

## Ramsey spectroscopy with nonclassical light sources

Girish S. Agarwal\*

Physical Research Laboratory, Navrangpura, Ahmedabad 380 009, India

Marlan O. Scully

Department of Physics, Texas A &amp; M University, College Station, Texas 77843

and Max-Planck Institut für Quantenoptik, Garching, Germany

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A first-principle calculation of the signal and the noise for Ramsey spectroscopy with nonclassical light sources is given. It is found that the signal-to-noise ratio can be enhanced by using fields with sub-Poissonian statistics.

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The Ramsey spectroscopy [1] is a very useful technique yielding high resolution, and has been extensively used in many different contexts [2,3]. The signal in such a setup has been calculated and interpreted in terms of the interference between two quantum-mechanical pathways [4]. In view of the current interest in doing spectroscopy [5] with squeezed light [6] and other types of nonclassical radiation [7] it is desirable to examine the use of nonclassical light in Ramsey spectroscopy. In particular it would be interesting to find out if the signal-to-noise ratio can be improved using nonclassical sources. The noise arises from the statistics of the atomic beam, statistics of the field used to excite the atoms, and intrinsic quantum noise [8]. In this paper we calculate the signal-to-noise ratio for the standard two-field Ramsey method. We also discuss how the use of sub-Poissonian light can lead to an improvement in the signal-to-noise ratio.

The general two-field setup is schematically illustrated in Fig. 1. One monitors the excited-state population at the end of the second interaction region. Let  $p_e$  be the probability of finding the atom in the excited state at  $2\tau+T$ . The signal will then be

$$S = \sum_{i=1}^N p_e^i = Np_e, \quad (1)$$

where the summation is over all the atoms, and we have assumed all atoms are equivalent so the sum just yields a factor of  $N$ . Note that in the spin language (1) can be expressed as

$$S = \left\langle \sum_i (S_i^Z + \frac{1}{2}) \right\rangle. \quad (2)$$

Let us define a detection operator  $D$  as  $\sum_i (S_i^Z + \frac{1}{2})$ . The fluctuation in the signal can then be obtained from

$$\begin{aligned} (\Delta S)^2 &= \langle D^2 \rangle - \langle D \rangle^2 \\ &= \sum_{ij} \langle (S_i^Z + \frac{1}{2})(S_j^Z + \frac{1}{2}) \rangle - \left( \sum_i \langle S_i^Z + \frac{1}{2} \rangle \right)^2 \\ &= \sum_{ij} (\langle S_i^Z S_j^Z \rangle - \langle S_i^Z \rangle \langle S_j^Z \rangle). \end{aligned} \quad (3)$$

We assume that at  $t=0$ , all atoms are uncorrelated. The excitation by an external field will not change the correlation characteristics, and therefore

$$\langle S_i^Z S_j^Z \rangle = \langle S_i^Z \rangle \langle S_j^Z \rangle, \quad i \neq j. \quad (4)$$

On combining (4) with (3) and on using  $\langle (S_i^Z)^2 \rangle = \frac{1}{4}$  we obtain

$$(\Delta S)^2 = \frac{N}{4} - N \langle S_i^Z \rangle^2 = \frac{N}{4} - N(p_e - \frac{1}{2})^2 = Np_e(1 - p_e). \quad (5)$$

The signal-to-noise ratio then becomes

$$\frac{S}{N} = \frac{S}{\Delta S} = \frac{Np_e}{\sqrt{Np_e(1 - p_e)}} = \sqrt{N} \left( \frac{p_e}{(1 - p_e)} \right)^{1/2}. \quad (6)$$

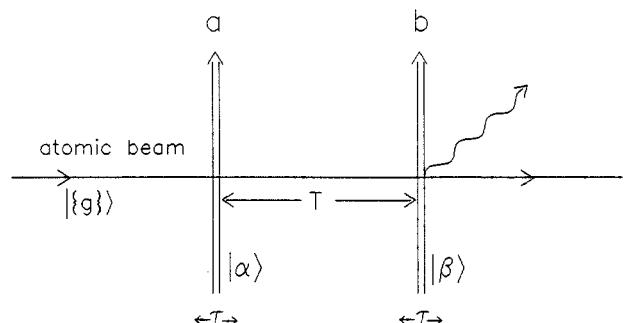


FIG. 1. Schematic illustration of the two-field interferometer. The fields  $a$  and  $b$  could also represent two cavities. The states  $|\alpha, \beta\rangle$  are used only as an intermediate step in the calculation.

\*Electronic address: gsa@prl.ernet.in

It should be borne in mind that  $p_e$  depends on the strength of the fields, the time separation between the two regions, and the atomic detuning factor. Note that (6) is derived under the following conditions: (i) the external field is treated as a semiclassical field; (ii) the initial state of the atoms is an uncorrelated state. It is interesting to observe that noise limits similar to (6) have been obtained for atomic interferometers [8].

Next we generalize these results by treating the excitation fields quantum mechanically. The quantum-mechanical generalization is needed for several reasons: (i) there are proposals in the literature where two micromaser cavities [9] provide two different regions of interaction, (ii) the use of nonclassical fields should improve upon the signal-to-noise ratio as given by (6), and (iii) the quantum treatment will enable us to choose a kind of nonclassical field that can enhance the resolution of the signal.

We start with the quantum-mechanical derivation of the signal. Let the fields in the two regions be denoted by the annihilation operators  $a$  and  $b$ , respectively. The interaction Hamiltonian in the interaction picture can be written as

$$\begin{aligned} H_1(t) = & \sum_i \hbar (g_i S_i^+ a e^{i\Delta t} + \text{H.c.}) \theta(\tau - t) \\ & + \sum_i \hbar (G_i S_i^+ b e^{i\Delta t} + \text{H.c.}) \theta(t - T - \tau) \\ & \times \theta(T + 2\tau - t), \quad \Delta = \omega_O - \omega_I. \end{aligned} \quad (7)$$

The  $\theta$ -step functions in (7) give the regions of time where the fields are acting and  $g_i, G_i$  give the coupling constants in the two zones. The detuning between the field and the atom is represented by  $\Delta$ . The initial state of the atomic system is denoted by  $|\{g\}\rangle$ , i.e., all the atoms are in the ground state. We keep the initial state  $|\alpha, \beta\rangle$  of the two quantized fields arbitrary. We will keep the analysis simple by assuming that  $\tau$  is small so that one can use perturbation theory. The wave function at time  $t$ , to the lowest order, can be written as

$$|\psi(t)\rangle \approx |\{g\}\rangle |\alpha, \beta\rangle - \frac{i}{\hbar} \int_0^t H_1(t') dt' |\{g\}\rangle |\alpha, \beta\rangle. \quad (8)$$

The final quantity of interest is the excitation probability, and thus we need the projection of  $|\psi(t)\rangle$  onto a state in which at least one atom is excited. Let  $|e_i\rangle$  be the atomic state in which the  $i$ th atom is excited. Using (8) in (7) and on simplification we obtain

$$\begin{aligned} \langle e_i | \psi(t) \rangle = & -i g_i a \frac{(e^{i\Delta\tau-1})}{i\Delta} |\alpha, \beta\rangle \\ & - i G_i b e^{i\Delta(T+\tau)} \frac{(e^{i\Delta\tau-1})}{i\Delta} |\alpha, \beta\rangle \\ \cong & -i f(\tau) [g_i a + G_i b e^{i\Delta T}] |\alpha, \beta\rangle, \end{aligned} \quad (9)$$

where

$$f(\tau) = \frac{e^{i\Delta\tau} - 1}{i\Delta}, \quad T \gg \tau; \quad f(\tau) \rightarrow 1 \quad \text{if } \Delta\tau \ll 1. \quad (10)$$

The probability of finding the  $i$ th atom in the excited state is obtained from

$$p_e = \text{Tr}_f \langle e_i | \psi(t) \rangle \langle \psi(t) | e_i \rangle, \quad (11)$$

where  $\text{Tr}_f$  denotes trace over the field variables. On using (9) in (11) we find the result

$$p_e = |f(\tau)|^2 \langle (g_i^* a^+ + G_i^* b^+ e^{-i\Delta T}) (g_i a + G_i b e^{i\Delta T}) \rangle \quad (12)$$

$$\begin{aligned} = & |f(\tau)|^2 \{ \langle a^+ a \rangle |g_i|^2 + |G_i|^2 \langle b^+ b \rangle \\ & + (g_i^* G_i e^{i\Delta T} \langle a^+ b \rangle + \text{c.c.}) \}. \end{aligned} \quad (13)$$

The interference term will survive only if

$$\langle a^+ b \rangle \neq 0. \quad (14)$$

If the two fields are derived from the same source, and if  $g_i = G_i$ , then

$$p_e = |f(\tau)|^2 \langle a^+ a \rangle |g_i|^2 |(1 + e^{i\Delta T})|^2. \quad (15)$$

The last factor in (15) contains the well-known interference fringe pattern [1,2].

We next examine the fluctuations in the signal. As discussed earlier, the detection operator is  $D = \sum_i (S_i^Z + \frac{1}{2})$  and the calculation of fluctuations will require the calculation of finding  $i$ th and  $j$ th atoms excited at the same time. It should be noted that even though the atomic state at  $t=0$  is an uncorrelated state, it does not remain so because of the quantized treatment of the field and because all atoms see the same field. Let  $|e_i, e_j\rangle$  be the state of the atom where the  $i$ th and  $j$ th atoms are in the excited state and the remaining atoms are in the ground state. We thus need to calculate

$$p_{ij} = \text{Tr}_f \langle e_i, e_j | \psi(t) \rangle \langle \psi(t) | e_i, e_j \rangle. \quad (16)$$

The fluctuation in the signal [cf. (3)] will be given by

$$\begin{aligned} (\Delta S)^2 = & \sum_{ij} \langle (S_i^Z + \frac{1}{2}) (S_j^Z + \frac{1}{2}) \rangle - N^2 p_e^2 \\ = & N p_e - N^2 p_e^2 + \sum_{i \neq j} p_{ij}, \end{aligned} \quad (17)$$

where  $p_e$  is given by (13).

We next consider the evaluation of the quantity  $p_{ij}$ . The two atoms can be found simultaneously in the excited state via four different pathways: (i) both atoms get excited in the first zone, (ii) both atoms get excited in the second zone, (iii) the  $i$ th atom gets excited in the first zone and the  $j$ th atom gets excited in the second zone, and (iv) pathway (iii) with  $i$  and  $j$  interchanged. To obtain  $p_{ij}$  we expand the wave function to second order in the perturbation,

$$|\psi(t)\rangle \approx |\{g\}\rangle |\alpha, \beta\rangle - \frac{i}{\hbar} \int_0^t H_1(t') dt' |\{g\}\rangle |\alpha, \beta\rangle + \left( \frac{-i}{\hbar} \right)^2 \int_0^t dt' \int_0^{t'} dt'' H_1(t') H_1(t'') |\{g\}\rangle |\alpha, \beta\rangle. \quad (18)$$

We can now write the contributions of the four different pathways as

$$\langle e_i e_j | \psi(t) \rangle = I_1 + I_2 + I_3 + I_4. \quad (19)$$

Here

$$I_1 = \left( -\frac{i}{\hbar} \right)^2 \int_0^\tau dt' \int_0^{t'} dt'' \langle e_i e_j | H_1(t') H_1(t'') | \{g\} \rangle | \alpha, \beta \rangle, \quad (20)$$

which on substituting (7) reduces to ( $\Delta\tau \ll 1$ )

$$I_1 = -\tau^2 g_i g_j a^2 | \alpha, \beta \rangle. \quad (21)$$

Similarly  $I_2$  is found to be

$$I_2 = -\tau^2 e^{2i\Delta T} G_i G_j b^2 | \alpha, \beta \rangle. \quad (22)$$

The amplitude corresponding to excitation of the  $i$ th atom in zone 1 and the  $j$ th atom in zone 2 will be

$$I_3 = \left\langle e_i e_j \left| -\frac{i}{\hbar} \right|^2 \int_0^\tau dt_1 H_1(t_1) \times \int_{T+\tau}^{T+2\tau} H_1(t_2) dt_2 | \{g\} \right\rangle | \alpha, \beta \rangle. \quad (23)$$

This involves nonoverlapping time intervals. The final result is

$$I_3 = -\tau^2 e^{i\Delta T} g_i G_j a b | \alpha, \beta \rangle. \quad (24)$$

Similarly one can show that

$$I_4 = -\tau^2 e^{i\Delta T} G_i g_j a b | \alpha, \beta \rangle. \quad (25)$$

On combining (19)–(25) we obtain

$$\begin{aligned} \langle e_i, e_j | \psi(t) \rangle = & -\tau^2 (g_i g_j a^2 + G_i G_j e^{2i\Delta T} b^2 + e^{i\Delta T} g_i G_j a b \\ & + e^{i\Delta T} G_i g_j a b) | \alpha, \beta \rangle. \end{aligned} \quad (26)$$

On using (26) in (16) we get

$$p_{ij} = \tau^4 \langle A_i^+ A_j^+ A_i A_j \rangle, \quad (27)$$

where

$$A_i = (g_i a + G_i b e^{i\Delta T}). \quad (28)$$

The fluctuation in the signal (17) can now be expressed as

$$(\Delta S)^2 = N p_e - N^2 p_e^2 + \sum_{i \neq j} \tau^4 \langle A_i^+ A_j^+ A_i A_j \rangle. \quad (29)$$

This is our final formula for the fluctuation of the signal. We examine several special cases. Let us set all coupling constants the same, i.e.,  $g_i = G_i = g$ . We further assume that the fields in the two regions are derived from the same source. Then (29) simplifies to

$$(\Delta S)^2 = N p_e - N^2 p_e^2 + \tau^4 |1 + e^{i\Delta T}|^2 |g|^4 \langle a^{+2} a^2 \rangle N(N-1). \quad (30)$$

On using (15), Eq. (30) can be written in an instructive form,

$$(\Delta S)^2 = N p_e - N^2 p_e^2 + p_e^2 N(N-1) \frac{\langle a^{+2} a^2 \rangle}{\langle a^+ a \rangle^2}. \quad (31)$$

The quantum statistics [10] enter through the last factor in (31). For a *coherent* field  $\langle a^{+2} a^2 \rangle = \langle a^+ a \rangle^2$  and then (31) reduces to the result obtained earlier [Eq. (5)],

$$(\Delta S_e)^2 = N p_e (1 - p_e). \quad (32)$$

On introducing the  $Q$  parameter [7] of the field

$$Q = \frac{\langle a^{+2} a^2 \rangle - \langle a^+ a \rangle^2}{\langle a^+ a \rangle}, \quad (33)$$

the noise (31) can be expressed as

$$(\Delta S)^2 = (\Delta S_e)^2 + \frac{Q}{\langle a^+ a \rangle} p_e^2 N(N-1). \quad (34)$$

For a *sub-Poissonian* field  $Q$  is negative and hence there would be *noise reduction*. This reduction will be most significant if  $N p_e \sim \langle a^+ a \rangle$ ; whence  $(\Delta S)^2 \sim (\Delta S_e)^2 (1 + Q)$ .

Note also that by considering the incoming atomic beam in a squeezed state [11] rather than in the ground state it is possible to improve the signal-to-noise ratio. Since this issue has already been discussed in the literature [12,13], we do not pursue it further.

In conclusion we have shown how the use of a nonclassical light source can be useful in the context of Ramsey spectroscopy. Finally the analysis of this paper can be generalized to obtain a signal-to-noise ratio to higher order in perturbation theory. One can even obtain an exact result; however, due to its complexity we do not pursue it here.

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