PHASE CONJUGATE OPTICS

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ABSTRACT

A brief review of some of the important developments in the relatively new field of phase conjugate optics is presented.

INTRODUCTION

HISTORICAL development of physics shows the crucial role played by discoveries in optics. The techniques discovered in optics have found tremendous applications in other fields of physics. Here we mention a few important ones. Photon counting techniques, first used for the study of statistical properties of light produced by thermal and nonthermal sources, have been successfully used in precise studies of the phase transition behaviour of a system near the critical point. Generalizations of the intensity correlation technique of Hanbury-Brown and Twiss have been used in hadron interferometry[1]. Quantum noise techniques[2] which were developed first in connection with problems in quantum optics, are finding many interesting applications in the detection of gravitational radiation. Some years back, an entirely new optical techniques was discovered by Zeldovich and coworkers[3,4] by which it was possible to correct the distortions or abbreviations imparted on the optical wavefront. This technique, now known as the phase conjugacy technique, has been extensively developed[5-8] over the last few years in view of its tremendous potential for applications in a wide variety of fields such as optical communication, plasma fusion, and the design of better laser cavities[7] and optical interferometers[9].

Let me now explain briefly what a phase conjugated signal is. Consider an optical wave moving in z direction.

$$E(\vec{r},t) = \exp(ikz)\,\epsilon(\vec{r},\omega)\exp(-i\omega t) \qquad (1.1)$$

The operation of phase conjugacy, say in the plane $z=0$, would produce a wave $E^{(c)}(\vec{r},t)$ given by

$$E^{(c)}(\vec{r},t) = \epsilon^{*}(\vec{r},\omega)\exp(-ikz - i\omega t) \qquad (1.2)$$

The new wave is propagating in $-z$ direction with an amplitude that is equal to the complex conjugate of the amplitude ϵ of original wave. The applications of the phase conjugate signals arise due to the possibility of correcting for the distortions in the original wave. This is shown schematically in figure 1, where an incident wave $\epsilon^{(i)}$ passes through a distorting medium and is conjugated at the plane $z=z_1$ to produce a wave $\epsilon^{(c)}$. On passing through the scatterer again, the field in the space R^- (far away from the scatter-

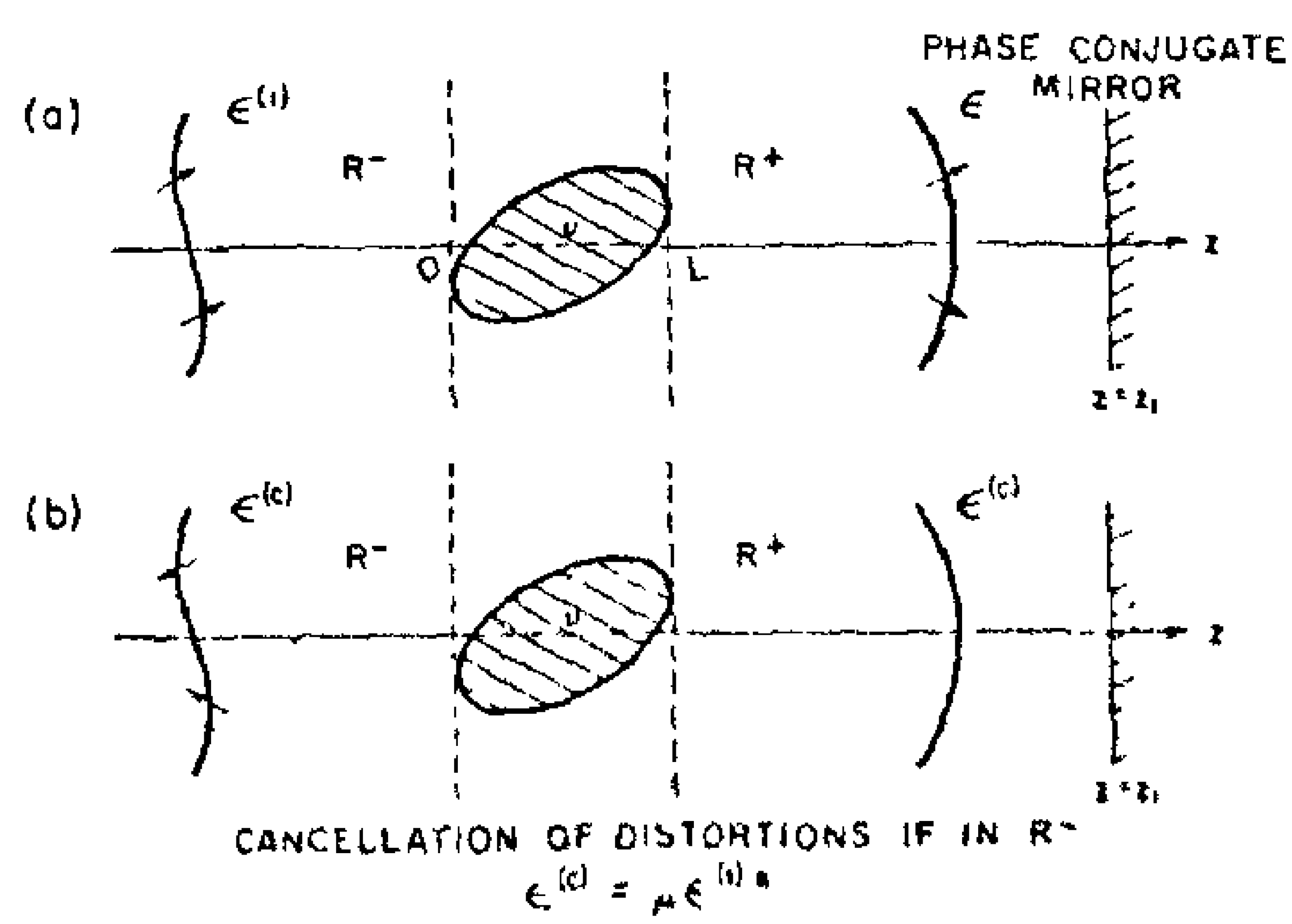

Figure 1. Schematic illustration of the correction of distortions by phase conjugation.

ing region) should become $\mu\epsilon^{(i)*}$, where μ is a parameter characterizing losses or gain at the phase conjugate mirror. If this is so, then the effects of the distorting medium would have been completely cancelled out. It is obvious from the above that there is very interesting physics associated with the problem of phase conjugacy and the correction of distortions. Two detailed questions that one has to answer at this stage are: (i) How is the transformation of the fields from $\epsilon \to \epsilon^*$ achieved? (ii) Given that conjugate fields can be produced, what are the conditions on the characteristics of the incident field, scattering medium, and the phase conjugate device, under which we can completely correct for the distortions imparted by the medium? We will see that the first question can be answered by looking into the detailed nonlinear interaction between the atoms/molecules and electromagnetic fields; whereas in order to answer (ii) we have to go into the details of the electromagnetic scattering theory in the presence of a phase conjugate mirror (PCM). Before discussing the physics of the problem we will present two simple novel applications of phase conjugacy from among the myriad possibilities.

Consider the question of the delivery of energy from a laser system to a target as, for example, in laser fusion. The application of phase conjugacy

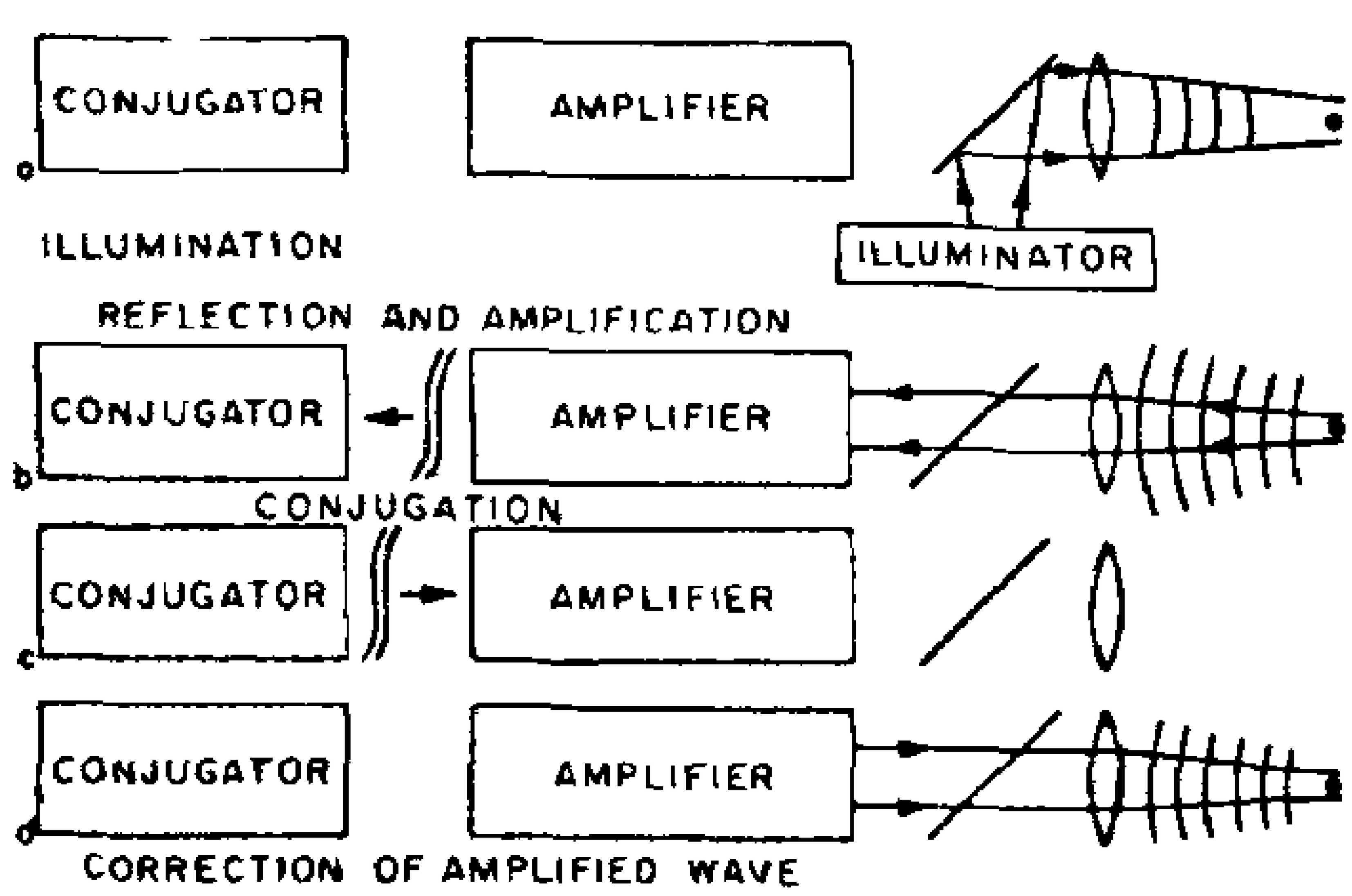

Figure 2. A possible application of phase conjugacy in laser fusion (after Giuliano, Ref. 10)—(a) illumination of the pellet with a probe beam, (b) reflection and transmission resulting in a distorted wave, (c) conjugation, (d) a second passage through the distorting medium; thus producing a coherent illumination of the pellet.

to such a system was first considered by Giuliano[10] In the usual systems the beam goes through a complex optical and electronic system and obviously suffers considerable distortion, beam spread, etc., resulting in appreciable loss of energy before the target is reached. Obviously if one could correct for distortions, then one would have an intense pulse at the target, i.e. the wavefront incident on the target would be a replica of the reference wavefront that radiated originally from the target. Figure 2 shows how this could possibly be achieved by phase conjugation.

Another important application of phase conjugacy, suggested by Yariv[5], is in the transmission of images without distortions. For example, if an image is transmitted through an optical fiber, then the image will be distorted due to the optical inhomogeneities in the fiber. However, if we have a phase conjugation device at the end of the fiber and if we let the resultant image transmit through an identical optical fiber, then at the end of the second fiber, the image should be distortion-free since on propagation through the second optical fiber, the distortion introduced in the first fiber would have been cancelled.

ELECTROMAGNETIC THEORY OF DISTORTION CORRECTIONS BY PHASE CONJUGATION

In this section we find, by studying the electromagnetic scattering problem[11,12], the conditions under which complete cancellation of distortions takes place. We assume that a field $\vec{E}^{(i)}$ is incident on the distorting medium which is taken to be characterized by an inhomogeneous refractive index. The phase conjugation is done on the plane $z = z_1$. The incident field is assumed to contain no evanescent waves. We also assume that the phase conjugate mirror is such that complete reversal of polarization takes place, *i.e.* if $\vec{E}$ is the field before conjugation, then the field after conjugation is

$$\vec{E}^{(c)}\exp(-i\omega t) = \vec{E}^*\exp(-i\omega t) \qquad (2.1)$$

Zeldovich and Shkunov[4] have proposed several schemes by which a complete reversal of polarization can be obtained. It is well known that in any scattering process a homogeneous wave can be converted into a evanescent wave and vice versa. We further assume (cf. figure 1) that the distances of the mirror (in R^{+}) and of the observation plane (located in R^{-}) from the scatterer are such that any evanescent waves produced in scattering do not contribute to the fields at PCM and at the observation point. Given the above conditions, one has to calculate the field distribution everywhere. The field distribution can be shown to be determined from the solution of the integral equation

$$\vec{E}(\vec{r},\omega) = (1+\mu\hat{C})\vec{E}^{(i)}(\vec{r},\omega) + \int d^3r'\,\overleftrightarrow{G}_c(\vec{r},\vec{r}',\omega)\,\frac{n^2(r')-1}{4\pi}\cdot\vec{E}(\vec{r}',\omega) \qquad (2.2)$$

where the Green's dyadic $\overleftrightarrow{G}_c$ that takes into account the presence of PCM, is given by

$$\overleftrightarrow{G}_c(\vec{r},\vec{r}',\omega) = \theta(z-z')(1+\mu\hat{C})\overleftrightarrow{G}^{(H)}_{>}(\vec{r},\vec{r}') + \theta(z',z)\overleftrightarrow{G}^{(H)}_{<}(\vec{r},\vec{r}')(1-\mu\hat{C}) + \overleftrightarrow{G}^{(I)}(\vec{r},\vec{r}'), \qquad (2.3)$$

with $\hat{C}$ denoting the operation of complex conjugation, *i.e.*

$$\hat{C}\varphi(r) = \varphi^*(r). \qquad (2.4)$$

For $\mu = 0$, (2.3) reduces to the free space Green's function

$$\overleftrightarrow{G}(\vec{r},\vec{r}',\omega) = \left\{\overleftrightarrow{I} + \frac{1}{k^2}\vec{\nabla}\vec{\nabla}\right\}\frac{\exp(ik|\vec{r}-\vec{r}'|)}{|\vec{r}-\vec{r}'|}$$

$$= \overleftrightarrow{G}^{(H)}(\vec{r},\vec{r}',\omega) + \overleftrightarrow{G}^{(I)}(\vec{r},\vec{r}',\omega),$$

$$\overleftrightarrow{G}^{(H)}(\vec{r},\vec{r}',\omega) = \left\{\overleftrightarrow{I} + \frac{1}{k^2}\vec{\nabla}\vec{\nabla}\right\}\iint_{\kappa<k}\frac{d^2\kappa}{2\pi w}\times$$

$$\exp[i\vec{\kappa}\cdot(\vec{r}-\vec{r}') + iw\,|z-z'|$$

$$\overleftrightarrow{G}^{(I)}(\vec{r},\vec{r}',\omega) = \left\{\overleftrightarrow{I} + \frac{1}{k^2}\vec{\nabla}\vec{\nabla}\right\}\iint_{\kappa>k}\frac{d^2\kappa}{2\pi w}$$

$$\exp[i\vec{\kappa}\cdot(\vec{r}-\vec{r}') - w\,|z-z'|\,,$$

$$w^2 = k^2 - \kappa^2 \qquad (2.5)$$

Here the Green's function $\overleftrightarrow{G}^{(H)}$ represents the contributions from all the homogeneous waves whereas $\overleftrightarrow{G}^{(I)}$ is the contribution from all the evanescent waves. The appearance of $\overleftrightarrow{G}^{(I)}$ in (2.3) accounts for the conversion, inside the scatterer, of homogeneous waves into evanescent waves and vice versa.

The meaning of the various terms in the integral equation will be clear from the examination of some of the lower-order terms: This we depict schematically in figures 3 and 4. These figures

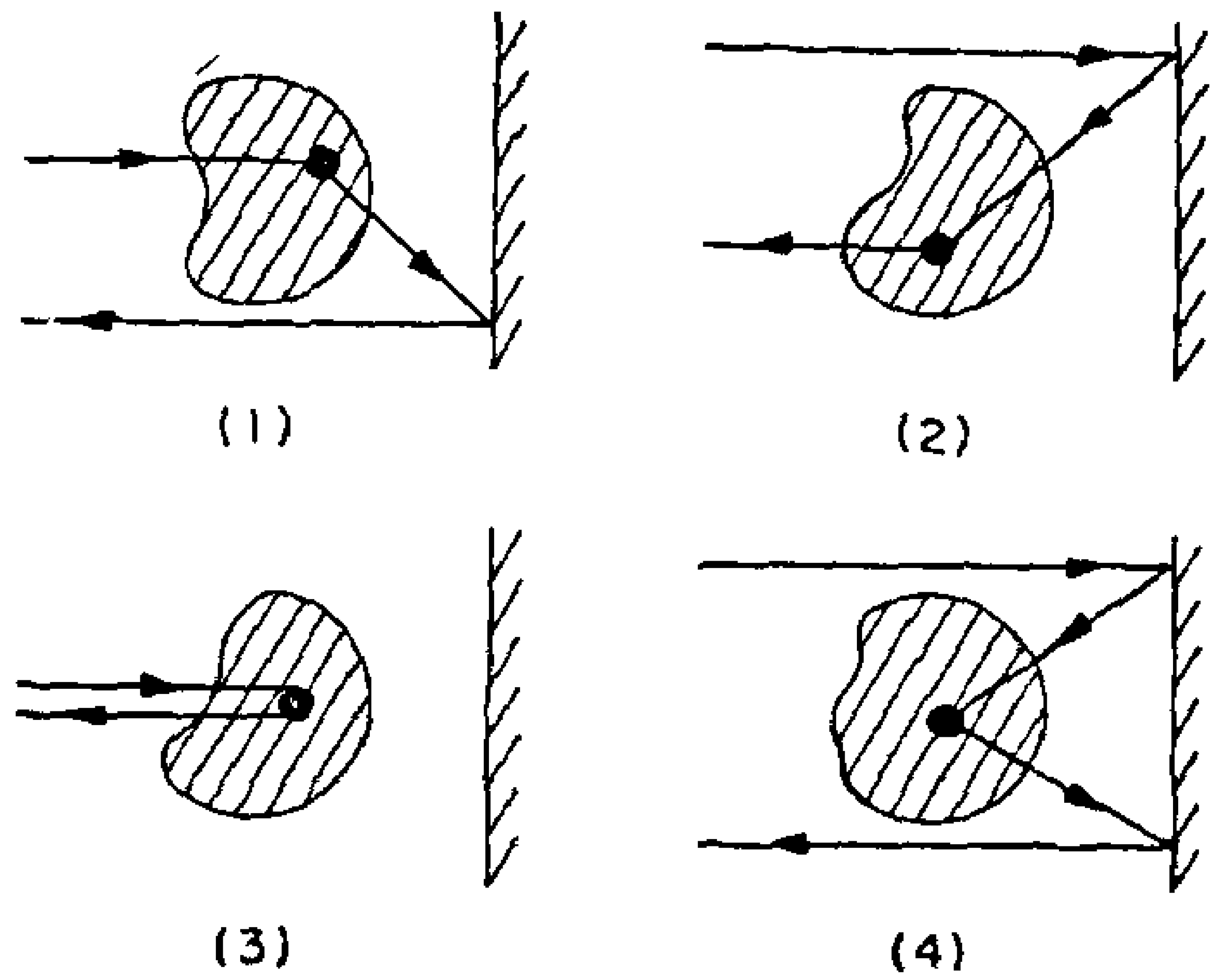

Figure 3. Schematic illustration of first-order scattering in presence of a PCM. Diagrams (3) and (4) correspond to back scattering. In particular, in diagram (4), phase conjugation is used twice.

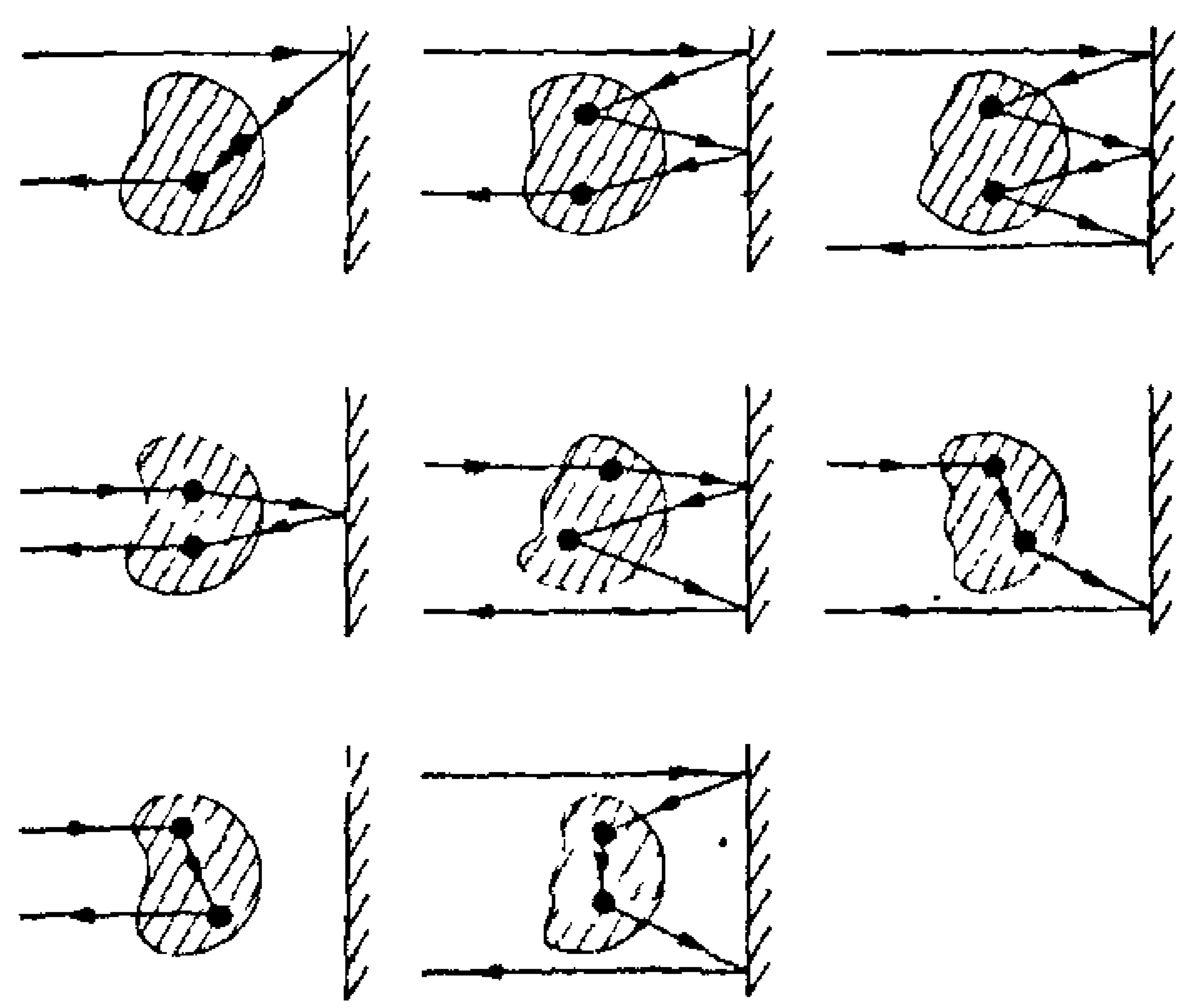

Figure 4. Various scattering and conjugations in second-order scattering. Total scattering contribution, *i.e.* sum of all the eight diagrams zero if the phase conjugation is done without losses or gains.

show clearly various scatterings and conjugations involved in each order of perturbation theory. It is possible to prove[12] using (2.2) that if $|\mu| = 1$, and the scatterer is nonabsorbing, then the field, in $R^{'}$, at a point far away from the scatterer is

$$\vec{E} = \vec{E}^{(i)}(\vec{r}, \omega) + \vec{E}^{(i)*}(\vec{r}, \omega)\,\mu \,. \qquad (2.6)$$

This leads to the important result that—if the phase conjugation is done with *no less or gains* ($|\mu| = 1$) and with complete polarization reversal, then the field distribution in the left half space shows no effects of the distorting medium, *i.e.* the effects of the medium have cancelled each other and it is possible to recover the original undistorted wave. This is an exact result and holds to all orders in scattering potential. The value of μ depends on the particular experimental arrangement and a wide range of values have been reported. For very small values of μ, cancellation of distortions has been shown[12] to take place under the conditions: (i) scatterer is weak and nonabsorbing; (ii) back scattering is negligible; (iii) effects of evanescent waves outside the scatterer are negligible. We have thus demonstrated how the electromagnetic scattering theory can be used in the study of the correction of distortions.

GENERATION OF PHASE CONJUGATE SIGNALS

Various schemes for the generation of phase conjugate signals have been used in the past. These include the ones based on difference frequency generation[5], four-wave mixing[6], stimulated Brillouin scattering[3], etc. In this section we discuss two of these schemes:

Phase conjugation by difference frequency mixing

Consider a crystal lacking inversion symmetry and let two fields at the frequencies ω and 2ω be incident on such a crystal

$$E_1 = \epsilon_1 \exp(-i\omega t + ik_1 z),$$
$$E_2 = \epsilon_2 \exp(-2i\omega t + ik_2 z), \qquad (3.1)$$

where k_1 and k_2 are, respectively, the propagation vectors of the beams at ω and 2ω, ϵ_1 and ϵ_2 are the slowly varying field envelops. The lowest-order nonlinear polarization produced in such a crystal would be second order in the fields. In particular, it is possible to produce the signals at the difference frequency $2\omega - \omega = \omega$ because of the nonlinear polarization

$$P(\vec{r}, \omega) = \chi^{(2)} E_2 E_1^{*}$$
$$= \chi^{(2)} \epsilon_2 \epsilon_1^{*} \exp(-i\omega t + i(k_2 - k_1)z]. \qquad (3.2)$$

The appearance of a complex conjugate of the envelope ϵ_1 of the field at ω, in (3.2) is to be noted. If the direction of propagation is such that the phase matching condition is satisfied $k_2 = 2k_1$, then the nonlinear polarization (3.2) leads to a signal at ω that is propagating in the z direction. Such a signal on reflection with an ordinary mirror, will lead to the phase conjugate signal[5] *i.e.* a signal whose envelope is complex conjugate of the original envelope ϵ_1 and which is propagating in the $-z$ direction. The phase matching condition $k_2 = 2k_1$, however, puts severe restrictions on the usefulness of this technique.

Phase conjugation by four-wave mixing

We next discuss the process of phase conjugation based on four-wave mixing. This turns out to be a versatile technique and is free from the complications of the requirements of phase matching. In order to simplify the analysis, let us consider the simplest case of degenerate four-wave mixing. Take the interaction of a medium with two strong pump beams propagating in the directions $\vec{k}_1$ and $\vec{k}_2 = -\vec{k}_1$ and a signal field in the direction $\vec{k}_3$. All the fields are at the frequency ω. The lowest-order nonlinear interaction among the three fields and the medium can be described by a third-order nonlinear susceptibility[13], $\chi^{(3)}$, which leads to *a term* in the polarization of the form

$$P = \chi^{(3)} \epsilon_1 \epsilon_2 \epsilon_3^{*} \exp(i\vec{k}_1\cdot\vec{r} + i\vec{k}_2\cdot\vec{r} - i\vec{k}_3\cdot\vec{r}) \exp(-i\omega t) \quad (3.3$$

where field envelopes are taken to be slowly varying functions of spatial coordinates. Since the two pump beams are counter propagating $\vec{k}_2 = -\vec{k}_1$ the nonlinear polarization (3.3) leads to a wave at ω, propagating in the direction $-\vec{k}_3$, which has an envelope function that is a complex conjugate of the signal's envelope ϵ_3. We thereby see that the four-wave mixing leads very easily to the process of phase conjugation. The usual problems of phase matching do not arise since the two pump beams have been taken to be counter propagating. The process of phase conjugation using four-wave mixing has another interesting feature first noted by Yariv and Pepper[5], namely, that it is possible to have a case in which the intensity of the conjugate wave exceeds that of the input wave. In fact one can show by solving Maxwell equations under the approximation of slowly varying envelopes that the intensity of the conjugate wave ϵ_4 is given by

$$|\epsilon_4|^2 = (\tan^2 |\kappa| L)\, |\epsilon_3|^2, \tag{3.4}$$

where

$$|\kappa| = \left| \frac{2\pi\omega}{cn} \chi^{(3)} \epsilon_1 \epsilon_2 \right| , \tag{3.5}$$

with n giving the refractive index at ω. Equation (3.4) obviously shows amplification for $\pi/4 < |\kappa| L < 3\pi/4$ and oscillation for $|\kappa| L = \pi/2$. It should be noted that in general $\chi^{(3)}$ is quite small and hence in order to have efficient phase conjugation it may be necessary to use, for example, the resonant enhancement[14,15] of $\chi^{(3)}$ by tuning ω close to one of the atomic transition frequencies. Experiments[15,16] in various vapours have already been reported. It is interesting to note that at this stage of the phase conjugation process, the physics of the optical resonant processes enters. The further enhancement due to the strong pump beams[14] (which lead to, among other things, AC Stark effect) have also been reported.

So far, we have kept aside the polarization properties of the conjugate wave, which generally would be different from those of the incident wave. As mentioned earlier, a complete polarization reversal is ideal for the cancellation of all the distortion effects. The complete polarization reversal could be achieved by arranging the experimental geometry[4] and by choosing the polarization of the pump beams suitably. For example, consider an isotropic medium characterized by the susceptibility

$$\chi^{(3)}_{ijkl} = \chi_1 \delta_{ij}\delta_{kl} + \chi_2 \delta_{ik}\delta_{jl}, \tag{3.6}$$

where the constants χ_1 and χ_2 depend on the nature of the medium. Consider now two strong counterpropagating beams along the z axis with polarization along the x axis. Let the signal wave be travelling in the x direction with polarization in the yz plane; then using (3.6) one easily sees that the third-order nonlinear polarization has the form

$$\vec{P} = 2\chi_2 \epsilon_1 \epsilon_2 \epsilon_3^* \exp\{-i\vec{k}_3 \cdot \vec{r} - i\omega t\}, \tag{3.7}$$

leading thus to complete polarization reversal. A similar polarization reversal scheme by Lam *et al.*[17] considers the importance of the induced coherence between Zeeman sublevels in determining the polarization characteristics of the conjugate wave and demonstrated this by experimental studies on the Na D_2 line. The complete polarization reversal implies, for example, that a right-hand circularly polarized wave will result in a conjugate wave that is also right-hand circularly polarized—which is in contrast to the case of an ordinary mirror.

From the foregoing it should be clear that the physics associated with the entire area of phase conjugate optics is very interesting. Some of the questions being currently investigated are: How do the spatial and temporal coherence characteristics of the signals affect the phase conjugation and the correction of distortions? What happens when the distorting medium is turbulent in nature? Resonant processes involving multiphoton processes[18] are also being examined in an attempt to improve the frequency and other spatial characteristics of the conjugation process. Another class of problems that are likely to have far-reaching consequences concern the interaction of atoms/molecules with radiation fields in laser cavities with phase conjugate mirrors. Our recent studies[19] in connection with the dipole radiation in the presence of phase conjugate mirrors show that there is very interesting inhibition of dipole radiation. This in turn can be shown to lead to the inhibition of spontaneous emission thereby implying that there is reduction of zero point fluctuations in such situations. Such a reduction of quantum fluctuations may have important applications in the design of optical interferometers, which in turn may be used to detect very weak radiation, such as gravitational radiation.

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