^2)^{3/2}} \times$$

$$\left[2(1+\sigma) - \frac{3(11+2\sigma)}{1+D^2} + \frac{45}{(1+D^2)^2}\right] \tag{46}$$

$$A = \frac{4\pi\mu h^3}{(7-2\sigma)M}. \tag{47}$$

Figure 4 shows the variation of the vertical displacement (uplift) with epicentral distance. For a single force, a centre of dilatation and a tensile dislocation, the vertical displacement is always an uplift. However, from equation (29), we note that, for a dipole the vertical displacement vanishes at

$$D = \left(\frac{3-2\sigma}{2\sigma}\right)^{1/2},$$

which yields $D = 2.24$ for $\sigma = 0.25$.

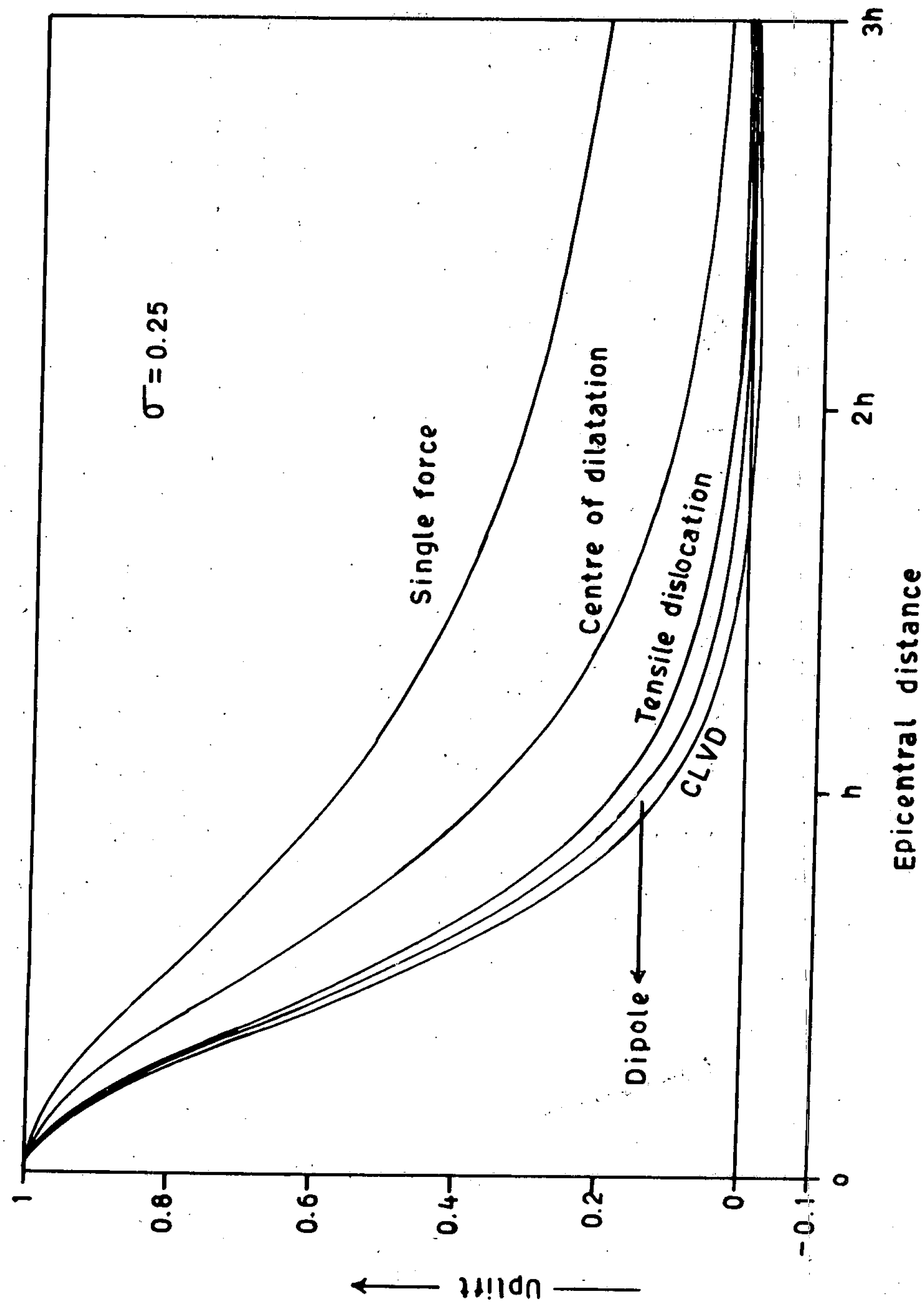

Fig. 4–Variation of the dimensionless vertical displacement (uplift, W) with epicentral distance. For each source, the sources strength is so normalised as to make $W = 1$ at $r = 0$.

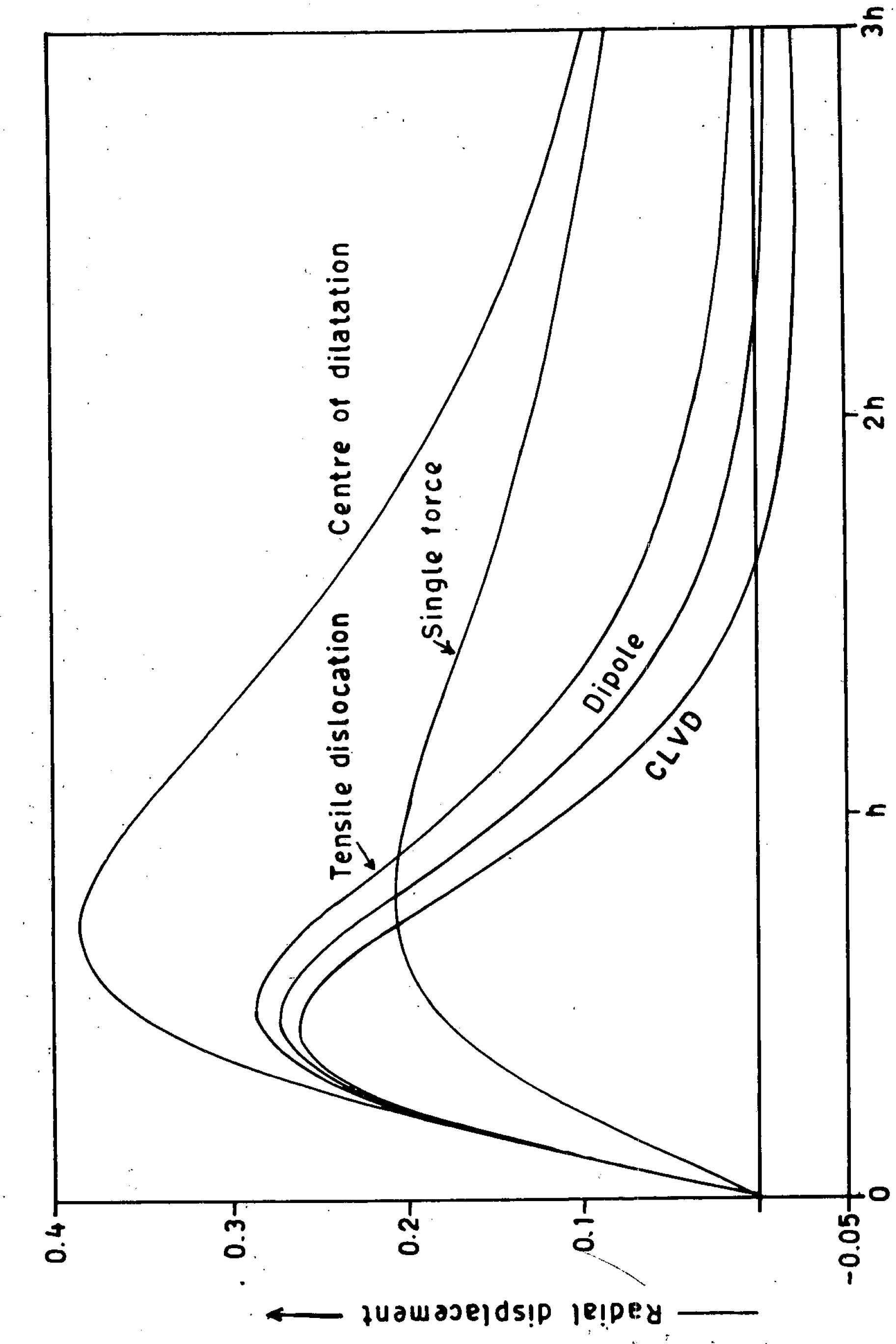

Fig. 5– Variation of the dimensionless radial displacement (U) with epicentral distance.

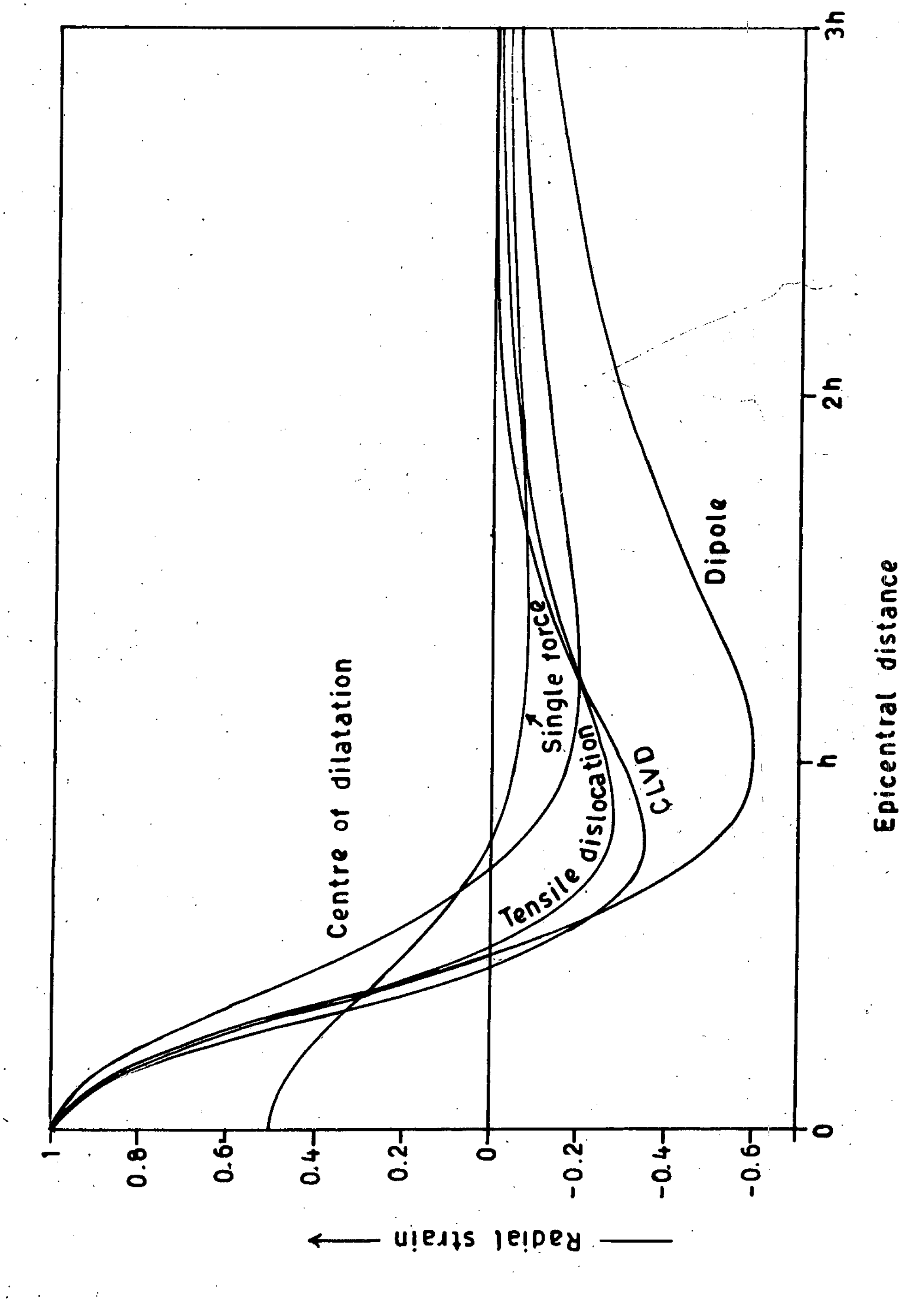

Fig. 6– Variation of the radial strain (E11) with epicentral distance.

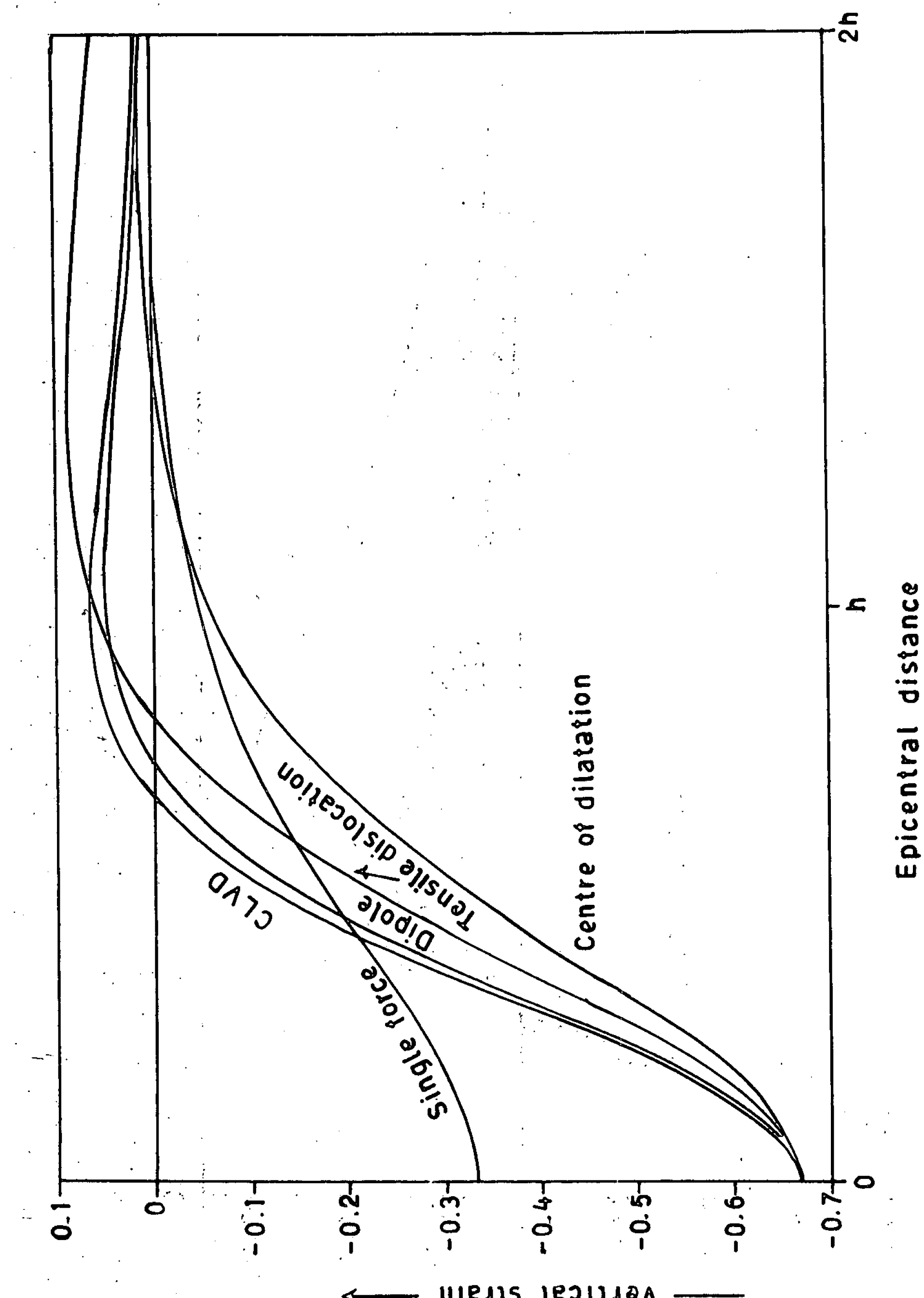

Fig. 7– Variation of the vertical strain (E_{33}) with epicentral distance.

Similarly, for a CLVD, the vertical displacement vanishes at

$$D = \left(\frac{7 - 2\sigma}{2(1 + \sigma)} \right)^{1/2}$$

which gives $D = 1.61$ for $\sigma = 0.25$.

Figure 5 shows the variation of the radial displacement with epicentral distance. In this case also the radial displacement does not change sign for a single force, a centre of dilatation or a tensile dislocation. However, for a dipole it vanishes at $D = 2.24$ and for a CLVD, it vanishes at $D = 1.61$. In each case, the radial displacement is zero at the epicentre and has a single minimum. Table 1 gives the maximum value of U and the epicentral distance at which this maximum is attained. It may be noted that the source strength for each source is so normalized that $W = 1$ at the epicentre.

Table 1 –The maximum value of the radial displacement U and the epicentral distance $D = \frac{r}{h}$ at which it is attained for the five sources in a Poissonian half space ($W_{max} = 1$, $\sigma = 0.25$)

Source	U_{max}	D
Centre of dilatation	0.385	0.71
Tensile dislocation	0.286	0.50
Dipole	0.273	0.47
CLVD	0.262	0.45
Single force	0.207	0.79

From Table 1, we note that out of the five source models considered, the centre of dilatation is the most efficient and the single force is the least efficient in generating the radial displacement. Figures 6 and 7 depict the variation of the radical and vertical strains with distance.

Conclusions : (a) Out of the five axially symmetric sources considered, the centre of dilatation is the most efficient and the single force is the least efficient in generating the horizontal displacement.

(b) The vertical displacement due to a dipole, a tensile dislocation or a CLVD decays much faster than the vertical displacement due to a centre of dilatation.

(c) A centre of dilatation, a tensile dislocation or a single force predicts uplift at all epicentral distances; but a CLVD or a dipole predicts subsidence after a certain epicentral distance. Similarly, the radial displacement due to a centre of dilatation, a tensile dislocation or a single force is outward at all epicentral distances but the radial displacement due to a CLVD or a dipole becomes inward after a certain epicentral distance.

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