Lasing without inversion in the absence of a coherent coupling field

Gautam Vemuri,* K. V. Vasavada, and G. S. Agarwal†
Department of Physics, Indiana University—Purdue University at Indianapolis, 402 North Blackford Street, Indianapolis, Indiana 46202-3273

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We analyze the inversionless gain in a three-level ladder system by replacing the usual coherent coupling field with an incoherent field. Surprisingly, it is found that one can obtain inversionless amplification of a weak probe even in the absence of a coherent field in the model. We conclude that gain is determined by the ensemble average of the product of the two-photon coherence and the "effective Rabi frequency" of the field. Thus, even though the incoherent pump reduces the two-photon coherence, gain can be restored by choosing sufficiently high strengths of the incoherent field.

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Most models of lasing without inversion (LWI) utilize a coherent pump to prepare the atom-field system, such that a weak probe can be amplified on one of the transitions, even in the absence of population inversion on that transition [1–5]. The gain mechanism is explained on the basis of coherence induced between two atomic levels by the coherent field, such that the inversionless gain is predominantly determined by the two-photon coherence [6]. The noise properties of various laser fields in a LWI configuration have not received much attention, except for a few recent works which suggested that replacing the incoherent pump in traditional LWI schemes by a spectrally colored pump can provide significant gain enhancements [7]. Scully and co-workers [8] investigated the consequences of relative phase fluctuations between two coupling fields in a double-lambda LWI model and concluded that any phase fluctuations in the coupling fields will reduce the available gain. Gong and Xu [9] reported a study where the coherent coupling field was replaced by a phase-diffusing field and their conclusion was similar to that of Scully and co-workers. A phase-diffusing field can be viewed as a partially coherent field, especially if its bandwidth is comparable to the atomic widths. It is therefore not surprising that these authors found gain even when the coherent field was replaced by a stochastic field.

In this paper we show that the coherence effects that lead to LWI are preserved even in the absence of any coherent coupling field in the model. Specifically, we show that inversionless amplification can be obtained even if the coherent coupling field is replaced by a chaotic field, and elucidate the mechanism behind this unexpected result. To be specific, consider the three-level ladder system of Fig. 1, with ground state $|3\rangle$ and two excited states $|1\rangle$ and $|2\rangle$. The $|1\rangle \leftrightarrow |3\rangle$ transition is dipole forbidden, while the $|1\rangle \leftrightarrow |2\rangle$ transition is the lasing transition. This upper transition, at frequency ω_{12}, has a radiative decay rate of $2\gamma_1$ and the lower transition, at frequency ω_{23} has a width of $2\gamma_2$. In most models, the $|2\rangle \leftrightarrow |3\rangle$ transition is coupled by a coherent pump (at frequency ω_2), an incoherent pump is used to transfer population from $|2\rangle$ to $|1\rangle$, and a weak probe (at frequency ω_1) is scanned across this upper transition and its absorption or gain monitored. The strong coherent field leads to Stark splitting of the lower atomic levels, creating dressed states, which are linear combinations of the bare states $|2\rangle$ and $|3\rangle$. The Stark split levels have an energy separation equal to the Rabi frequency of the coherent pump, and when the probe is tuned to one of these dressed levels it experiences gain.

In most reports on LWI to date, the coherent field has been considered an essential component for obtaining gain. If this coherent pump is replaced by a phase-diffusing field, there would be a concomittant reduction in gain, which can be easily understood in terms of reduction in the two-photon coherence [8,9]. However, one expects a dramatically different result if a chaotic field replaces the coherent pump. It is well known that in the presence of a chaotic field, coherence (or Rabi) effects are not visible, e.g., the sidebands in the Mollow triplet vanish for a chaotic field because the Rabi frequency itself is fluctuating and on average its effects are not revealed in the observable [10]. One might thus conclude that there should be no gain for chaotic coupling fields, since LWI is based on coherence phenomena.

In this paper, we address the consequences of utilizing an incoherent coupling field in LWI, by comparing the gain when the coherent field is replaced by either a phase-diffusing field or a chaotic field. The former is known to accurately reproduce the behavior of intensity stabilized, single-mode lasers operating well above threshold, while the

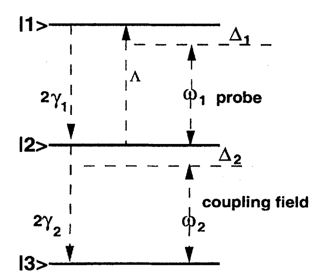

FIG. 1. Schematic representation of a three-level ladder system with ground state $|3\rangle$ and two excited states $|1\rangle$ and $|2\rangle$. The spontaneous decay rates from $|1\rangle$ to $|2\rangle$ and $|2\rangle$ to $|3\rangle$ are $2\gamma_1$ and $2\gamma_2$, respectively. ω_1 is the probe frequency, ω_2 is the central frequency of the coupling field, and Λ is the incoherent pump rate.

*Electronic address: gvemuri@indyvax.iupui.edu
†Permanent address: Physical Research Laboratory, Ahmedabad 380009, India.

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latter can be used to describe the output of multimode or pulsed lasers. The most striking result we find is that it is possible to realize gain in LWI configurations, even without any coherent pump in the model. Further, if the incoherent field has a bandwidth of the order of the atomic radiative width, the gain one obtains is comparable to that with a coherent pump. This is important from an experimental point of view, since it is much easier to produce a spectrally colored pump than a purely coherent pump.

The semiclassical Hamiltonian for the atomic system of Fig. 1, with a coherent coupling field, can be used to derive the time evolution of the relevant density matrix equations, which are

$$\dot{\rho}_{11}=-2\gamma_1\rho_{11}+ig\rho_{21}-ig^*\rho_{12}-2\Lambda(\rho_{11}-\rho_{22}) \;, \quad (1a)$$

$$\dot{\rho}_{12}=-(\gamma_1+\gamma_2+i\Delta_1)\rho_{12}+ig(\rho_{22}-\rho_{11})-iG_2^*\rho_{13} -2\Lambda\rho_{12}, \quad (1b)$$

$$\dot{\rho}_{13}=-(\gamma_1+i\Delta_1+i\Delta_2)\rho_{13}+ig\rho_{23}-iG_2\rho_{12}-\Lambda\rho_{13}, \quad (1c)$$

$$\dot{\rho}_{22}=2\gamma_1\rho_{11}-2\gamma_2\rho_{22}-ig\rho_{21}+ig^*\rho_{12}+iG_2\rho_{32}-iG_2^*\rho_{23} +2\Lambda(\rho_{11}-\rho_{22}) \;, \quad (1d)$$

$$\dot{\rho}_{23}=-(\gamma_2+i\Delta_2)\rho_{23}+ig^*\rho_{13}+iG_2(\rho_{33}-\rho_{22})-\Lambda\rho_{23}, \quad (1e)$$

$$\dot{\rho}_{33}=2\gamma_2\rho_{22}-iG_2\rho_{32}+iG_2^*\rho_{23}, \quad (1f)$$

where G_2 is the Rabi frequency of the coherent pump, Δ_2 $(=\omega_{23}-\omega_2)$ is its detuning from the lower transition, Δ_1 $(=\omega_{12}-\omega_1)$ is detuning of probe from upper transition, Λ is the incoherent pump rate, and g is the Rabi frequency of the probe.

The gain G is given by calculation of the density matrix element ρ_{12}, and explicitly is

$$G=-\text{Im}\left[\frac{\rho_{12}\gamma_1}{g}\right] \;, \quad (2)$$

where G is in units of the weak-field resonant absorptivity. We use the same parameters as in Ref. [7], which are $G_2=14.3\gamma_1$, $\Delta_2=25.1\gamma_1$, $\gamma_2=5.45\gamma_1$, $\Lambda=1.7\gamma_1$, and $g=0.2\gamma_1$, and for which the maximum gain one obtains is approximately 0.0042.

We now apply a stochastic coupling field on the lower transition, and so replace G_2 by $G_c(t)e^{i\phi(t)}$, where the time-dependent amplitude or phase is a Gaussian-Markovian random process. For chaotic fields $[\phi(t)=0]$, $G_c(t)$ has zero mean and an autocorrelation function of the form

$$\langle G_c(t)G_c^*(t')\rangle = D\Gamma e^{-\Gamma|t-t'|} \;. \quad (3)$$

In Eq. (3), D is the strength of the noise, Γ is its bandwidth, and the product $D\Gamma$ can be identified physically with the intensity of the field. A chaotic field as defined here has a Lorentzian spectral profile with a full width at half maximum (FWHM) of 2Γ. If the coupling field is taken to be a phase diffusing field, then we define Ω $[=G_c(t)]$ as its Rabi frequency [11], and $\mu(t)$ as the frequency fluctuations

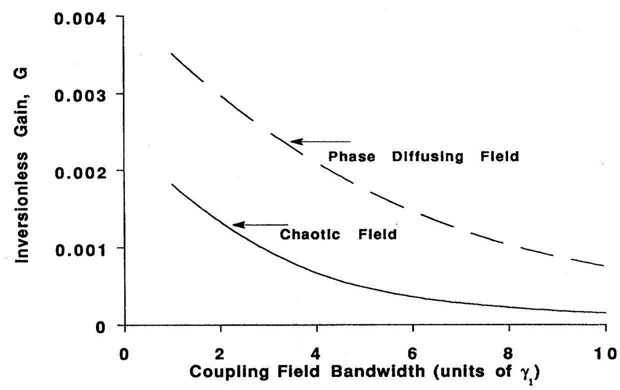

FIG. 2. Inversionless gain as a function of the coupling field bandwidth. For chaotic fields, $D\Gamma=(14.3\gamma_1)^2$, and for phase diffusing fields, $\beta=100\gamma_1$. Other parameters as in text.

$[d\phi/dt=\mu(t)]$, where the ensemble average $\langle\mu(t)\rangle=0$ and $\mu(t)$ has an autocorrelation function of the form

$$\langle\mu(t)\mu(t')\rangle = b\beta e^{-\beta|t-t'|} \;. \quad (4)$$

Here b is the strength of the frequency noise, β is the bandwidth of the noise, and for $\beta \gg b$, Eq. (4) reduces to a δ-correlated function, with the resulting field having a Lorentzian spectral profile with a FWHM of $2b$.

Equation (1), modified to include a fluctuating coupling field, is solved using Monte Carlo methods [12], and we compare the gain for two situations—when the driving field has chaotic fluctuations and when it has phase fluctuations. In order to make a fair comparison, it is necessary that the fields in both models have identical Rabi frequencies, bandwidths, and band shapes. The chaotic field, as defined in Eq. (3), always has a Lorentzian line shape with a bandwidth equal to 2Γ, and the "effective Rabi frequency" is given by $\sqrt{D\Gamma}$. For the phase diffusion field, if we choose $\beta \gg b$, we ensure a Lorentzian field line shape with a bandwidth equal to $2b$, while the Rabi frequency is simply given by Ω. Thus, if we choose Γ equal to b, β much larger than b, and $\sqrt{D\Gamma}$ equal to Ω, the chaotic and phase-diffusing fields would have identical excitation strengths, bandwidths, and line shapes.

Figure 2 depicts the main result of this work, the maximum inversionless gain as a function of the coupling field bandwidth for chaotic and phase-diffusing fields. The gain shown is the ensemble average over several hundred independent iterations, each with a different set of random numbers, to reduce the errors due to small number statistics. It is quite clear that there are dramatic differences in the two cases, which become less pronounced as one gets to larger bandwidths. For a purely incoherent pump, the probe response is identical for the two models, and as expected, there is no gain. The most interesting regime is where the field bandwidth is comparable to the atom relaxation rate. This is the regime where most lasers operate, and such bandwidths are easy to achieve. When the coupling field bandwidth is equal to 1 (in units of γ_1), the gain due to a phase diffusion field ($G \sim 0.0035$) is not significantly different from that with a coherent pump ($G \sim 0.0042$). For the same field bandwidth, though the gain due to a chaotic field is less than that from a phase-diffusing field by a factor of 2 ($G \sim 0.002$), it is surprising that there is any gain at all. Since a chaotic field is expected to erase all coherence effects, the fact that one can realize gain even in the presence of a chaotic field is unexpected.

Figure 2 shows results for values of coupling field bandwidths starting at 1 (in units of γ_1). The Monte Carlo technique does not permit calculation of gain for zero-field bandwidths due to the nature of the numerical algorithm [12]. However, one can easily determine this gain analytically. For the phase-diffusion model, the limit of zero bandwidth is the coherent field, for which the gain is 0.0042. However, for the chaotic field, zero bandwidth does not correspond to a coherent field. Instead, the intensity of the field I has a probability distribution $P(I)$ that is given by

$$P(I)=\frac{1}{\langle I\rangle} e^{-I/\langle I\rangle} , \tag{5}$$

where $\langle I\rangle$ is the mean intensity. The gain is obtained by averaging ρ_{12} over the intensity distribution of Eq. (5), and we determine the gain, from such an averaging, to be approximately 0.0025. We emphasize here that while the gain for nonzero bandwidths has been calculated via the Monte Carlo method, the gain for zero bandwidths has been calculated by solving for ρ_{12} analytically (to first order in g) and then averaging it over the chaotic field probability distribution. We mention this to point out that the unexpected gain for chaotic fields is not a numerical artifact of the Monte Carlo procedure.

We now analyze the density-matrix equations to elucidate the source of this gain with chaotic fields. If we accept the notion that the two-photon coherence ρ_{13} determines gain, then we find that even for moderate bandwidths, this coherence is almost zero and hence cannot give rise to gain. However, a closer inspection of Eq. (1) [Eq. (1b) in particular] indicates that the dominant term in the density-matrix equations that produces gain is the $\langle G_c^*(t)\rho_{13}\rangle$ term, and not $\langle\rho_{13}\rangle$ by itself (angular brackets denote ensemble averages). If a coherent coupling field is applied, this product can be separated into the product of the Rabi frequency and the two-photon coherence. However, if an incoherent field is applied, it is the ensemble average of the above product which must be examined. Thus, even though $\langle\rho_{13}\rangle$ itself is very small when the field is incoherent, if G_c is chosen large

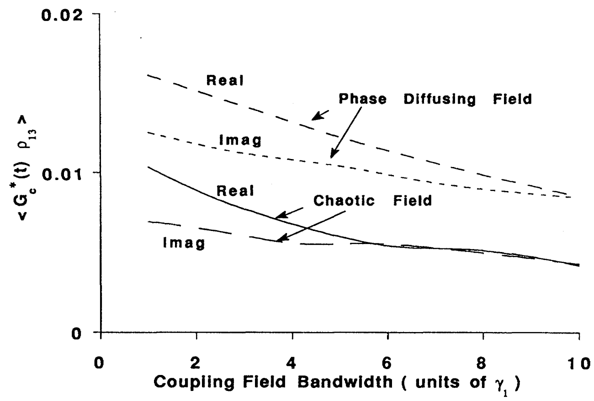

FIG. 3. Real and imaginary part of $\langle G_c^*(t)\rho_{13}\rangle$ as a function of coupling field bandwidth. The noise parameters are the same as in Fig. 2, and other parameters as in text.

enough such that the product $\langle G_c^*(t)\rho_{13}\rangle$ is large, one can still realize gain. In Fig. 3 we show the real and imaginary parts of $\langle G_c^*(t)\rho_{13}\rangle$, which have significant nonzero values, even though $\langle\rho_{13}\rangle$ by itself is almost zero. Clearly then, it is not ρ_{13} which is the important term, but rather the product $G_c^*(t)\rho_{13}$ which provides gain.

In summary, we have shown that it is possible to preserve atomic coherence effects even in the absence of a coherent coupling field. This point has been demonstrated by showing that one can realize inversionless gain by using incoherent coupling fields. The origin of this coherence preservation has been traced to the fact that it is not just the two-photon coherence which is responsible for gain in LWI, but rather the ensemble average of the product of the two-photon coherence and effective Rabi frequency of the incoherent coupling field. This implies that one can use a stochastic field to realize LWI, as long as reduction in the two-photon coherence is compensated by an increase in strength of the field. As expected, we do find that for a given field bandwidth, band shape, and Rabi frequency, the phase-diffusing field is more effective in producing gain than the chaotic field. The predictions of this work can be experimentally tested via laser noise engineering techniques developed by Elliott and coworkers [11,13].

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