Modulated Hanle signals in partially coherent fields

R. Saxena and G. S. Agarwal
School of Physics, University of Hyderabad, Hyderabad 500001, India
(Received 29 April 1980; revised manuscript received 9 July 1981)

The fluorescence emitted by atoms whose excited states have Zeeman structure is studied for the case of excitation by partially coherent optical sources whose amplitudes are weakly modulated at a low frequency Ω. The temporal fluctuations in lasers are taken into account exactly by using the phase-diffusion model of the laser. Analytical results for the modulated fluorescence are presented for the case of weak fields. It is shown that the shape of the fluorescent signal is extremely sensitive to the bandwidth of the field; for example, one gets resonances at a Larmor frequency equal to $\nu=\pm\Omega/2(0,\pm\Omega)$ for broadband (monochromatic) fields. Various limiting cases are treated, and in the limit of very broadband fields, results reduce to those of Corney and Series. The fluorescence is studied both as a function of the modulation frequency and as a function of the magnetic field. Numerical studies of the modulated Hanle signals in intense fields reveal resonant structures in the modulation-frequency scan at $\Omega=0,\pm\alpha_0,\pm 2\alpha_0(\alpha_0^2=\nu^2+2\alpha^2)$ due to the dynamical Stark splitting of the various energy levels, with the strength of the resonances depending on the direction of detection.

I. INTRODUCTION

In the 1920's the resonance effects that appear when light is scattered by atoms in a magnetic field were studied in great detail by Hanle[1] who showed that the observed fluorescence exhibits resonant behavior as the static magnetic field is varied, the width of the resonance curve being determined by the natural lifetime of the atoms in the excited state. These techniques (also known as zero-magnetic-field level-crossing experiments) have also been used to study Zeeman and hyperfine structure of excited and ground levels of atoms, together with measurements of radiative and interatomic collisional relaxation rates.[2] Recently, an optical analog of the Hanle effect was proposed.[3–6] In this method suitably polarized radiation produces dynamic Stark shifts of the energy levels coupled by the field, thus lifting the degeneracy of the coupled excited state and replacing the external magnetic field in the usual Hanle effect. Once this is achieved the properties of the magnetic sublevels can be investigated by using another laser field coupling the shifted energy levels, the fluorescence signals now being observed as a function of the intensity or frequency of the initial radiation field used to produce the light shifts. The resonances in the optical Hanle effect were found to occur in the same manner as those observed in the zero-magnetic-field level-crossing experiments. Yet another important and useful variation of the usual Hanle effect consists of the excitation of atoms by light whose intensity is periodically modulated in time, the degeneracy of the energy levels having been lifted by a static magnetic field as before.[7,8] The intensity of the fluorescent light is itself modulated at the same frequency as the exciting light while the amplitude of modulation exhibits resonant structures when the frequency of modulation is equal to the Zeeman splitting between the levels. The width of these resonances was once again determined by the natural lifetime of atoms in the excited state when the source used for preparing the atoms in a coherent superposition of the Zeeman sublevels was a weak, broadband source.

In this paper we study in detail the problem of the Hanle effect excited by an intensity modulated source of arbitrary bandwidth. For weak fields the shape as well as the linewidth of the resonances observed in the magnetic scan of the modulated signals is shown to be critically dependent on the bandwidth of the exciting source and its frequency of modulation. The results of a previous study of the problem[7,8] using a weak, incoherent broadband pump are obtained as a limiting case of our general theory, while excitation by a truly monochromatic laser leads to entirely new results. We also study the experimentally equivalent possibility of scanning the modulated fluorescence by tuning the fre-

quency of modulation of the exciting source at a fixed strength of the static magnetic field, producing the Zeeman splitting of the excited state. Well-defined resonances in the amplitude of modulation are obtained when the frequency of modulation is equal to the Larmor precessional frequency of the atoms in the magnetic field (monochromatic source), or when the frequency of modulation is zero or equal to twice the Larmor frequency (broadband source). Section II contains the mathematical formulation of the problem valid for arbitrary strength and bandwidth of the exciting field. In Sec. III analytical expressions are obtained for the signals in weak, partially coherent laser beams. Various limiting cases in laser bandwidth are explicitly studied with numerical plots for the general case. The modulated Hanle signals in a partially coherent laser beam of *arbitrary* intensity are studied numerically using the formulation of Sec. II. Similar studies of modulated fluorescence from a single two-level atom have revealed the presence of resonances in the amplitude of modulation when the frequency of modulation of the exciting source was equal to the Rabi frequency of the atom in the intense, resonant laser beam.[9–12] The modulated fluorescence in an optical double-resonance experiment[13] also exhibits resonances corresponding to the dynamic Stark splitting of the various energy levels of the composite system consisting of the atom and the intense, coherent laser field. We show that the modulation-frequency scans of the Hanle signals in intense laser beams contain similar information, there being resonances at $\Omega=0,\pm\alpha_0,\pm 2\alpha_0$.

II. FORMULATION OF PROBLEM

We consider here the typical atomic beam Hanle-type experiment,[14] shown in Fig. 1(a)—an atomic beam travels along the Z axis and a constant magnetic field $\vec{H}$ is applied in the same direction. The atoms are irradiated by a fluctuating laser beam (whose intensity is time modulated) propagating along the Y direction and having a linear polarization $\hat{X}$ parallel to the X axis. We can detect the fluorescence emitted along the Z axis with a linear polarization $\hat{X}[\equiv L(\hat{X})]$ or the fluorescence emitted along the X axis with a linear polarization $\hat{Y}[=L(\hat{Y})]$. For simplicity we restrict our study of the Hanle effect to transitions between levels with angular momentum $J=0$ and $J=1$. The static magnetic field $\vec{H}$ splits the latter into three Zeeman sublevels with $m_J=0,\pm 1$. However, the plane-polarized laser beam for our geometry couples the nondegenerate ground state $|J=0,m_J=0\rangle$ to the $m_J=\pm 1$ sublevels of the excited state ($J=1$) only, the laser being near resonance with the atomic transition $|J=0,m_J=0\rangle \leftrightarrow |J=1,m_J=0\rangle$ which is forbidden. Thus we can neglect the existence of $m_J=0$ excited sublevels in further consideration and the relevant atomic levels are shown in Figure 1(b) where the labels 1, 2, and 3 refer, respectively, to the $|J=1,m_J=1\rangle$, $|J=1,m_J=-1\rangle$, and $|J=0,m_J=0\rangle$ levels, Δ is the Zeeman splitting or Larmor frequency of the excited state and δ is the detuning of the laser beam of mean frequency ω_L from the atomic transition

$$|J=0,m_J=0\rangle \leftrightarrow |J=1,m_J=0\rangle \ .$$

Taking $\hbar\equiv 1$, the energy of the atomic states $|1\rangle$, $|2\rangle$, and $|3\rangle$ are $\omega_{13}(=\omega_L+\Delta-\delta)$, $\omega_{23}(=\omega_L-\Delta-\delta)$, and zero, respectively.

In order to take into account the temporal fluctuations present in the laser, we treat the laser electric field $\vec{E}(t)$ as a classical random variable. For a

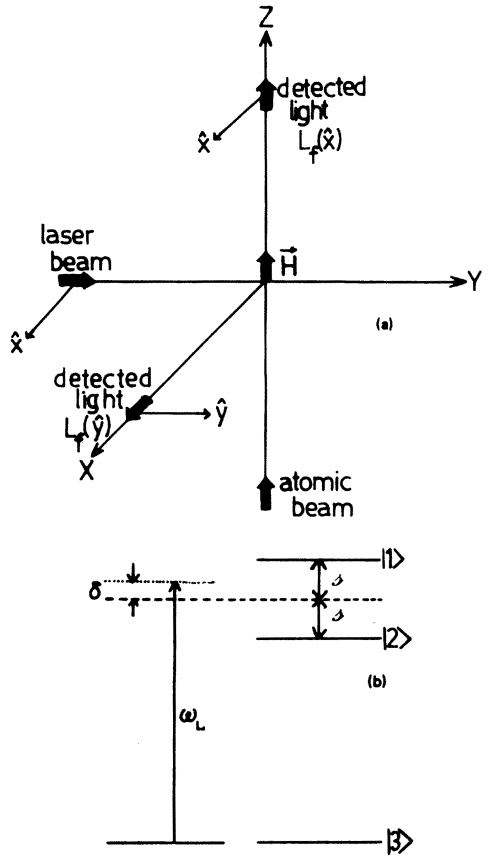

FIG. 1. (a) Schematic representation of experimental geometry. (b) Schematic representation of the relevant energy levels.

laser well above threshold $\vec{E}(t)$ can be written as

$$\vec{E}(t)=\tfrac{1}{2}\vec{\mathscr{E}}_0 e^{-i[\omega_L t+\Phi(t)]}+\text{c.c.}\ , \tag{2.1}$$

where ω_L is the mean frequency and $\Phi(t)$ is the phase, and the laser field is assumed to be very well stabilized in amplitude. For the fluctuations of the field we adopt the phase-diffusion model for the laser fluctuations. As for this model the correlation functions of the electric field are known to all orders. For this model the amplitude of the laser beam is a deterministic variable independent of time, as in expression (2.1), while the phase undergoes diffusion

$$\dot{\Phi}(t)=\mu(t),\quad \Phi(0)=\Phi_0\ , \tag{2.2a}$$

where Φ_0 is uniformly distributed between 0 and 2π and $\mu(t)$ is a δ-correlated Gaussian random process with

$$\begin{aligned}&\langle\mu(t)\rangle=0\ ,\\ &\langle\mu(t_1)\mu(t_2)\rangle=2\gamma_c\delta(t_1-t_2)\ .\end{aligned} \tag{2.2b}$$

Here the angular brackets denote the ensemble average with respect to the distribution of random process $\mu(t)$ and γ_c^{-1} is the correlation time for laser amplitude fluctuations. The electric field of the fluctuating laser whose intensity and hence, amplitude are time modulated at low frequencies can be written as

$$\vec{E}(t)=\tfrac{1}{2}\vec{\mathscr{E}}_0(1+a\cos\Omega t)e^{-i[\omega_L t+\Phi(t)]}+\text{c.c.}\ , \tag{2.3}$$

where Ω is the modulating frequency and a the index of modulation. The modulation of the exciting source is assumed to be weak, i.e., $a<<1$. Thus, the incident field is comprised of three frequencies, viz., ω_L, $\omega_L+\Omega$, and $\omega_L-\Omega$. The amplitude of the latter two components is $\frac{1}{2}a$ times that of the central component and hence, is much smaller. For a monochromatic laser ($\gamma_c\to 0$) the three components will be well separated, this being also true for a source with a bandwidth much smaller than the modulation frequency ($\gamma_c/\Omega<<1$). If $\gamma_c/\Omega>>1$ there will be considerable overlap between the three components. In the absence of laser detuning ($\delta=0$) and $\mathscr{s}=\Omega$, the shifted components $\omega_L+\Omega$ and $\omega_L-\Omega$ are exactly resonant with the $|1\rangle\leftrightarrow|3\rangle$ and $|2\rangle\leftrightarrow|3\rangle$ transitions, respectively, while the central component ω_L is on resonance with the

$$|J=0,m_J=0\rangle\to|J=1,m_J=0\rangle$$

transition [refer to Fig. 1(b)]. The average intensity produced by such a source will be

$$\begin{aligned}I(t)&=\tfrac{1}{2}\mathscr{E}_0^2(1+a\cos\Omega t)^2\\ &\approx\tfrac{1}{2}\mathscr{E}_0^2(1+2a\cos\Omega t)\ ,\end{aligned}$$

since the modulation index a is considered to be weak. Throughout this paper we will ignore the terms of order a^2. It is to be noted that as long as one continues to ignore the terms of the order of a^2 the modulation of the intensity and the amplitude are equivalent. In the dipole approximation the Hamiltonian for the interaction between the atom and the external field is

$$H_{\text{ext}}(t)=-\vec{d}\cdot\vec{E}(t)\ , \tag{2.4}$$

where $\vec{d}$ is the atomic dipole moment operator having only off-diagonal elements. The equations for the evolution of the diagonal and off-diagonal matrix elements are easily obtained as

$$\begin{aligned}
\frac{\partial\tilde{\rho}_{11}}{\partial t}&=-2\gamma_1\tilde{\rho}_{11}+[i\alpha_1(t)(1+a\cos\Omega t)\tilde{\rho}_{31}+\text{c.c.}]\ ,\\
\frac{\partial\tilde{\rho}_{12}}{\partial t}&=-(\gamma_1+\gamma_2+2i\mathscr{s})\tilde{\rho}_{12}+i\alpha_1(t)(1+a\cos\Omega t)\tilde{\rho}_{32}-i\alpha_2^*(t)(1+a\cos\Omega t)\tilde{\rho}_{13}\ ,\\
\frac{\partial\tilde{\rho}_{13}}{\partial t}&=-[\gamma_1+i(\mathscr{s}-\delta)]\tilde{\rho}_{13}+i\alpha_1(t)(1+a\cos\Omega t)(\tilde{\rho}_{33}-\tilde{\rho}_{11})-i\alpha_2(t)(1+a\cos\Omega t)\tilde{\rho}_{12}\ ,\\
\frac{\partial\tilde{\rho}_{22}}{\partial t}&=-2\gamma_2\tilde{\rho}_{22}+[i\alpha_2(t)(1+a\cos\Omega t)\tilde{\rho}_{32}+\text{c.c.}]\ ,\\
\frac{\partial\tilde{\rho}_{23}}{\partial t}&=-[\gamma_2-i(\mathscr{s}+\delta)]\tilde{\rho}_{23}+i\alpha_2(t)(1+a\cos\Omega t)(\tilde{\rho}_{33}-\tilde{\rho}_{22})-i\alpha_1(t)(1+a\cos\Omega t)\tilde{\rho}_{21}\ ,\\
\frac{\partial\tilde{\rho}_{33}}{\partial t}&=2\gamma_1\tilde{\rho}_{11}+2\gamma_2\tilde{\rho}_{22}+[i\alpha_1^*(t)(1+a\cos\Omega t)\tilde{\rho}_{13}+i\alpha_2^*(t)(1+a\cos\Omega t)\tilde{\rho}_{23}+\text{c.c.}]\ ,
\end{aligned} \tag{2.5}$$

where the incoherent terms correspond to spontaneous emission,[15] and where the oscillations at optical frequencies have been removed by transforming to the slowly varying quantities

$$\tilde{\rho}_{ii}=\rho_{ii},\quad \tilde{\rho}_{13}=\rho_{13}e^{i\omega_L t},$$
$$\tilde{\rho}_{12}=\rho_{12},\quad \tilde{\rho}_{23}=\rho_{23}e^{i\omega_L t}, \tag{2.6}$$

and neglecting terms with $e^{\pm 2i\omega_L t}$ [rotating-wave approximation (RWA)]. The parameters $\alpha_1(t)$, $\alpha_2(t)$ are defined by

$$2\alpha_1(t)=\vec{d}_{13}\cdot\vec{\mathscr{E}}_0 e^{-i\Phi(t)},$$
$$2\alpha_2(t)=\vec{d}_{23}\cdot\vec{\mathscr{E}}_0 e^{-i\Phi(t)}, \tag{2.7}$$

and $2\gamma_1$ ($2\gamma_2$) represents the transition probability per unit time for spontaneously emitting a photon in the transition $|1\rangle\rightarrow|3\rangle$ ($|2\rangle\rightarrow|3\rangle$). Equation (2.5) involves the phase $\Phi(t)$ in a nonlinear fashion. By a redefinition of the variables the equations may be written as a linearized set of equations. On introducing the variables

$$\sigma_1=\tilde{\rho}_{11},\quad \sigma_2=\tilde{\rho}_{12},$$
$$\sigma_3=\tilde{\rho}_{13}e^{i\Phi(t)},\quad \sigma_4=\sigma_2^*,$$
$$\sigma_5=\tilde{\rho}_{22},\quad \sigma_6=\tilde{\rho}_{23}e^{i\Phi(t)},$$
$$\sigma_7=\tilde{\rho}_{31}e^{-i\Phi(t)},\quad \sigma_8=\sigma_6^*, \tag{2.8}$$

and the normalization condition $\tilde{\rho}_{11}+\tilde{\rho}_{22}+\tilde{\rho}_{33}=1$, the coupled Eq. (2.5) may be written in matrix form as

$$\dot{\sigma}(t)=[A-i\mu(t)B]\sigma(t)+I$$
$$+a\cos\Omega t[C\sigma(t)+I], \tag{2.9}$$

where $\sigma(t)$ and I are the following column matrices of order 8:

$$\sigma(t)=\begin{bmatrix}\sigma_1(t)\\ \sigma_2(t)\\ \sigma_3(t)\\ \sigma_4(t)\\ \sigma_5(t)\\ \sigma_6(t)\\ \sigma_7(t)\\ \sigma_8(t)\end{bmatrix},\quad I=\begin{bmatrix}0\\ 0\\ i\alpha_1\\ 0\\ 0\\ i\alpha_2\\ -i\alpha_1^*\\ -\alpha_2^*\end{bmatrix}, \tag{2.10}$$

while A, B, and C are the following square (8×8) matrices:

$$A=\begin{bmatrix}
-2\gamma_1 & 0 & -i\alpha_1^* & 0 & 0 & 0 & i\alpha_1 & 0\\
0 & -\gamma_1-\gamma_2-2i\delta & -i\alpha_2^* & 0 & 0 & 0 & 0 & i\alpha_1\\
-2i\alpha_1 & -i\alpha_2 & -\gamma_1-i(\delta-\delta) & 0 & -i\alpha_1 & 0 & 0 & 0\\
0 & 0 & 0 & -\gamma_1-\gamma_2+2i\delta & 0 & -i\alpha_1^* & i\alpha_2 & 0\\
0 & 0 & 0 & 0 & -2\gamma_2 & -i\alpha_2^* & 0 & i\alpha_2\\
-i\alpha_2 & 0 & 0 & -i\alpha_1 & -2i\alpha_2 & -\gamma_2+i(\delta+\delta) & 0 & 0\\
2i\alpha_1^* & 0 & 0 & +i\alpha_2^* & i\alpha_1^* & 0 & -\gamma_1+i(\delta-\delta) & 0\\
i\alpha_2^* & i\alpha_1^* & 0 & 0 & 2i\alpha_2^* & 0 & 0 & -\gamma_2-i(\delta+\delta)
\end{bmatrix},$$

$$B_{33}=B_{66}=-B_{77}=-B_{88}=-1,$$
$$C_{ii}=0,\quad C_{ij}=A_{ij}, \tag{2.11}$$

the remaining elements of B being zero. Equation (2.9) has the form of the standard equation of the multiplicative stochastic processes discussed in detail in Ref. (16) from which it follows that the average of σ over the distribution of Φ satisfies the exact equation

$$\langle\dot{\sigma}\rangle=A\langle\sigma\rangle-\gamma_c\Lambda\langle\sigma\rangle+I$$
$$+a\cos\Omega t(C\langle\sigma\rangle+I), \tag{2.12}$$
$$\Lambda_{ii}=1\quad\text{for}\quad i=3,6,7,8;$$

the remaining elements of Λ being zero. Thus, the diagonal elements of the density matrix are unaffected by the phase fluctuations present in the laser beam, while the effect on the off-diagonal elements is to alter the decay rates as

$$\gamma_1\rightarrow\gamma_1+\gamma_c,$$
$$\gamma_2\rightarrow\gamma_2+\gamma_c. \tag{2.13}$$

This is an example of the substitution rule which has been emphasized in the context of optical resonance.[17,18]

We are interested in the shape of the detection

signals $L(\hat{X})$ and $L(\hat{Y})$ obtained when one measures the total intensity of the modulated fluorescence along the Z and X directions with linear polarizations $\hat{X}$ and $\hat{Y}$, respectively. These can be easily related[14] to the populations of the Zeeman sublevels $|1\rangle$, $|2\rangle$, and the Zeeman coherence between these two levels

$$L\{\begin{matrix}\hat{X}\\ \hat{Y}\end{matrix}\} \propto \rho_{11}+\rho_{22}\pm 2\,\mathrm{Re}\rho_{12}\ , \tag{2.14}$$

where we have made use of the following relationship (phases of wave functions properly chosen) for the dipole matrix elements between a Zeeman sublevel and the ground state

$$\vec{d}_{13}=R(\hat{X}-i\hat{Y}),\quad \vec{d}_{23}=R(\hat{X}+i\hat{Y})\ . \tag{2.15}$$

The ensemble-averaged signals will then be

$$L\{\begin{matrix}\hat{X}\\ \hat{Y}\end{matrix}\} \propto \langle\sigma_1\rangle+\langle\sigma_5\rangle\pm 2\,\mathrm{Re}\langle\sigma_2\rangle\ , \tag{2.16}$$

where we have made use of Eq. (2.8). Thus, in order to study the detection signals we must obtain the solutions of Eq. (2.12). These equations constitute a set of eight coupled equations for the ensemble-averaged quantities $\langle\sigma\rangle$ and can be solved by straightforward Laplace transform techniques. However, for general values of the parameters occurring in the problem the solutions will require the inversion of 8×8 matrices which is cumbersome. We will obtain analytical expressions for the intensity of the modulated fluorescence at steady states for weak laser fields, studying limiting cases in laser bandwidth in detail with numerical plots for sources of arbitrary bandwidth. The case of the fields with arbitrary intensity and bandwidth will be studied numerically.

III. MODULATED HANLE SIGNALS IN PARTIALLY COHERENT FIELDS

A. Weak fields

In this subsection we consider the optical source used to prepare the atom in a coherent superposition of the Zeeman sublevels as a weak, partially coherent one. Since the depth of modulation of the exciting source is assumed to be very weak ($a << 1$), the solutions of the coupled Eqs. (2.12) may be studied within the framework of a perturbation theory. Thus, we may expand the ensemble averaged $\langle\sigma\rangle$ in a power series with a as the expansion parameter

$$\langle\sigma\rangle=\langle\sigma\rangle^{(0)}+\langle\sigma\rangle^{(1)}+\langle\sigma\rangle^{(2)}+\cdots\ . \tag{3.1}$$

We further assume the exciting source to be weak ($\alpha/\gamma << 1$), and study the signals which are linear in the laser intensity, i.e., to second order in α. Within these approximations we obtain the following results for the required density-matrix elements [the subscript (superscript) denotes the order with respect to the field (modulation)]:

$$\langle\tilde{\rho}_{11}(\infty)\rangle^{(1)}_{(2)}=\frac{a\alpha^2}{2}\,\frac{e^{i\Omega t}}{2\gamma+i\Omega}\left[\frac{1}{\gamma+\gamma_c+i(\delta-\delta)}+\frac{1}{\gamma+\gamma_c-i(\delta-\delta)}+\frac{1}{\gamma+\gamma_c+i[\Omega+(\delta-\delta)]}\right.$$
$$\left.+\frac{1}{\gamma+\gamma_c+i[\Omega-(\delta-\delta)]}\right]+\text{c.c.}\ , \tag{3.2}$$

$$\langle\tilde{\rho}_{22}(\infty)\rangle^{(1)}_{(2)}=\frac{a\alpha^2}{2}\,\frac{e^{i\Omega t}}{2\gamma+i\Omega}\left[\frac{1}{\gamma+\gamma_c+i(\delta+\delta)}+\frac{1}{\gamma+\gamma_c-i(\delta+\delta)}+\frac{1}{\gamma+\gamma_c+i[\Omega+(\delta+\delta)]}\right.$$
$$\left.+\frac{1}{\gamma+\gamma_c+i[\Omega-(\delta+\delta)]}\right]+\text{c.c.}\ , \tag{3.3}$$

$$\langle\tilde{\rho}_{12}(\infty)\rangle^{(1)}_{(2)}=\frac{a\alpha^2}{4}\,\frac{e^{i\Omega t}}{\gamma+i(\delta+\Omega/2)}\left[\frac{1}{\gamma+\gamma_c+i(\delta+\delta)}+\frac{1}{\gamma+\gamma_c+i(\Omega+\delta+\delta)}\right]$$
$$+\frac{a\alpha^2}{4}\,\frac{e^{-i\Omega t}}{\gamma+i(\delta-\Omega/2)}\left[\frac{1}{\gamma+\gamma_c+i(\delta+\delta)}+\frac{1}{\gamma+\gamma_c+i(\delta+\delta-\Omega)}\right]+\cdots\ , \tag{3.4}$$

where the ellipses represent terms with $\delta \rightarrow -\delta$.

The fluorescent light will thus be modulated at the same frequency as the exciting light, while the amplitude of modulation will exhibit resonances. The modulated fluorescence may be observed with a phase-sensitive detector, which enables the cosine and sine component of the fluorescence to be studied separately. The resonances in the amplitude of the sine or cosine component may be studied by varying the magnetic field (hence the Larmor frequency or Zeeman splitting of the excited state) for a fixed frequency of modulation, or by changing the modulation frequency of the exciting source for a fixed strength of the static magnetic field. In general, the magnetic-field scan of the modulated fluorescence will exhibit resonances whenever the Zeeman splitting has the values $\mathscr{s} = \pm\delta, \Omega \pm \delta$, $-\Omega \pm \delta$, $\pm\Omega/2$. The resonance at $\mathscr{s} = \pm\Omega/2$ has a width γ, while the width of the remaining resonances is dependent on the laser bandwidth and is given by $\gamma + \gamma_c$. The shape of these resonances may be absorption or dispersion with considerable overlap depending on the magnitude of the parameters Ω, δ, γ, γ_c. The resonances at $\mathscr{s} = \delta, \pm\Omega + \delta$ $(-\delta, \pm\Omega - \delta)$ correspond to the matching of the atomic energy level $|J=1, m_J=1\rangle$ ($|J=1, m_J=-1\rangle$) with the three frequencies present in the exciting light. The resonances at $\mathscr{s} = \pm\Omega/2$ arise due to the interference between the probability amplitudes of the photon emissions in the transitions

$$|J=1, m_J=1\rangle \leftrightarrow |J=0, m_J=0\rangle$$

and

$$|J=1, m_j=-1\rangle \leftrightarrow |J=0, m_j=0\rangle \ .$$

We first study the conventional magnetic-field scan of the modulated Hanle signals.

1. Magnetic-field scan of signals

Further simplification of (3.4) for the case of resonant excitation ($\delta = 0$) yields

$$\begin{aligned} L\{\hat{x}\} = {} & 2a\alpha^2 \cos\Omega t \left[\left(\frac{4\gamma}{4\gamma^2+\Omega^2} \mp \frac{4\gamma_c}{4\gamma_c^2+\Omega^2} \right) \mathscr{L}_0 + \left(\frac{2\gamma}{4\gamma^2+\Omega^2} \mp \frac{2\gamma_c}{4\gamma_c^2+\Omega^2} \right) (\mathscr{L}_1 + \mathscr{L}_2) \right. \\ & \left. - \left(\frac{\Omega}{4\gamma^2+\Omega^2} \mp \frac{\Omega}{4\gamma_c^2+\Omega^2} \right) (\mathscr{D}_1 - \mathscr{D}_2) \pm \frac{4\gamma_c}{4\gamma_c^2+\Omega^2} (\mathscr{L}_3 + \mathscr{L}_4) \right] \\ & + 2a\alpha^2 \sin\Omega t \left[\left(\frac{2\Omega}{4\gamma^2+\Omega^2} \pm \frac{2\Omega}{4\gamma_c^2+\Omega^2} \right) \mathscr{L}_0 + \left(\frac{\Omega}{4\gamma^2+\Omega^2} \mp \frac{\Omega}{4\gamma_c^2+\Omega^2} \right) (\mathscr{L}_1 + \mathscr{L}_2) \right. \\ & \left. + \left(\frac{2\gamma}{4\gamma^2+\Omega^2} \mp \frac{2\gamma_c}{4\gamma_c^2+\Omega^2} \right) (\mathscr{D}_1 - \mathscr{D}_2) \pm \frac{4\gamma_c}{4\gamma_c^2+\Omega^2} (\mathscr{D}_3 - \mathscr{D}_4) \right] , \end{aligned} \tag{3.5}$$

where

$$\begin{aligned} & \mathscr{L}_0 = \frac{\gamma+\gamma_c}{(\gamma+\gamma_c)^2+\mathscr{s}^2} , \quad \mathscr{D}_0 = \frac{\mathscr{s}}{(\gamma+\gamma_c)^2+\mathscr{s}^2} , \quad \mathscr{L}_1 = \frac{\gamma+\gamma_c}{(\gamma+\gamma_c)^2+(\mathscr{s}+\Omega)^2} , \quad \mathscr{D}_1 = \frac{\mathscr{s}+\Omega}{(\gamma+\gamma_c)^2+(\mathscr{s}+\Omega)^2} , \\ & \mathscr{L}_2 = \frac{\gamma+\gamma_c}{(\gamma+\gamma_c)^2+(\mathscr{s}-\Omega)^2} , \quad \mathscr{D}_2 = \frac{\mathscr{s}-\Omega}{(\gamma+\gamma_c)^2+(\mathscr{s}-\Omega)^2} , \quad \mathscr{L}_3 = \frac{\gamma}{\gamma^2+(\mathscr{s}+\Omega/2)^2} , \quad \mathscr{D}_3 = \frac{\mathscr{s}+\Omega/2}{\gamma^2+(\mathscr{s}+\Omega/2)^2} , \\ & \mathscr{L}_4 = \frac{\gamma}{\gamma^2+(\mathscr{s}-\Omega/2)^2} , \quad \mathscr{D}_4 = \frac{\mathscr{s}-\Omega/2}{\gamma^2+(\mathscr{s}-\Omega/2)^2} . \end{aligned} \tag{3.6}$$

Thus, there are resonances in the amplitude of the sine and cosine component of the modulated fluorescence at field values ($\mathscr{s} \equiv gH$) $H = 0, \pm\Omega/2g, \pm\Omega/g$ where g is the gyromagnetic ratio. The width of the resonances at field values $0, \pm\Omega/g$ is affected by the bandwidth of the fluc-

tuating laser beam, the laser bandwidth (γ_c) simply adding to half the radiative bandwidth (γ) to give the width ($\gamma+\gamma_c$) of these resonances. However, the resonances obtained when the modulation frequency is equal to the separation between the Zeeman sublevels ($\Omega=\pm 2\delta$) have a width γ associated with the spontaneous decay of the excited state *only*. As may be seen from (3.5), the weight factor of each resonant structure depends critically on the bandwidth of the exciting source besides its frequency of modulation. It should be borne in mind that one expects additional resonances in higher orders of perturbation theory, i.e., terms of order higher than a. The resonances in the modulated signals could be understood as follows—the resonances at $\delta=\pm\Omega$ essentially arise due to the absorption of photon with frequency $\omega_L\pm\Omega$. Note that the intensity of emission has contributions from the populations $\tilde{\rho}_{11}$ and $\tilde{\rho}_{22}$ of the excited states and the population of the excited states is given by the absorption line shape which is known to have a width ($\gamma+\gamma_c$) in the limit of weak fields [cf. the damping term in the $\langle\sigma_3\rangle$ Eq. (2.12)]. Therefore, the modulated signals at $\delta=\pm\Omega$ have absorptive and dispersive type of contributions with width ($\gamma+\gamma_c$). The resonances at $\delta=\pm\Omega/2$ arise due to the interference term $\mathrm{Re}\tilde{\rho}_{12}$ in the intensity of fluorescence where $\mathrm{Re}\tilde{\rho}_{12}$ represents the optical coherence between the two magnetic sublevels induced by the exciting field. In the rotating frame $\mathrm{Re}\tilde{\rho}_{12}$ has the oscillation frequency 2δ and the damping parameter associated with $\tilde{\rho}_{12}$ is 2γ. Therefore, this leads to extra resonances at $\delta=\pm\Omega/2$ with width γ. We now consider the following limiting cases explicitly.

i. Broadband excitation. If the optical source used to prepare the atoms in a coherent superposition of the Zeeman sublevel is a weak, incoherent broadband lamp such as a discharge tube ($\gamma_c\to\infty$ such that α^2/γ_c remains a constant equal to say β), then (3.5) reduces to

$$L\{\overset{\hat{x}}{\hat{y}}\}=4a\beta\cos\Omega t\left[\frac{4\gamma}{4\gamma^2+\Omega^2}\pm\frac{1}{2}\left[\frac{\gamma}{\gamma^2+(\delta+\Omega/2)^2}+\frac{\gamma}{\gamma^2+(\delta-\Omega/2)^2}\right]\right]$$
$$+4a\beta\sin\Omega t\left[\frac{2\Omega}{4\gamma^2+\Omega^2}\pm\frac{1}{2}\left[\frac{\delta+\Omega/2}{\gamma^2+(\delta+\Omega/2)^2}-\frac{\delta-\Omega/2}{\gamma^2+(\delta-\Omega/2)^2}\right]\right], \tag{3.7}$$

which agrees with the results of Corney and Series[7] for signals detected perpendicular to $\vec{H}$ [which corresponds to the latter choice of sign in expression (3.7)], the incident light being linearly polarized at right angles to the magnetic field direction. The result was later verified experimentally by Corney.[8] Thus, as the magnetic field is scanned around zero value resonances appear in the amplitude of modulation for field values $H=\pm\Omega/2g$, the shape of these resonances being Lorentzian (dispersionlike) for the cosine (sine) component of the fluorescence, the width being determined by the natural width of the excited state. In general, for small frequencies of modulation the resonances at $\delta=\pm\Omega/2$ in the amplitude of the cosine or sine component overlap. Use of higher frequency of modulation of the exciting source ($\Omega/\gamma=10$, as in Figs. 2–5) results in well-defined Lorentzian (dispersion) shaped resonances at $\delta=\pm\Omega/2=\pm 5$ for the cosine (sine) component of the fluorescence detected parallel or perpendicular to the magnetic field when $\gamma_c/\gamma>>1$ (see the dot-dash curves of Figs. 2–5 which correspond to $\gamma_c=5\gamma$). The signals detected along the two directions are similar in shape but inverted about the horizontal axis (compare Fig. 2 with Fig. 4, Fig. 3 with Fig. 5), as may also be seen from (3.7), this result having been confirmed experimentally by Corney.[8]

ii. Monochromatic excitation. For a strictly monochromatic laser beam ($\gamma_c=0$), expression (3.5) simplifies to

$$L\{\overset{\hat{x}}{\hat{y}}\}=2a\alpha^2\cos\Omega t\left[\frac{4\gamma}{4\gamma^2+\Omega^2}\mathscr{L}_0+\frac{2\gamma}{4\gamma^2+\Omega^2}(\mathscr{L}_1+\mathscr{L}_2)-\left[\frac{\Omega}{4\gamma^2+\Omega^2}\mp\frac{1}{\Omega}\right](\mathscr{D}_1-\mathscr{D}_2)\right]$$
$$+2a\alpha^2\sin\Omega t\left[\left[\frac{2\Omega}{4\gamma^2+\Omega^2}\pm\frac{2}{\Omega}\right]\mathscr{L}_0+\left[\frac{\Omega}{4\gamma^2+\Omega^2}\mp\frac{1}{\Omega}\right](\mathscr{L}_1+\mathscr{L}_2)+\frac{2\gamma}{4\gamma^2+\Omega^2}(\mathscr{D}_1-\mathscr{D}_2)\right]. \tag{3.8}$$

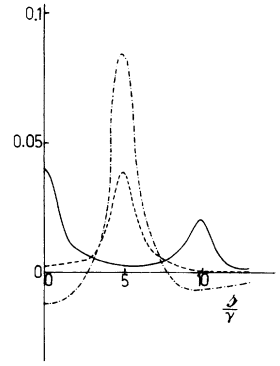

FIG. 2. Magnetic-field scan of the amplitude of the cosine component of the modulated fluorescence detected parallel to $\vec{H}$ for $\Omega/\gamma=10$ and for the following values of the bandwidth parameter: (a) ——, $\gamma_c=0$; (b) - - - - -, $\gamma_c=\gamma$; (c) - · - · -, $\gamma_c=5\gamma$.

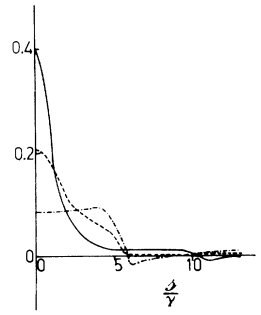

FIG. 3. Magnetic-field scan of the amplitude of the sine component of the modulated fluorescence detected parallel to $\vec{H}$ for the same values of the parameters as in Fig. 2.

Hence, in contrast to the broadband excitation, we expect resonances in the magnetic field scan of the modulated fluorescence whenever $\delta=0,\pm\Omega$. The physical origin of these resonances may be understood as follows: At $\delta=0$ the relevant magnetic sublevels of the $J=1$ state have the energies $\omega_L\pm\delta$. The three components of the exciting light are $\omega_L,\omega_L\pm\Omega$. Thus, the central component excites the atoms resonantly when $\delta=0$ while the shifted components are off resonance. At $\delta=\pm\Omega$ the shifted components are in resonance with the $|1\rangle\leftrightarrow|3\rangle$, $|2\rangle\leftrightarrow|3\rangle$ transitions, respectively, but ω_L is off resonance. The resonance at $\delta=\pm\Omega/2$ disappears since for monochromatic excitation none of these components is at resonance. In fact, as discussed earlier, this resonance arises from $\mathrm{Re}\tilde{\rho}_{12}$ in the intensity of the fluorescence light and is a measure of the optical coherence between two magnetic sublevels. This coherence can be shown to be directly proportional to the bandwidth of the exciting field and hence, vanishes in the monochromatic case. Of course, if one allows for the detuning of the applied field and if this is set at say $\Omega/2$, then it follows from the discussion following (3.4) that the magnetic field scan will reveal resonances at $\delta=\pm\Omega/2=\pm\delta$ even in the monochromatic case. The resonance at $\delta=0$ is Lorentzian while the resonances at $\delta=\pm\Omega$ have both Lorentzian and dispersion parts with different weight factors. When $\Omega/\gamma=10$ (Figs. 2–5) the resonances are clearly resolved. Use of a higher value of modulation frequency also has the added advantage of making the weight factors of the Lorentzian and dispersion shaped parts of the resonances at $\delta=\pm\Omega$ very different from each other so that either one of them is predominantly present at $\delta=\pm\Omega$, thus making the interpretation of results easy. The signals detected along and perpendicular to the magnetic field are no longer similar in shape for a particular component of the modulated fluorescence. For instance, the resonance at $\delta=\Omega$ in the amplitude of the cosine component is a Lorentzian for $L(\hat{x})$ (solid curve of Fig. 2) and is predominantly dispersion shaped for $L(\hat{y})$ (solid curve of Fig. 4), as may also be verified from (3.8).

iii. Intermediate case. For an optical source with bandwidth of the same order of magnitude as the radiative width of the excited state ($\gamma_c\sim\gamma$), the expressions (3.5) for the signals simplify to

$$
\begin{aligned}
L(\hat{x}) &\propto \cos\Omega t\,\frac{4\gamma}{4\gamma^2+\Omega^2}(\mathscr{L}_3+\mathscr{L}_4)+\sin\Omega t\left[\frac{4\Omega}{4\gamma^2+\Omega^2}\mathscr{L}_0+\frac{4\gamma}{4\gamma^2+\Omega^2}(\mathscr{D}_3-\mathscr{D}_4)\right],\\
L(\hat{Y}) &\propto \cos\Omega t\left[\frac{8\gamma}{4\gamma^2+\Omega^2}\left[\mathscr{L}_0+\tfrac{1}{2}(\mathscr{L}_1+\mathscr{L}_2)-\tfrac{1}{2}(\mathscr{L}_3+\mathscr{L}_4)\right]-\frac{2\Omega}{4\gamma^2+\Omega^2}(\mathscr{D}_1-\mathscr{D}_2)\right]\\
&\quad+\sin\Omega t\left[\frac{2\Omega}{4\gamma^2+\Omega^2}(\mathscr{L}_1+\mathscr{L}_2)+\frac{4\gamma}{4\gamma^2+\Omega^2}(\mathscr{D}_1-\mathscr{D}_2-\mathscr{D}_3+\mathscr{D}_4)\right].
\end{aligned}
\tag{3.9}
$$

Thus, for signals detected along the magnetic-field direction the amplitude of the cosine component has a

Lorentzian shaped resonance at $\delta=\pm\Omega/2$ (broken curve of Fig. 2), while for the sine component there are resonances at $\delta=0,\pm\Omega/2$, the weight of the Lorentzian peak at $\delta=0$ (broken curve of Fig. 3) being much greater than the dispersion shaped resonance at $\delta=\pm\Omega/2$ for large frequencies of modulation. Magnetic-field scan of fluorescence detected perpendicular to $\vec{H}$ reveals resonant structures corresponding to $\delta=0,\pm\Omega/2,\pm\Omega$ with considerable overlap even for $\Omega/\gamma=10$, as may be seen from the broken curves of Figs. 4 and 5.

2. *Modulation-frequency scan of signals*

We have seen that when atoms whose excited states have Zeeman structures are excited by light whose intensity is modulated in time, then the fluorescent light modulated at the same frequency exhibits resonances. These resonances may also be studied by tuning the modulation frequency of the exciting source for a fixed strength of the static magnetic field. Simplification of Eqs. (3.2)–(3.4) for the case of resonant excitation yields

$$
\begin{aligned}
L\{{}^{S}_{P}\} = {} & 2a\alpha^2\cos\Omega t\left[\left\{\frac{\gamma+\gamma_c}{(\gamma+\gamma_c)^2+\delta^2}-\frac{\gamma-\gamma_c}{(\gamma-\gamma_c)^2+\delta^2}\right\}[2\mathscr{L}'_0\pm(\mathscr{L}'_3+\mathscr{L}'_4)]\right.\\
& \mp\left\{\frac{\delta}{(\gamma+\gamma_c)^2+\delta^2}-\frac{\delta}{(\gamma-\gamma_c)^2+\delta^2}\right\}(\mathscr{D}'_3-\mathscr{D}'_4)\\
& \left.+\frac{\gamma-\gamma_c}{(\gamma-\gamma_c)^2+\delta^2}[\mathscr{L}'_1+\mathscr{L}'_2\pm(\mathscr{L}'_1+\mathscr{L}'_2)]+\frac{\delta}{(\gamma-\gamma_c)^2+\delta^2}[\mathscr{D}'_1-\mathscr{D}'_2\mp(\mathscr{D}'_1-\mathscr{D}'_2)]\right]\\
& +2a\alpha^2\sin\Omega t\left[\left\{\frac{\gamma+\gamma_c}{(\gamma+\gamma_c)^2+\delta^2}-\frac{\gamma-\gamma_c}{(\gamma-\gamma_c)^2+\delta^2}\right\}[2\mathscr{D}'_0\pm(\mathscr{D}'_3+\mathscr{D}'_4)]\right.\\
& \pm\left\{\frac{\delta}{(\gamma+\gamma_c)^2+\delta^2}-\frac{\delta}{(\gamma-\gamma_c)^2+\delta^2}\right\}(\mathscr{L}'_3-\mathscr{L}'_4)\\
& \left.+\frac{\gamma-\gamma_c}{(\gamma-\gamma_c)^2+\delta^2}[\mathscr{D}'_1+\mathscr{D}'_2\pm(\mathscr{D}'_1+\mathscr{D}'_2)]-\frac{\delta}{(\gamma-\gamma_c)^2+\delta^2}[\mathscr{L}'_1-\mathscr{L}'_2\mp(\mathscr{L}'_1-\mathscr{L}'_2)]\right],
\end{aligned}
\tag{3.10}
$$

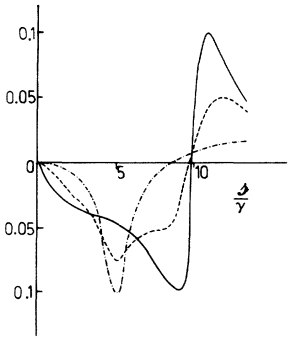

FIG. 4. Magnetic-field scan of the amplitude of the cosine component of the modulated fluorescence detected perpendicular to $\vec{H}$ for the same values of the parameters as in Fig. 2.

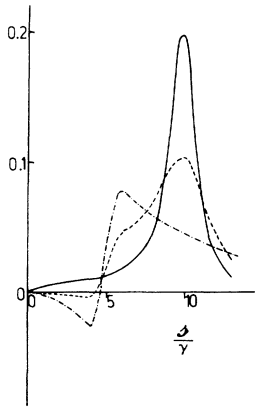

FIG. 5. Magnetic-field scan of the amplitude of the sine component of the modulated fluorescence detected perpendicular to $\vec{H}$ for the same values of the parameters in Fig. 2.

where

$$\mathscr{L}'_0=\frac{2\gamma}{4\gamma^2+\Omega^2}\,,\quad \mathscr{D}'_0=\frac{\Omega}{4\gamma^2+\Omega^2}\,,$$
$$\mathscr{L}'_1=\mathscr{L}_1\,,\quad \mathscr{D}'_1=\mathscr{D}_1\,,$$
$$\mathscr{L}'_2=\mathscr{L}_2\,,\quad \mathscr{D}'_2=-\mathscr{D}_2\,, \tag{3.11}$$
$$\mathscr{L}'_3=\tfrac{1}{2}\mathscr{L}_3\,,\quad \mathscr{D}'_3=\tfrac{1}{2}\mathscr{D}_3\,,$$
$$\mathscr{L}'_4=\tfrac{1}{2}\mathscr{L}_4\,,\quad \mathscr{D}'_4=-\tfrac{1}{2}\mathscr{D}_4\,.$$

Hence, there are resonances in the amplitude of modulation of the fluorescent light whenever the frequency of modulation is zero or equal to the Larmor precessional frequency or twice of it, i.e., at $\Omega=0,\pm\omega,\pm2\omega$. The bandwidth of the fluctuating laser beam effects the width of the resonance at $\Omega=\pm\omega$ only, the remaining resonances having a width determined by the lifetime of the excited state only. The shape of the detected signals is critically dependent on the relative magnitude of the Larmor frequency and the decay constants. For small strengths of the magnetic field so that the separation between the Zeeman sublevels is of the same order as the natural width ($\omega/\gamma\sim1$), there is considerable overlap of the resonances. At higher magnetic-field strengths the resonances are clearly resolved and the effect of source bandwidth more pronounced, as will be shown by the study of the following limiting cases.

i. Broadband excitation. The signals (3.10) reduce to

$$\begin{aligned}L(\hat{X})&\propto\cos\Omega t(2\mathscr{L}'_0+\mathscr{L}'_3+\mathscr{L}'_4)\\&\quad+\sin\Omega t(2\mathscr{D}'_0+\mathscr{D}'_3+\mathscr{D}'_4)\\L(\hat{Y})&\propto\cos\Omega t(2\mathscr{L}'_0-\mathscr{L}'_3-\mathscr{L}'_4)\\&\quad+\sin\Omega t(2\mathscr{D}'_0-\mathscr{D}'_3-\mathscr{D}'_4)\,.\end{aligned} \tag{3.12}$$

Thus, when the optical source used to excite the atoms is a weak, incoherent broadband one the modulated fluorescence exhibits resonances at $\Omega=0,\pm2\omega$ of width 2γ. These resonances are Lorentzian (dispersion) shaped for the amplitude of the cosine (sine) component of the modulated fluorescence. The plot of the amplitude of the cosine component (which is chosen for illustrating the main features) detected parallel and perpendicular to $\vec{H}$ is shown in Figs. 6 and 7, respectively. The dot-dash curves of Figs. 6 and 7 correspond to $\gamma_c=5\gamma$ when the magnetic field is held constant at $\omega/\gamma=5$.

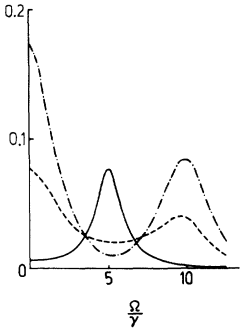

FIG. 6. Modulation-frequency scan of the amplitude of the cosine component of the modulated fluorescence detected parallel to H for $\omega/\gamma=5$ and for the following values of the bandwidth parameter: (a) ——, $\gamma_c=0$; (b) - - - -, $\gamma_c=\gamma$; (c) - · - · -, $\gamma_c=5\gamma$.

ii. Monochromatic excitation. The detection signals are

$$\begin{aligned}L(\hat{x})&\propto\cos\Omega t(\mathscr{L}'_1+\mathscr{L}'_2)+\sin\Omega t(\mathscr{D}'_1+\mathscr{D}'_2)\,,\\L(\hat{Y})&\propto\cos\Omega t(\mathscr{D}'_1-\mathscr{D}'_2)+\sin\Omega t(\mathscr{L}'_2-\mathscr{L}'_1)\,.\end{aligned} \tag{3.13}$$

In contrast to (i) [Eqs. (3.12)], resonances are now obtained at $\Omega=\pm\omega$ of width γ, the two detection signals being entirely different for a particular component of the fluorescence as may be seen from the solid curves of Figs. 6 and 7 corresponding to $\gamma_c=0$.

iii. Intermediate case. For an optical source of bandwidth of the same order as the width of the excited state ($\gamma_c\sim\gamma$), the detection signals are no longer so simple:

$$\begin{aligned}L(\hat{x})&\propto\cos\Omega t\left[\frac{2\gamma}{4\gamma^2+\omega^2}(2\mathscr{L}'_0+\mathscr{L}'_3+\mathscr{L}'_4)+\frac{4\gamma^2}{\omega(4\gamma^2+\omega^2)}(\mathscr{D}'_3-\mathscr{D}'_4)\right]\\&\quad+\sin\Omega t\left[\frac{2\gamma}{4\gamma^2+\omega^2}(2\mathscr{D}'_0+\mathscr{D}'_3+\mathscr{D}'_4)-\frac{4\gamma^2}{\omega(4\gamma^2+\omega^2)}(\mathscr{L}'_3-\mathscr{L}'_4)\right],\\L(\hat{Y})&\propto\cos\Omega t\left[\frac{2\gamma}{4\gamma^2+\omega^2}(2\mathscr{L}'_0-\mathscr{L}'_3-\mathscr{L}'_4)-\frac{4\gamma^2}{\omega(4\gamma^2+\omega^2)}(\mathscr{D}'_3-\mathscr{D}'_4)+\frac{2}{\omega}(\mathscr{D}'_1-\mathscr{D}'_2)\right]\\&\quad+\sin\Omega t\left[\frac{2\gamma}{4\gamma^2+\omega^2}(2\mathscr{D}'_0-\mathscr{D}'_3-\mathscr{D}'_4)+\frac{4\gamma^2}{\omega(4\gamma^2+\omega^2)}(\mathscr{L}'_3-\mathscr{L}'_4)-\frac{2}{\omega}(\mathscr{L}'_1-\mathscr{L}'_2)\right].\end{aligned} \tag{3.14}$$

Thus, for detection along the magnetic-field direction there are resonances at $\Omega=0,\pm 2\mathcal{S}$. The resonance at $\Omega=0$ is Lorentzian (dispersion) in shape for the cosine (sine) component of the fluorescence, while the resonances at $\Omega=\pm 2\mathcal{S}$ have both Lorentzian and dispersion parts with different weight factors such that either one predominates when $\mathcal{S}>>\gamma$ (broken curve of Fig. 6 corresponds to the amplitude of the cosine component). The modulated fluorescence detected perpendicular to the magnetic-field direction exhibits resonances at $\Omega=0,\pm\mathcal{S},\pm 2\mathcal{S}$ so that for the case studied ($\mathcal{S}/\gamma=5$) there is considerable overlap of these resonances (broken curve of Fig. 7).

Intense fields

It is no longer possible to carry out the above procedure for intense laser beams of arbitrary bandwidth so that we have the general solutions from (2.12) to first order in a (the depth of modulation of the exciting laser is always assumed to be weak),

$$\langle\sigma(\infty)\rangle^{(1)}=\frac{a}{2}[(i\Omega-A+\gamma_c\Lambda)^{-1}e^{i\Omega t} + (-i\Omega-A+\gamma_c\Lambda)^{-1}e^{-i\Omega t}] \times[C\langle\sigma(\infty)\rangle^{(0)}+I]\ . \tag{3.15}$$

In order to obtain analytical expressions for the signals we would have to invert 8×8 matrices and find their products, which though possible in principle is extremely cumbersome and would yield complicated expressions which lend no insight into the problem. Hence, we study the modulated Hanle signals in intense laser beams numerically. For intense laser fields ($\alpha>>\gamma,\gamma_c,\mathcal{S},\delta$) one expects the resonances in the amplitude of modulation to yield the energy spectra of the composite system consisting of the atom and the coherent laser field, i.e., resonances are expected in the modulated fluorescence corresponding to the dynamical stark splitting of the various energy levels. An idea of the position of the resonant components may be obtained by finding the eigenvalues of the resonant part of the Hamiltonian (without the damping term), which now requires dealing with 3×3 matrix. The resonant Hamiltonian may be written as

$$H=\begin{bmatrix}\mathcal{S}-\delta & 0 & -\alpha\\ 0 & -\mathcal{S}-\delta & -\alpha\\ -\alpha & -\alpha & 0\end{bmatrix} \tag{3.16}$$

and the secular equation $|H-\lambda\hat{1}|=0$ yields the following cubic equation for the eigenvalues:

$$\lambda(\lambda+\mathcal{S}+\delta)(\lambda-\mathcal{S}+\delta)-2\alpha^2(\lambda+\delta)=0\ . \tag{3.17}$$

For arbitrary values of the detuning there are no simple factorizations of the cubic equation and the roots have a complicated form. In the special case of the saturating laser field being resonant with the atomic transition $|J=0,m_J=0\rangle\rightarrow|J=1,m_J=0\rangle$ ($\delta=0$), the eigenvalues have the simple form

$$\lambda=0,\pm\alpha_0,\quad \alpha_0^2\equiv\mathcal{S}^2+2\alpha^2\ . \tag{3.18}$$

In the magnetic-field scan of the signals we expect the correspoding resonances to show up at Larmor frequency equal to $\mathcal{S}=\pm(\Omega^2/4-2\alpha^2)^{1/2}$, $\pm(\Omega^2-2\alpha^2)^{1/2}$. If $\Omega/\alpha<<1$ the modulated signals are similar in shape to the weak-field case so that the modulation frequency must be fixed at a much

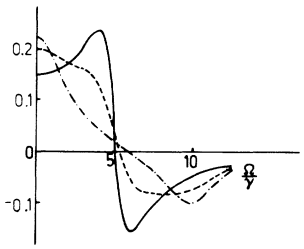

FIG. 7. Modulation-frequency scan of the amplitude of the cosine component of the modulated fluorescence detected perpendicular to H for the same values of the parameters as in Fig. 6.

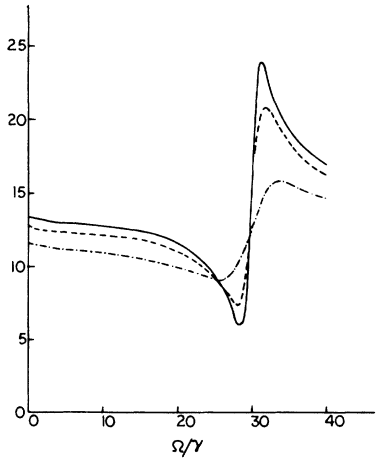

FIG. 8. Same as in Fig. 6 but for $\alpha/\gamma=10$.

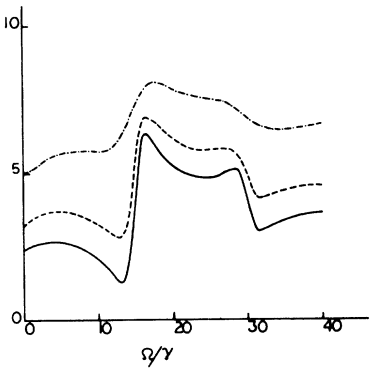

FIG. 9. Same as in Fig. 7 but for $\alpha/\gamma=10$.

higher value than the Rabi frequency for these peaks to be well resolved. When the modulated fluorescence is scanned as a function of the modulation frequency (some typical scans shown in Figs. 8 and 9), resonances are expected at $0,\pm\alpha_0,\pm 2\alpha_0$ provided of course their weight is not zero. The resonances at $\Omega=0,\pm\alpha_0,\pm 2\alpha_0$ are the analogs of the resonances at $\Omega=0,\pm\delta,\pm 2\delta$ in the weak fields. The weight factors of these lines can be calculated using secular approximation as was done in Ref. (13) and yields the results in agreement with numerical computations shown in Figs. 8 and 9.

The signal detected perpendicular to the direction of the magnetic field exhibits well-defined resonant structures at $\Omega=\pm\alpha_0,\pm 2\alpha_0$, while the fluorescence detected along the magnetic field exhibits sharp resonances at $\Omega=\pm 2\alpha_0$ only as shown in Figs. 8 and 9. These resonances are found to be Lorentzian (dispersion) in shape for the sine (cosine) component of the modulated fluorescence in both the directions. The effect of laser bandwidth is to alter the peak height as well as the width of the resonances.

One of us (R.S.) is grateful to the Council of Scientific and Industrial Research, New Delhi for the award of Junior Research Fellowship.

[1]W. Hanle, Z. Phys. 30, 93 (1924).
[2]A. Corney, in *Atomic and Laser Spectroscopy* (Oxford University Press, London, 1977), Chap. 15 and references cited therein.
[3]V. P. Kaftandjian and L. Klein, Phys. Lett. 62A, 317 (1977).
[4]V. P. Kaftandjian, L. Klein, and W. Hanle, Phys. Lett. 65A, 188 (1978).
[5]C. Delsart, J. C. Keller, and V. P. Kaftandjian, in *Laser Spectroscopy IV,* edited by H. Walther and K. W. Rothe (Springer, Berlin, 1979), p. 618.
[6]P. Anatha Lakshmi and G. S. Agarwal, Phys. Rev. A 23, 2553 (1981).
[7]A. Corney and G. W. Series, Proc. Phys. Soc. London 83, 207 (1964); 83, 331 (1964).
[8]A. Corney, J. Phys. B 1, 458 (1968).
[9]L. Armstrong and S. Feneuille, J. Phys. B 8, 546 (1975).
[10]S. Feneuille, M. G. Schweighofer, and G. Oliver, J. Phys. B 9, 2003 (1976).
[11]W. A. McClean and S. Swain, J. Phys. B 9, 2011 (1976); Opt. Commun. 25, 113 (1978).
[12]R. Saxena and G. S. Agarwal, J. Phys. B 12, 1939 (1979).
[13]R. Saxena and G. S. Agarwal, J. Phys. B 13, 453 (1980).
[14]P. Avan and C. Cohen-Tannoudji, J. Phys. (Paris) 36, L85 (1975); J. Phys. B 10, 171 (1977).
[15]G. S. Agarwal, in *Springer Tracts in Modern Physics,* edited by G. Höhler (Springer, New York, 1974), Vol. 70, Sec. 6.
[16]G. S. Agarwal, Phys. Rev. A 18, 1490 (1978).
[17]J. H. Eberly, in *Laser Spectroscopy IV,* edited by H. Walther and K. W. Rothe (Springer, Berlin, 1979), p. 80.
[18]G. S. Agarwal, J. Opt. Soc. Am. 70, 591 (1980).