Strong Coupling Induced Splitting of Dynamical Response of the Optical Parametric Oscillator

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A new quantum regime of the optical parametric oscillator is reported. The strong coupling between the single photon pump mode and the signal-idler modes leads to splitting of the transmission resonance of a cavity containing an optical parametric oscillator. The characteristics of the transmission doublet are obtained.

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It is now well established both theoretically [1,2] and experimentally [3,4] that the dynamical response of atoms to weak external fields is critically dependent on the coupling constant g of the atoms with the cavity field. In a high Q cavity such that $g \gg \omega/2Q, \gamma$ (where ω and γ are respectively the atomic transition frequency and the lifetime), the transmission spectra exhibit a doublet. The strong coupling changes each transmission resonance into a doublet. Such splitting of the transmission resonance has been referred to as the vacuum field Rabi splitting. The term vacuum field referring to the vacuum of the cavity field. The splitting takes place even if there is no cavity field at $t = 0$. Clearly there should be many other systems which would display a different type of dynamical response in a high Q cavity. I have found that the very well studied case of optical parametric oscillator (OPO) exhibits strong coupling induced splitting of dynamical response in a high Q cavity. Most previous studies [5] have examined OPO either in linearized or in quasilinearized approximation. The study of the strong interaction regime with one or few photons is totally new and leads to an analog of vacuum field Rabi splitting. I describe the physics of this system in the strong coupling regime and show how this results in the splitting of the response function. I also show that the observation of this splitting should be possible using the type of cavities used in recent experiments [3].

Consider for simplicity the case of degenerate parametric oscillator. The Hamiltonian can be written in the form

$$H_0 = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b + \hbar g(b^{\dagger 2} a + a^\dagger b^2), \quad (1)$$

where the annihilation and creation operators a and $a^\dagger$ (b and $b^\dagger$) refer to the pump mode (signal-idler mode). The pump mode is driven by a weak external field of frequency ω_l so that total Hamiltonian H' becomes

$$H' = H_0 - \hbar(G^* a e^{-i\omega_l t} + \text{c.c.}), \quad (2)$$

where G describes the coupling of the cavity mode a to external field. In order to keep the analysis as simple as possible we choose $\omega_a = 2\omega_b$ and we will work in a frame rotating with the frequency ω_l of the external driving field. In the rotating frame the effective Hamiltonian can be written as

$$H = \hbar\delta a^\dagger a + \hbar\frac{\delta}{2} b^\dagger b + \hbar g(b^{\dagger 2} a + a^\dagger b^2) - \hbar(G^* a + G a^\dagger), \quad \delta = \omega_a - \omega_l. \quad (3)$$

In addition we include the leakage of photons from the cavity—the leakage as usual is described by a master equation. The final master equation describing the dynamics of OPO is

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}[H,\rho] - \kappa_a(a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a) - \kappa_b(b^\dagger b\rho - 2b\rho b^\dagger + \rho b^\dagger b), \quad \kappa_\alpha = \frac{\omega_\alpha}{2Q_\alpha}. \quad (4)$$

Most of the existing literature [5] on OPO discusses two different regimes—below threshold where a in H_0 is replaced by the external field or above threshold where a quasilinearization is used. Within these approximations one studies the quantum characteristics of the field produced by OPO. In this paper we concentrate on the external field which is essentially a field at one photon level and examine the quantum dynamics. Clearly the structure of Eq. (3) implies that at one photon level of the external field the states that need to be considered are $|1,0\rangle, |0,2\rangle$. Here $|n,m\rangle$ represents a Fock state with $n(m)$ photons in the mode $a(b)$. The relevant eigenstates of H (with $G = 0$) and energies are

$$|\psi_0\rangle = |0,0\rangle, \quad \mathcal{E}_0 = 0,$$

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|1,0\rangle \pm |0,2\rangle), \quad \mathcal{E}_\pm = \pm\sqrt{2}\hbar g. \quad (5)$$

A single photon of the pump field can be absorbed *via two pathways* as shown in Fig. 1 $|\psi_0\rangle \rightarrow |\psi_+\rangle$; $|\psi_0\rangle \rightarrow |\psi_-\rangle$. Each pathway leads to resonance at $\delta = \mathcal{E}_+/\hbar$ or $\delta = \mathcal{E}_-/\hbar$. Thus the transmission resonance will be split into a doublet provided that $\sqrt{2}g$ is larger than the width of each peak. The width of each peak will be determined by κ_a and κ_b. We have thus found the strong coupling induced splitting of the dynamical response of OPO. To estimate the width of each peak we have to solve the master equation (4). Note the following *decay* channels:

$$|1,0\rangle \xrightarrow{\kappa_a} |0,0\rangle,$$

$$|0,2\rangle \xrightarrow{\kappa_b} |0,1\rangle \xrightarrow{\kappa_b} |0,0\rangle. \quad (6)$$

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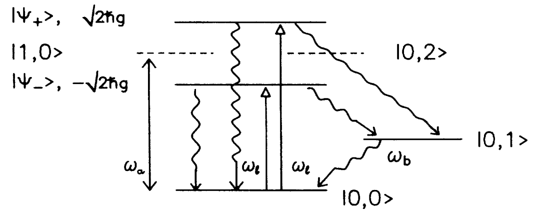

FIG. 1. Schematic representation of various energy levels and the transitions. The solid lines give the two pathways for the absorption of external photon. The wavy lines give the cavity induced decays.

The extra decay channel $|0,1\rangle$ makes the OPO problem a little more complex. The density matrix equation (4) can be solved in the truncated Hilbert space consisting of four bare states $|0,0\rangle$, $|0,1\rangle$, $|0,2\rangle$, and $|1,0\rangle$ by converting (4) into a matrix equation [total number of unknowns (16)]. This solution will yield among other things the mean number of photons $\langle a^\dagger a\rangle$. A plot is shown in Fig. 2. The numerical results are given for small values of G so that our truncation procedure holds. As argued above, the numerical solution exhibits strong coupling induced splitting of the dynamical response. The linewidth at half maximum is 3κ (for the special case $\kappa_a = \kappa_b$). This can be understood by examining the linewidths associated with the transitions $|\psi_\pm\rangle \rightarrow |\psi_0\rangle$. By using Eq. (4) one can show that

$$\dot{\rho}_{\pm,0} = -\left(i\delta + ig\sqrt{2} + \frac{(\kappa_a + 2\kappa_b)}{2}\right)\rho_{\pm,0} + \cdots . \quad (7)$$

The halfwidth at half maximum is equal to $\frac{1}{2}(\kappa_a + 2\kappa_b)$. Note further that the decay of the bare states is given by

$$\dot{\rho}_{10,10} = -2\kappa_a\rho_{10,10}\,,$$
$$\dot{\rho}_{02,02} = -4\kappa_b\rho_{02,02}\,, \quad (8)$$

and thus the *net linewidth is the average of the decay rates of the bare states* $|1,0\rangle$ and $|0,2\rangle$.

We next examine the experimental feasibility of the predicted effect. The nonlinear d coefficient [6] for a crystal like $LiNbO_3$ is 6.25×10^{-12} m/V $\sim 1.5 \times 10^{-8}$ esu. The g coefficient will be $(2d/\hbar)(2\pi\hbar\omega/V)^{1/2}V(\pi\hbar\omega/V)$. The factor $(2\pi\hbar\omega/V)^{1/2}$ comes from the quantization of each field. The extra volume factor is from integration over the range of interaction. For cavities of the kind used in recent experiments [3,7] by Thompson and co-workers and with a focused pump beam and for a nonlinear crystal of length 1 cm, $V \sim (\pi\omega_0^2/4)l = (\pi/4)(50 \times 10^{-4})^2(l) \sim 2 \times 10^{-5}$ cm^3. This gives us $g \lesssim (2\pi)10^5$ Hz and thus very high Q cavities (cf. Refs. [3,7]) should make observation of the predicted effect possible.

There are of course other possibilities such as using crystals with much higher nonlinear coefficient. An interesting possibility suggested to us corresponds to using a crystal fiber in an open resonator so that one could make use of the transverse guiding. In this case the interaction region can be kept quite small leading to large coupling without affecting the cavity Q.

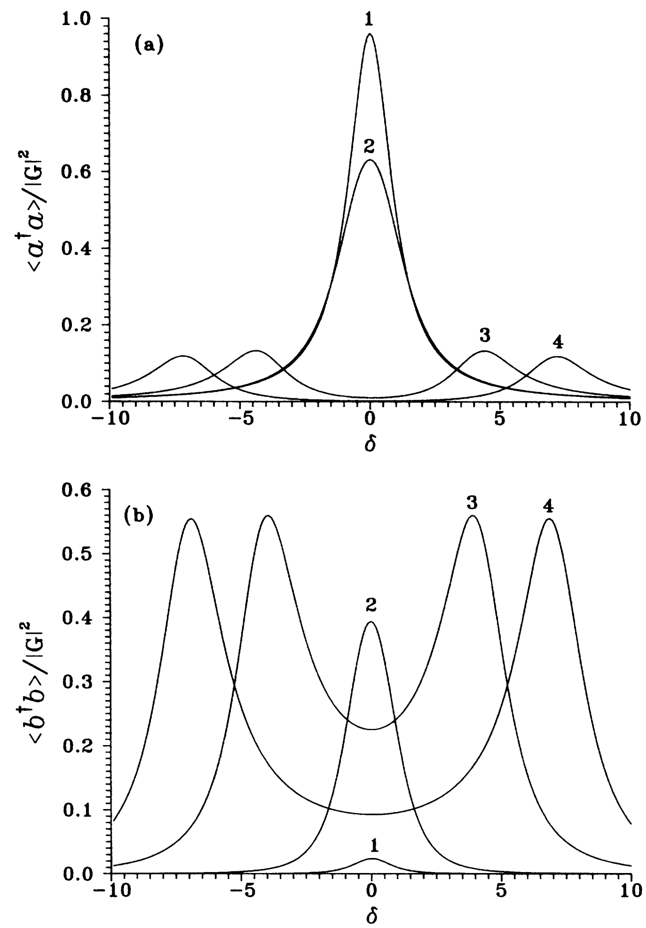

FIG. 2. The response of the OPO, i.e., the mean number of photons (a) pump mode and (b) signal mode as a function of the frequency of the externally applied field $\delta = \omega_a - \omega_l$ for $\kappa_a = \kappa_b$ and for different values of the nonlinear coupling parameter $g = 0.1(1), 0.5(2), 3.0(3), 5.0(4)$.

Note that the OPO Hamiltonian when linearized results in $H \propto (b^{\dagger 2} + b^2)$ which has no normalizable eigenstates. The underlying group is SU(1,1) which is a noncompact group. The corresponding atomic problem with $H_1 = g(S^+a + S^-a^\dagger)$ under linearization (i.e., approximating atomic operators by bosons) goes over to $H_1 \rightarrow g(b^\dagger a + a^\dagger b)$ which has well defined eigenstates. The underlying group is SU(2). It is thus quite interesting that the full OPO Hamiltonian has well defined states [8] and the transitions among the low lying states can lead to splitting of the dynamical response.

Thus in conclusion I have shown how the strong interaction between the pump and signal and idler modes can lead to the splitting of the dynamical response [9] function of OPO. I have further shown how the linewidth is related to the average of the linewidths corresponding to decay of pump and idler photons.

These results for OPO are the counterparts [2,10,11] of the corresponding results for atoms interacting strongly with the cavity field.

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