# Einstein-Podolsky-Rosen paradox for continuous variables using radiation fields in the pair-coherent state

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We show how the Einstein-Podolsky-Rosen paradox for continuous variables can be tested using the quadrature amplitudes of a radiation field in the pair-coherent state. Correlated pairs of photons are produced by two competing nonlinear processes—four-wave mixing and two-photon absorption.

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## I. INTRODUCTION

Einstein-Podolsky-Rosen [1] (EPR) argued that a quantum-mechanical description of a physical system is incomplete. Their argument is based on three assumptions or premises. One is realism, the doctrine that regularities in the observed phenomena are caused by some physical reality whose existence is independent of human observers, i.e., "if without in any way disturbing a system we can predict with certainty (with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." The second assumption says that inductive inference is a valid mode of reasoning. The third premise states that no influence of any kind can propagate faster than the speed of light, i.e., there is no action at a distance, which means that when a measurement is performed, it is done in such a way that measurement on the first system does not disturb the second system.

The system which EPR considered consists of two spatially separated particles. These particles show a high degree of correlation between their positions as well as their momenta. This implies that by measuring the position of particle 1, one can predict with certainty what value of position will particle 2 possess in case it is measured immediately. According to the third premise, the prediction for the position of particle 2 is made without in any way disturbing the particle. This led EPR to infer that the position of particle 2 has a definite preassigned value. Since the momenta of the two particles are also correlated by an analogous argument, one may predict the value of the momentum of particle 2 by measuring the momentum of particle 1. Thus one can infer that the momentum of particle 2 has a definite preassigned value. Thus, to quote EPR, "in accordance with our criterion of reality, in the first case we must consider the quantity **Q** (i.e., position) as being an element of reality; in the second case the quantity $P$ (i.e., momentum) is an element of reality. But, as we have seen, both wave functions $\psi_k$ and $\phi_r$ belong to the same reality."

The EPR argument deals with continuous variables. Bohm [2] later presented spin versions of the EPR paradox. Bell [3] derived a set of inequalities which should be obeyed if the assumption of local realism holds. Much of the experimental and theoretical work has been done with quantum correlations between spins or photons. Reid [4] in an important paper pointed out that EPR's original example involving the positions and momenta of two correlated particles can be realized by using the quadratures of the fields produced in the down-conversion process in a cavity. Here an input photon is converted into correlated signal ($a$ mode) nd idler ($b$ mode) photons. Initially there are no photons in either the signal or the idler mode. For the down-conversion process, there are strong correlations between the signal and the idler quadratures $(X_s, Y_s)$ and $(X_i, Y_i)$. The strong correlation (the degree of which depends on the cavity parameters, pump amplitudes, etc.), for example, can be used to infer the quadrature $X_s$ from knowledge of the quadrature $X_i$. Reid and co-workers discussed in quantitative terms the EPR paradox for continuous variables based on local realism. This is different from the work of Bohm, Bell, and others [5–8] which concentrates on discrete variables. Ou *et al.* [9] verified that there is indeed violation of the inequality which one would establish on the basis of local realism. It would be useful to find other examples where the EPR hypothesis of local realism for continuous variables can be tested. This is the purpose of this paper. Here we consider a pair of correlated photons produced by the competing nonlinear processes such as four-wave mixing and two-photon absorption. The outline of the paper is as follows: In Sec. II we introduce the characteristics of the system which is to be used for the study of the EPR paradox. Following Reid and Ou *et al.*, we present the mathematical criteria which can be used to test the EPR paradox. In Sec. III we derive explicit results for the violations of the predictions based on the EPR hypothesis of local realism. In Sec. IV we give numerical results. The results for the correlations in the pair-coherent state compare quite favorably with those produced in down conversion.

## II. EPR PARADOX IN TERMS OF CORRELATED QUADRATURE AMPLITUDES OF THE TWO-MODE RADIATION FIELD

Here we consider a situation where the roles of the position and momentum of the particles are played by the highly correlated quadrature amplitudes of the two-mode radiation field. Consider quantized radiation fields $\hat{E}_a$

 

and $\hat{E}_b$ of frequency $\omega_a$ and $\omega_b$, respectively. These fields can be written in terms of bosonic operators $\hat{a}$ and $\hat{b}$ as

$$\hat{E}_a = C[\hat{a}e^{-i\omega t} + \hat{a}^\dagger e^{i\omega t}] , \tag{2.1}$$

$$\hat{E}_b = C[\hat{b}e^{-i\omega t} + \hat{b}^\dagger e^{i\omega t}] , \tag{2.2}$$

where $C$, taken to be equal for both the modes, is a constant which has all the spatial factors. The quadrature amplitudes of the above fields are defined as

$$\hat{X}_\theta = \hat{a}e^{-i\theta} + \hat{a}^\dagger e^{i\theta} , \tag{2.3}$$

$$\hat{Y}_\phi = \hat{b}e^{-i\phi} + \hat{b}^\dagger e^{i\phi} . \tag{2.4}$$

The following special cases will be useful:

$$\begin{aligned} &\hat{X}_1 = \hat{X}_0 = \hat{a} + \hat{a}^\dagger , \\ &\hat{X}_2 = \hat{X}_{\pi/2} = (\hat{a} - \hat{a}^\dagger)/i , \end{aligned} \tag{2.5}$$

$$\begin{aligned} &\hat{Y}_1 = \hat{Y}_0 = \hat{b} + \hat{b}^\dagger , \\ &\hat{Y}_2 = \hat{Y}_{\pi/2} = (\hat{b} - \hat{b}^\dagger)/i . \end{aligned} \tag{2.6}$$

The commutation relations between $\hat{X}_1, \hat{X}_2$ and $\hat{Y}_1, \hat{Y}_2$ which follow from $[\hat{a}, \hat{a}^\dagger] = 1 = [\hat{b}, \hat{b}^\dagger]$ are

$$[\hat{X}_1, \hat{X}_2] = [\hat{Y}_1, \hat{Y}_2] = 2i . \tag{2.7}$$

This leads to the Heisenberg uncertainty principle

$$(\Delta\hat{X}_1)^2(\Delta\hat{X}_2)^2 \geq 1, \quad (\Delta\hat{Y}_1)^2(\Delta\hat{Y}_2)^2 \geq 1 . \tag{2.8}$$

Now we consider that the fields are prepared in such a way that there exists a strong correlation between quadrature amplitudes $\hat{X}_1, \hat{X}_2$ and $\hat{Y}_1, \hat{Y}_2$. The correlation coefficient $C_{\theta\phi}$ is defined as

$$C_{\theta\phi} = \frac{\langle \hat{X}_\theta \hat{Y}_\phi \rangle}{[\langle \hat{X}_\theta^2 \rangle \langle \hat{Y}_\phi^2 \rangle]^{1/2}} , \tag{2.9}$$

assuming that $\langle \hat{X}_\theta \rangle = \langle \hat{Y}_\phi \rangle = 0$. The correlation for some values of $\theta$ and $\phi$ is said to be perfect (100%) if $|C_{\theta\phi}| = 1$. Because of this correlation, the quadrature amplitude $\hat{X}_\theta$ can be inferred by measuring the corresponding amplitude $\hat{Y}_\phi$ or vice versa. In an ideal case, there should be perfect (100%) correlation between $\hat{X}_\theta$ and the corresponding $\hat{Y}_\phi$. In a realistic experimental situation, the correlation will not be perfect because of losses and finite detector efficiencies. We can only estimate $\hat{X}_\theta$ with certain accuracy. Thus the amplitudes $\hat{X}_\theta$ can be inferred by appropriate scale of the amplitudes $\hat{Y}_\phi$. Thus the estimated amplitudes $\hat{X}_1, \hat{X}_2$ are given by

$$\hat{X}_1^e = g_1 \hat{Y}_\phi , \tag{2.10}$$

$$\hat{X}_2^e = g_2 \hat{Y}_\phi . \tag{2.11}$$

This is fairly standard procedure, known as regression analysis [10] in statistical inference theory. Note that the commutation relations in (2.10) and (2.11) can be preserved only by adding the appropriate combinations of $\hat{X}$'s for, e.g., $\hat{X}_2^e$ in (2.10).

One can choose the scaling parameters $g_1$ and $g_2$ and the angle $\phi$ such that the amplitudes $\hat{X}_1, \hat{X}_2$ can be inferred to the highest possible accuracy. The deviation of the estimated amplitudes [as given by the Eqs. (2.10) and (2.11)] from the true amplitudes $\hat{X}_1, \hat{X}_2$ are determined by taking the difference $(\hat{X}_1 - \hat{X}_1^e)$ and $(\hat{X}_2 - \hat{X}_2^e)$, respectively. The average errors of the inferences are then given by

$$\begin{aligned} (\Delta_{\rm inf}\hat{X}_1)^2 &= \langle (\hat{X}_1 - \hat{X}_1^e)^2 \rangle \\ &= \langle (\hat{X}_1 - g_1 \hat{Y}_\phi)^2 \rangle , \end{aligned} \tag{2.12}$$

$$\begin{aligned} (\Delta_{\rm inf}\hat{X}_2)^2 &= \langle (\hat{X}_2 - \hat{X}_2^e)^2 \rangle \\ &= \langle (\hat{X}_2 - g_2 \hat{Y}_\phi)^2 \rangle . \end{aligned} \tag{2.13}$$

The values of $g_1, g_2$ are chosen by setting

$$\frac{\partial(\Delta_{\rm inf}\hat{X}_1)^2}{\partial g_1} = 0 = \frac{\partial(\Delta_{\rm inf}\hat{X}_2)^2}{\partial g_2} . \tag{2.14}$$

From Eqs. (2.12) and (2.13), one sees that

$$g_1 = \frac{\langle \hat{X}_1 \hat{Y}_\phi \rangle}{\langle \hat{Y}_\phi^2 \rangle} , \tag{2.15}$$

$$g_2 = \frac{\langle \hat{X}_2 \hat{Y}_\phi \rangle}{\langle \hat{Y}_\phi^2 \rangle} . \tag{2.16}$$

The value of the angle $\phi$ is decided by Eq. (2.9). It is the value for which the correlation coefficients $C_{1\phi}$ and $C_{2\phi}$ are maximum. Thus by the measurements of $g_1 \hat{Y}_\phi$ and $g_2 \hat{Y}_\phi$, the values of $\hat{X}_1$ and $\hat{X}_2$ are inferred with uncertainty $\Delta_{\rm inf}\hat{X}_1$ and $\Delta_{\rm inf}\hat{X}_2$, respectively. According to quantum mechanics, the quadrature amplitude operators $\hat{X}_1$ and $\hat{X}_2$ are noncommuting and hence they both cannot be specified simultaneously with accuracy greater than that allowed by the uncertainty relation given by Eq. (2.8). Now the EPR paradox occurs when [4,9]

$$(\Delta_{\rm inf}\hat{X}_1)^2 (\Delta_{\rm inf}\hat{X}_2)^2 < 1 . \tag{2.17}$$

Reid has provided an example of the EPR paradox by considering the correlations between the quadrature amplitudes of the signal and the idler modes of a nondegenerate parametric amplifier. The nondegenerate parametric amplifier can be modeled by an interaction Hamiltonian,

$$H_I = i\hbar\kappa(\hat{a}^\dagger \hat{b}^\dagger - \hat{a}\hat{b}) . \tag{2.18}$$

The term $\kappa$ (taken to be real) describes the nonlinear coupling coefficient proportional to the nonlinear susceptibility. The operators $\hat{a}$ ($\hat{a}^\dagger$) and $\hat{b}$ ($\hat{b}^\dagger$) are the boson annihilation (creation) operators for the signal and the idler mode, respectively. The evolution operator $\hat{U} = \exp(-iH_I t/\hbar)$ is found from Eq. (2.18) to be

$$\hat{U} = e^{\kappa t(\hat{a}^\dagger \hat{b}^\dagger - \hat{a}\hat{b})} . \tag{2.19}$$

Using the above equation, the quadrature amplitude $\hat{X}_\theta$ given by Eq. (2.5) after an interaction time $\tau = L/v$ with the medium is found to be

$$\hat{X}_1(L)=\hat{X}_1(0)\cosh r+\hat{Y}_1(0)\sinh r ,$$
$$\hat{X}_2(L)=\hat{X}_2(0)\cosh r-\hat{Y}_2(0)\sinh r ,$$
$$\hat{Y}_1(L)=\hat{Y}_1(0)\cosh r+\hat{X}_1(0)\sinh r ,$$
$$\hat{Y}_2(L)=\hat{Y}_2(0)\cosh r-\hat{X}_2(0)\sinh r , \quad (2.20)$$

where $r=\kappa t$. They calculate the correlation coefficient as given by Eq. (2.9) and show that the quadrature amplitudes $\hat{X}_1$ and $\hat{Y}_1$ and $\hat{X}_2$ and $-\hat{Y}_2$ are strongly correlated. They infer the signal amplitudes $\hat{X}_1$ and $\hat{X}_2$ by making measurements on $\hat{Y}_1$ and $-\hat{Y}_2$ of the idler amplitudes and calculate the error $\Delta_{\text{inf}}\hat{X}_1$ and $\Delta_{\text{inf}}\hat{X}_2$, in the inference. They found that

$$(\Delta_{\text{inf}}\hat{X}_1)^2(\Delta_{\text{inf}}\hat{X}_2)^2=\frac{1}{(\cosh 2r)^2} . \quad (2.21)$$

The EPR paradox occurs for the values of $r$ for which

$$(\Delta_{\text{inf}}\hat{X}_1)^2(\Delta_{\text{inf}}\hat{X}_2)^2<1 .$$

This has been experimentally verified by Ou *et al.* They observed in their experiment that

$$(\Delta_{\text{inf}}\hat{X}_1)^2(\Delta_{\text{inf}}\hat{X}_2)^2=0.70\pm0.01 .$$

## III. EPR PARADOX WITH A TWO-MODE FIELD IN A PAIR-COHERENT STATE

Here we consider a two-mode radiation field in the pair-coherent state and show that the conjugate quadrature amplitudes of both $\hat{a}$ and $\hat{b}$ modes are strongly correlated. The pair-coherent state is defined by the solution of the eigenvalue problem [11]

$$\hat{a}\hat{b}|\zeta\rangle=\zeta|\zeta\rangle ,$$
$$(\hat{a}^\dagger\hat{a}-\hat{b}^\dagger\hat{b})|\zeta\rangle=0 . \quad (3.1)$$

The two photons are either created together or annihilated together, leading to correlations between the two. The radiation fields in the pair-coherent state are shown to exhibit quantum features such as sub-Poissonian photon statistics, correlation in the photon number fluctuations in the two modes, squeezing and violations of quantum versions of Cauchy-Schwarz inequalities, etc. The solution to the eigenvalue problem (3.1) is

$$|\zeta\rangle=\sqrt{I_0(2\zeta)}\sum_{n=0}^{\infty}\frac{\zeta^n}{n!}|n,n\rangle , \quad (3.2)$$

where $I_n(x)$ is the modified Bessel function given by [12]

$$I_n(x)=\sum_{r=0}^{\infty}\frac{(x/2)^{n+2r}}{r!(n+r)!} . \quad (3.3)$$

We calculate the correlation coefficient as given by Eq. (2.9) for the radiation field in the pair coherent state.

$$C_{\theta\phi}=\frac{\langle\hat{X}_\theta\hat{Y}_\phi\rangle}{[\langle\hat{X}_\theta^2\rangle\langle\hat{Y}_\phi^2\rangle]^{1/2}} , \quad (3.4)$$

from Eqs. (3.1), (2.3), and (2.4) we see that

$$C_{\theta\phi}=\frac{\zeta[e^{-i(\theta+\phi)}+e^{+i(\theta+\phi)}]}{[(2\langle\hat{a}^\dagger\hat{a}\rangle+1)(2\langle\hat{b}^\dagger\hat{b}\rangle+1)]^{1/2}} . \quad (3.5)$$

Here we take $\zeta$ to be real. The average photon number in $a$ and $b$ modes is found to be [11]

$$\langle\hat{a}^\dagger\hat{a}\rangle=\langle\hat{b}^\dagger\hat{b}\rangle=\zeta\frac{I_1(2\zeta)}{I_0(2\zeta)} . \quad (3.6)$$

On using (3.6) in (3.5) the correlation coefficient $C_{\theta\phi}$ turns out to be

$$C_{\theta\phi}=\frac{2\zeta\cos(\theta+\phi)}{2\zeta[I_1(2\zeta)/I_0(2\zeta)]+1} . \quad (3.7)$$

To examine the degree of the correlation $C_{\theta\phi}$, we first consider the asymptotic limit for large $\zeta$. Using the expansion of Bessel function [12]

$$I_n(x)\simeq\frac{e^x}{\sqrt{2\pi x}}\left[1-\frac{(4n^2-1)}{8x}\right] , \quad (3.8)$$

Eq. (3.7) reduces to

$$C_{\theta\phi}=\cos(\theta+\phi)\left[1-\frac{1}{4\zeta}\right] . \quad (3.9)$$

This implies that for large $\zeta$, there exists perfect correlation for $\theta+\phi=0$, i.e., for $\theta=0$, $\phi=0$, perfect correlation exists between $\hat{X}_1$ and $\hat{Y}_1$; for $\theta=\pi/2$, $\phi=-\pi/2$, perfect correlation exists between $\hat{X}_2$ and $-\hat{Y}_2$. Thus, by performing measurements on $\hat{Y}_1$ and $\hat{Y}_2$, one can infer the values of $\hat{X}_1$ and $\hat{X}_2$. We calculate the scaling parameter $g_1$ and $g_2$ as given by Eqs. (2.15) and (2.16) which allows for the greatest accuracy in the determination of $\hat{X}_1$ and $\hat{X}_2$:

$$g_1=\frac{\langle\hat{X}_1\hat{X}_1\rangle}{\langle\hat{Y}_1^2\rangle}=\frac{2\zeta}{2[I_1(2\zeta)/I_0(2\zeta)]+1} \quad (3.10)$$

$$g_2=\frac{-\langle\hat{X}_2\hat{Y}_2\rangle}{\langle\hat{Y}_2^2\rangle}=\frac{-2\zeta}{2[I_1(2\zeta)/I_0(2\zeta)]+1} . \quad (3.11)$$

The error in inferring the values $\hat{X}_1$ and $\hat{X}_2$ as given by Eqs. (2.12) and (2.13) is then obtained by substituting the values of $g_1$ and $g_2$ as obtained above. The final result can be written as

$$(\Delta_{\text{inf}}\hat{X}_1)^2=\langle(\hat{X}_1-g_1\hat{Y}_1)^2\rangle =\langle\hat{X}_1^2\rangle-\frac{\langle\hat{X}_1\hat{Y}_1\rangle^2}{\langle\hat{Y}_1^2\rangle} =A-\frac{4\zeta^2}{A} , \quad (3.12)$$

$$(\Delta_{\text{inf}}\hat{X}_2)^2=\langle(\hat{X}_2-g_2\hat{Y}_2)^2\rangle =\langle\hat{X}_2^2\rangle-\frac{\langle\hat{X}_2\hat{Y}_2\rangle^2}{\langle\hat{Y}_1^2\rangle} =A-\frac{4\zeta^2}{A} , \quad (3.13)$$

where

$$A = 2\frac{I_1(2\zeta)}{I_0(2\zeta)}\zeta + 1 \ . \tag{3.14}$$

From Eqs. (3.12) and (3.13), we find that

$$(\Delta_{\rm inf}\hat{X}_1)^2(\Delta_{\rm inf}\hat{X}_2)^2 = A^2\left[1 - \frac{4\zeta^2}{A^2}\right] . \tag{3.15}$$

The EPR paradox occurs when

$$(\Delta_{\rm inf}\hat{X}_1)^2(\Delta_{\rm inf}\hat{X}_2)^2 < 1 \ , \tag{3.16}$$

i.e., when

$$A^2\left[1 - \frac{4\zeta^2}{A^2}\right] < 1 \ . \tag{3.17}$$

In Sec. IV we present the numerical results.

## IV. NUMERICAL RESULTS

Let $C$ stand for the correlation coefficient (3.7) with $\cos(\theta+\phi)=1$. In Fig. 1(a) we plot the quantity $(1-C^2)^2$ as a function of $\zeta$. This quantity decreases monotonically with $\zeta$ and tends to zero for large values of the parameter $\zeta$. In Fig. 1(b) we plot the quantity $\langle \hat{X}_1^2\rangle^2 = A^2$ as a function of $\zeta$. The parameter $A^2$ is related to the mean photon number which increases with $\zeta$. In fact for large $\zeta$, $A^2 \propto \zeta^2$, as shown in Fig. 1(b). It should be noted that the correlation coefficient represents the second-order statistical property of the field. It would be interesting to examine the higher-order correlation characteristics. In fact, one can calculate the joint probability distribution $P(X_1, Y_1)$ for the quadratures $X_1$ and $Y_1$. This distribution is defined as

$$P(X_1, Y_1) = |\langle X_1, Y_1|\zeta\rangle|^2 \ . \tag{4.1}$$

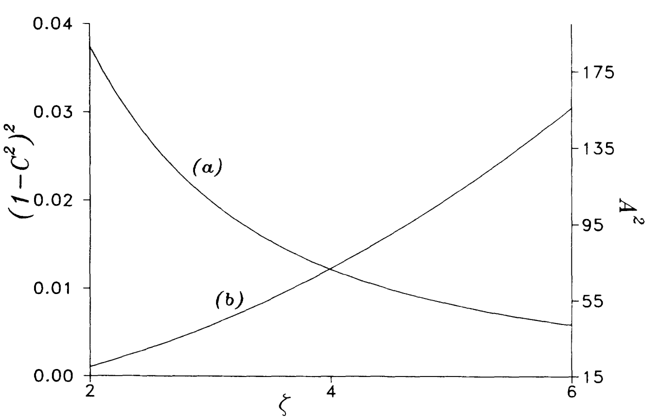


FIG. 1. (a) Plot of $(1-C^2)^2$ as a function of $\zeta$. Here the quantity $C$ is the correlation coefficient as given by Eq. (3.7) with $\cos(\theta+\phi)=1$. For large $\zeta$ values, this quantity tends to zero, implying that the perfect correlation exists between $X_\theta$ and $Y_\phi$. (b) Plot of the quantity $\langle \hat{X}_1^2\rangle$ or $\langle \hat{X}_2^2\rangle = A^2$ as a function of $\zeta$. Note that this quantity is related to the mean photon number.

Using Eq. (3.2) and the fact that $\langle X|n\rangle$ is a harmonic-oscillator wave function given in terms of Hermite polynomials as

$$\langle X|n\rangle = [2^n n!\sqrt{\pi}]^{-1/2} H_n(X) e^{-X^2/2} \ , \tag{4.2}$$

the joint probability is found to be

$$P(X_1, Y_1) = I_0(2\zeta)\left|\sum_{n=0}^{\infty}\frac{\zeta^n}{n!}\langle X_1|n\rangle\langle Y_1|n\rangle\right|^2$$
$$= I_0(2\zeta)\left|\sum_{n=0}^{\infty}\frac{\zeta^n H_n(X_1)H_n(Y_1)}{n!^2\sqrt{\pi}2^n} \times \exp[-(X_1^2+Y_1^2)/2]\right|^2 \ . \tag{4.3}$$

In Fig. 2(a) we plot the joint probability distribution

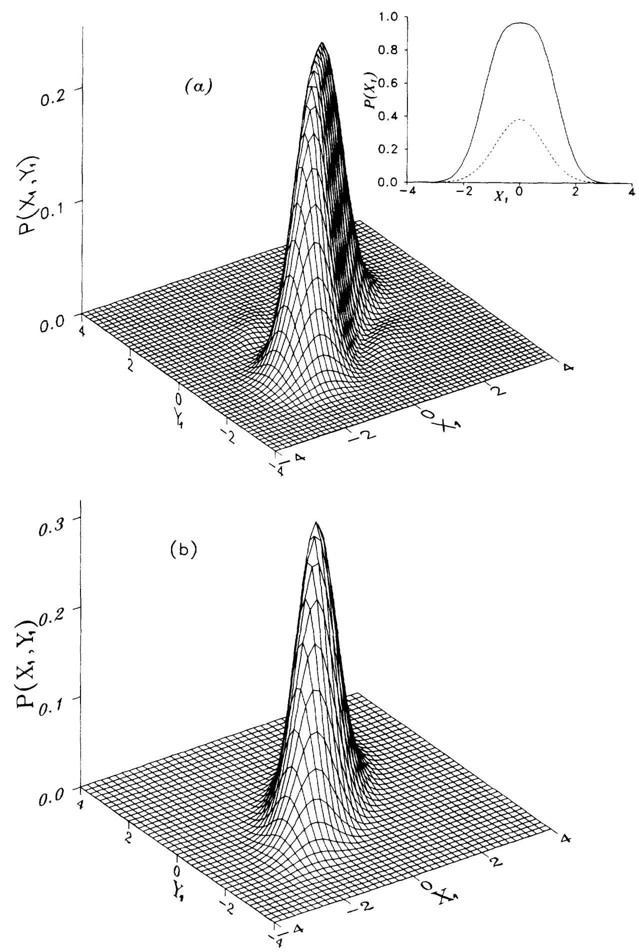


FIG. 2. (a) The joint probability distribution $P(X_1, Y_1)$ [Eq. (4.3)] as a function of $X_1$ and $Y_1$ for $\zeta=0.75$. For this value of $\zeta$ the error in inferring the values $X_1$ and $X_2$ is minimum, as can be seen in Fig. 4. The inset shows the unconditional distribution $P(X_1)$ for the field in the pair-coherent state (solid line) and the squeezed vacuum state (dashed line). (b) The joint probability $P(X_1, Y_1)$ [Eq. (4.4)] for the squeezed vacuum state with $r=0.46$.

$P(X_1, Y_1)$ for $\zeta=0.75$. The amount of squeezing for this $\zeta$ value is about 40%. We find that the same amount of squeezing can be produced for $r=0.46$ in the parametric amplifier. For comparison we plot in Fig. 2(b) the joint probability distribution $P(X_1, Y_1)$ for the field produced in the parametric amplifier for $r=0.46$. This distribution is given by

$$P(X_1, Y_1)=\frac{1}{\pi}\exp[2X_1Y_1\sinh 2r-(X_1^2+Y_1^2)\cosh 2r] . \tag{4.4}$$

It should be borne in mind that the joint probability distribution for the pair-coherent state is not Gaussian.

We display the unconditional distribution $P(X_1)$ for the field in the pair-coherent state and the squeezed vacuum state as an inset to Fig. 2(a). This distribution can be obtained by integrating the joint probability distribution $P(X_1, Y_1)$ over $Y_1$. The $P(X_1)$ for the field in the squeezed vacuum state turns out to be

$$P(X_1)=\frac{1}{\sqrt{\pi\cosh 2r}}\exp\left[\frac{-X_1^2}{\cosh 2r}\right] , \tag{4.5}$$

and $P(X_1)$ for the field in the pair-coherent state is found to be

$$P(X_1)=\frac{I_0(2\zeta)}{\sqrt{\pi}}\exp[-X_1^2]\sum_{n=0}^{\infty}\frac{|\zeta|^{2n}|H_n(X_1)|^2}{n!^3 2^n} . \tag{4.6}$$

The value of the uncertainty in the quadrature amplitude $X_1$, i.e., $(\Delta\hat{X}_1)^2$ can be obtained by measuring the full width at half maximum of this distribution. Similarly, the unconditional distribution $P(X_2)$ can be obtained, from which one can calculate $(\Delta\hat{X}_2)^2$.

We know from Sec. III that the quadrature amplitude $X_1(X_2)$ is correlated to $Y_1(-Y_2)$. Hence, it is useful to study their joint behavior, which can be done by considering the conditional probability distribution

$$P(X_1/Y_1)(P(X_2/(-Y_2)))$$

of $X_1(X_2)$ given $Y_1(-Y_2)$. The distribution $P(X_1/Y_1)$ can be obtained from the joint probability distribution $P(X_1, Y_1)$, as follows

$$P(X_1/Y_1)=\frac{P(X_1,Y_1)}{P(Y_1)} . \tag{4.7}$$

In Figs. 3(a) and 3(b) we display the conditional probability distribution $P(X_1/Y_1)$ for the field in the pair-coherent state and the squeezed vacuum state, respectively, for the same amount of squeezing as mentioned before. In Fig. 4 we plot the product $(\Delta_{\rm inf}\hat{X}_1)^2(\Delta_{\rm inf}\hat{X}_2)^2$ as a function of $\zeta$. We see that for some range of $\zeta$ values, $(\Delta_{\rm inf}\hat{X}_1)^2(\Delta_{\rm inf}\hat{X}_2)^2$ is less than unity, thus demonstrating the EPR paradox. We see from Fig. 4 that for large $\zeta$ values, the product $(\Delta_{\rm inf}\hat{X}_1)^2(\Delta_{\rm inf}\hat{X}_2)^2$ tends to 1. This is because the quantity $A^2$ in Eq. (3.17) increases faster than $(1-C^2)^2$; hence, the product $(\Delta_{\rm inf}\hat{X}_1)^2(\Delta_{\rm inf}\hat{X}_2)^2$ tends to 1. This can be seen from Figs. 1(a) and 1(b) where we have plotted $A^2$ and $(1-C^2)^2$ for large values

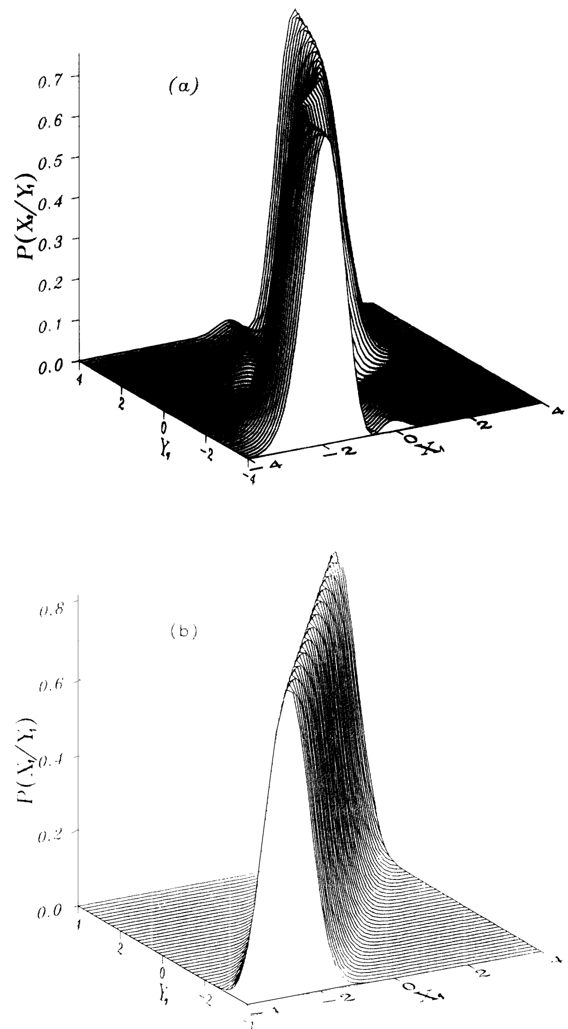


FIG. 3. (a) The conditional probability distribution $P(X_1/Y_1)$ for the field in the pair-coherent state for $\zeta=0.75$. (b) The same distribution for the field in the squeezed vacuum state.

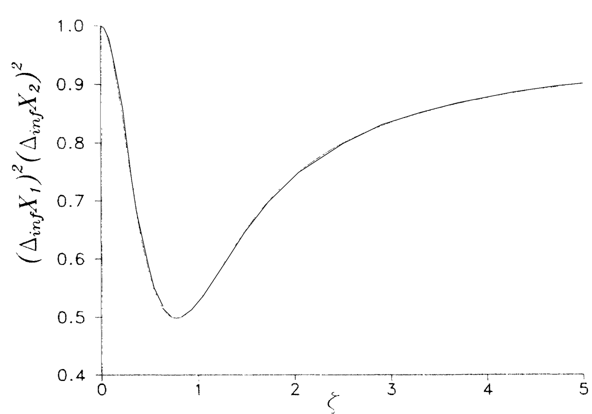


FIG. 4. The product $(\Delta_{\rm inf}\hat{X}_1)^2(\Delta_{\rm inf}\hat{X}_2)^2$ as a function of $\zeta$. The EPR paradox occurs when this product is less than 1.

of $\zeta$. From Fig. 4 we see that at $\zeta=0.75$, the product $(\Delta_{\rm inf}\hat{X}_1)^2(\Delta_{\rm inf}\hat{X}_2)^2$ is minimum, $\sim 0.49$. As mentioned before, the amount of squeezing for this $\zeta$ value is about 40%. We find that the same amount of squeezing can be produced for $r=0.46$ in the parametric amplifier [13]. For this $r$ the product $(\Delta_{\rm inf}\hat{X}_1)^2(\Delta_{\rm inf}\hat{X}_2)^2$ turns out to be $\sim 0.47$, which is quite comparable to that calculated using the field in the pair-coherent state. In conclusion, we have pointed out that the EPR argument of local realism for continuous variables can be tested via the correlation in quadrature amplitudes of the field in the pair-coherent state.

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[13] Note that in the parametric amplifier the quantity $1-C_1^2=\text{sech}^2 2r \approx 4/\zeta^2$ if we set $\zeta=e^{2r}$. However, for the pair-coherent state, $1-C_1^2 \sim O(1/\zeta)$. Thus, the correlation $C_1$ is much closer to 1 in the parametric amplifier than in the pair-coherent state. There is of course no reason to expect the two states to behave in a similar manner—there are important differences in the statistical characteristics, as discussed in Ref. [11].

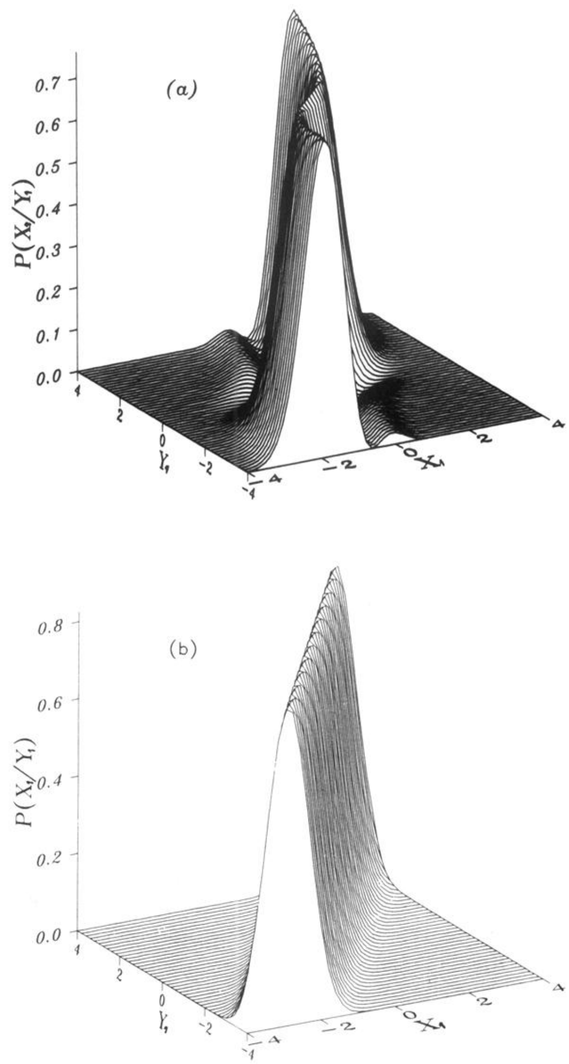


FIG. 3. (a) The conditional probability distribution $P(X_1/Y_1)$ for the field in the pair-coherent state for $\zeta=0.75$. (b) The same distribution for the field in the squeezed vacuum state.