Analysis of sub-Doppler linewidths in inversionless amplification

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(Received 17 May 1996)

In a recent experiment Zhu and Lin [Phys. Rev. A **53**, 1767 (1996)] reported gain linewidths, in a coherently pumped atomic system, which were much less than the Doppler width. We show that this observation is easily explained in terms of our work [Phys. Rev. A **53**, 2842 (1996)], which is generalized to correspond to the experimental situation by inclusion of incoherent pumping, and pump and probe laser linewidths.

PACS number(s): 42.50.Gy

Recently, Zhu and Lin reported the results of an experiment in a three-level Λ system, in which they observed amplification of a weak probe in the absence of population inversion in the bare states of the atom [1]. They found that the linewidth associated with the gain was much less than the Doppler width, even though the experiment was in a Doppler broadened vapor of atomic rubidium. In fact, these authors found that the measured linewidth was nearly equal to the natural linewidths of the stationary atom.

In a recent paper, we have proposed a scheme for obtaining sub-Doppler resolution at one transition of an inhomogeneously broadened atomic system by employing a suitable control field at a different transition. We presented a theoretical formalism that enables the observed sub-Doppler linewidths to be *analytically* predicted [2]. We briefly review our theory here, by considering electromagnetically induced transparency in an inhomogeneously broadened system. Subsequently we will show that the model can be modified to predict the sub-Doppler gain widths observed in Ref. [1]. Starting from the density matrix equations for a Λ system (see Fig. 1 in Ref. [2]; $|1\rangle$ is the excited state and $|3\rangle$ is the ground state), one can obtain the relevant induced polarization, ρ_{13}, at the transition to which the probe is coupled, as

$$\rho_{13}=\frac{g(\Delta_1-\Delta_2)}{|G|^2-i(\gamma_1+\gamma_2-i\Delta_1)(\Delta_1-\Delta_2)}\,, \tag{1}$$

where $2G$ is the Rabi frequency of the control laser, Δ_2 is its detuning from its transition, Δ_1 is the detuning of the probe from its transition, and γ_1 and γ_2 are decay rates of the excited state to each of the two lower states (all rates in units of γ_1). The imaginary part of Eq. (1) is related to the absorption of the probe. For a stationary atom, this expression leads to the usual Autler-Townes doublet, with maxima located at $\Delta_1=(\Delta_2/2)\pm\sqrt{\Delta_2^2+4|G|^2}$.

For moving atoms, one can modify the detuning terms in Eq. (1) as $\Delta_1\rightarrow\Delta_1+kv$ and $\Delta_2\rightarrow\Delta_2+kv$, where k is the wave vector of the probe and v is the z component of the atom velocity. For copropagating control field and probe, the term $(\Delta_1-\Delta_2)$ in Eq. (1) will have no velocity dependence. By examining the pole structure of the resulting expression for the polarization, it is possible to predict the linewidths of the peaks in the doublet. These widths, β, are given by

$$\beta=\frac{\gamma_1+\gamma_2+D}{2}\left(1\mp\frac{\Delta_2}{\sqrt{\Delta_2^2+4|G|^2}}\right)\,, \tag{2}$$

where D is the Doppler width. From Eq. (2), one can see that while one peak of the doublet is broadened, the other peak is *narrowed* by the same amount. Equation (2) shows that even in an inhomogeneously broadened medium, it is possible to obtain an analytic expression for the linewidths. It is also clear from Eq. (2) that the amount of narrowing, $1-\Delta_2/\sqrt{\Delta_2^2+4|G|^2}$, depends only on the Rabi frequency of the control field, and its detuning. Thus, to observe sub-Doppler widths, it is essential that the control field be detuned from resonance. It is precisely under this condition that Zhu and Lin observe a sub-Doppler width.

In the experiments of Ref. [1], the authors study amplification without inversion by utilizing an incoherent pump at the transition on which the probe is acting. Thus, to make an accurate comparison with the experiments requires a modification of our formalism to include the effects due to the incoherent pump, and the bandwidths of the control and probe lasers. As shown in Ref. [3], the bandwidth effects can be incorporated if the laser fluctuations are modeled by a phase-diffusion process. Following an analysis similar to that in Ref. [2], it can be shown that the probe response is now given by

$$\rho_{13}=\frac{gG\rho_{21}^0+ig(\delta-i\Delta_1+i\Delta_2)[1-(\gamma_2/q+3)\rho_{11}^0]}{|G|^2+(\mu-i\Delta_1)(\delta-i\Delta_1+i\Delta_2)}\,, \tag{3}$$

where ρ_{21}^0 and ρ_{11}^0 are the zeroth-order contributions of the relevant density matrix elements, and are given explicitly by

$$\rho_{21}^0=\frac{-iG\gamma_2(\alpha-i\Delta_2)}{\alpha|G|^2}\,\rho_{11}^0$$

and

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$$\rho_{11}^{0}=\frac{\alpha|G|^2\Lambda}{\Lambda\gamma_2(\alpha^2+\Delta_2^2)+\alpha|G|^2(\gamma_1+3\Lambda)}\ .$$

The constants in Eq. (3) are $\delta=\Gamma_c+\Gamma_p+\Lambda$, $\mu=\gamma_1+\gamma_2+\Gamma_p+2\Lambda$, $q=\alpha|G|^2/(\alpha^2+\Delta_2^2)$, and $\alpha=\gamma_1+\gamma_2+\Gamma_c+\Lambda$, where 2Λ is the incoherent pumping rate, and Γ_p and Γ_c are the bandwidths of the probe and control lasers, respectively.

Now the width (β) of the narrowed Autler-Townes peak, at $\Delta_1=\Delta_2/2+\sqrt{\Delta_2^2+4|G|^2}$, is given by the sum of two terms (one that is independent of the bandwidth of the control laser and the other that depends on this bandwidth),

$$\beta=\frac{(\mu+D)}{2}\left(1-\frac{\Delta_2}{\sqrt{\Delta_2^2+4|G|^2}}\right)+\frac{\delta}{2}\left(1+\frac{\Delta_2}{\sqrt{\Delta_2^2+4|G|^2}}\right)\ . \tag{4}$$

Equation (4), which is the central result of this comment, is a general expression for the linewidth of the observed gain feature and includes the contributions of the incoherent pump, the laser bandwidths, the Doppler broadening, and the atomic relaxation rates.

We now make a direct comparison between the predictions of Eq. (4) and the experimental observations reported in Ref. [1]. For the incoherent pumping rate, we utilize the information in Ref. [1] that the bandwidth of the pump, on resonance, is 30 MHz. From this we estimate that Λ is approximately 1 MHz. Thus, for the experimental parameters of control field Rabi frequency equal to 120 MHz, and detuning of 1.4 GHz, the first part of Eq. (4) predicts a width of less than 2 MHz. Thus, at large detunings of the control field, the major source of the observed linewidth in the experiments is the bandwidth of the control and probe lasers, i.e., the contribution from the second half of Eq. (4). Note that Eq. (4) implies that though one can reduce the Doppler width through intensity and frequency of the control field, there is no quenching of the laser noise linewidth, i.e., the contribution from δ.

A second feature noted in the experiments (see Fig. 6 in Ref. [1]) was that as the detuning of the pump is increased, the linewidth decreases and approaches the natural width. This trend is also evident from Eq. (4).

Thus the sub-Doppler widths observed by Zhu and Lin [1] are readily explained by the theoretical formalism developed here, and in Ref. [2].

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