Entangling nanomechanical oscillators in a ring cavity by feeding squeezed light

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Abstract. A scheme is presented for entangling two separated nanomechanical oscillators by injecting broad band squeezed vacuum light and laser light into the ring cavity. We work in the resolved sideband regime. We find that in order to obtain the maximum entanglement of the two oscillators, the squeezing parameter of the input light should be about 1. We report significant entanglement over a very wide range of power levels of the pump and temperatures of the environment.

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Contents

1. Introduction

It is well known that entanglement is a key resource for quantum information processing [1]. One now has fairly good understanding of how to produce entanglement among microscopic entities. In recent times there has been considerable interest in studying entanglement in mesoscopic and even microscopic systems [2, 3, 4, 5, 6]. Nanomechanical oscillators are beginning to be important candidates for the study of quantum mechanical features at mesoscopic scales. In fact the possibility of entangling two nanomechanical oscillators has been investigated from many different angles: such as entangling two mirrors in a ring cavity [7], entangling two mirrors of two independent optical cavities driven by a pair of entangled light beams [8], entangling two mirrors by using a double-cavity set up by driving with squeezed light [9], entangling two mirrors of a linear cavity driven by a classical laser field [10], entangling two mirrors in a ring cavity by using a phase-sensitive feedback loop [11], entangling two dielectric membranes suspended inside a cavity [12], and entangling two oscillators by entanglement swapping [13, 14]. Other proposals do not use cavity configurations but coupling to Cooper pair boxes [15]. Here we report a conceptually simple method to produce entanglement between two mirrors. Our proposal enables us to trace the physical origin of entanglement.

In this paper, we propose a scheme for entangling two movable mirrors of a ring cavity by feeding broad band squeezed vacuum light along with the laser light. The two movable mirrors are entangled based on their interaction with the cavity field. The achieved entanglement of the two movable mirrors depends on the degree of squeezing of the input light, the laser power, and the temperature of the movable mirrors. The feeding of the squeezed light has been considered to produce squeezing of a nanomechanical mirror [16, 17]. Further Pinard *et al.* [9] have considered entanglement of two mirrors in a double cavity configuration which is fed by squeezed light - one part of the cavity is fed by light squeezed in amplitude quadrature and the other is fed by light squeezed in phase quadrature. In contrast we consider a single mode ring cavity driven by a single component amplitude squeezed light. In our scheme the entanglement can be managed by an externally controllable field which is the squeezed light.

The paper is organized as follows. In section 2 we introduce the model, give the quantum Langevin equations, and obtain the steady-state mean values. In section 3 we derive the stability conditions, calculate the mean square fluctuations in the relative momentum and the total displacement of the movable mirrors. In section 4 we analyze how the entanglement of the movable mirrors can be modified by the squeezing parameter, the laser power, and the temperature of the environment. The parameters chosen in the paper are from a recent experiment on optomechanical normal mode splitting [18].

Before we present our calculations, we present a key idea behind our work. For a bipartite system, a sufficient criterion for entanglement is that the sum of continuous variables satisfies the inequality [19]

$$\langle(\Delta(q_1+q_2))^2\rangle + \langle(\Delta(p_1-p_2))^2\rangle < 2, \tag{1}$$

where q_j and p_j $(j=1,2)$ are the position and momentum operators for two particles, respectively. They obey the commutation relation $[q_j, p_k] = i\delta_{jk}$ $(j,k=1,2)$.

Mancini *et al.* [7] have derived another sufficient condition for bipartite entanglement, which requires the product of continuous variables satisfies the inequality

$$\langle(\Delta(q_1+q_2))^2\rangle\langle(\Delta(p_1-p_2))^2\rangle < 1. \tag{2}$$

In this paper, we will use equation (2) to show the entanglement between the two oscillating mirrors. Thus if we have a situation where the interaction occurs only via the relative coordinates q_1-q_2,p_1-p_2, then we can hold $\langle(\Delta(q_1+q_2))^2\rangle$ at its value, says $\simeq 1$, before interaction and if the interaction can make $\langle(\Delta(p_1-p_2))^2\rangle < 1$, then the inequality (2) would imply that the mirrors 1 and 2 are entangled. In the next section we discuss how this can be achieved by using a single mode ring cavity.

2. Model

The system under study, sketched in figure 1, is a ring cavity with one fixed partially transmitting mirror and two movable perfectly reflecting mirrors, driven by a laser with frequency ω_L. As the photons in the cavity with length L bounce off the movable mirrors, they will exert a radiation pressure force on the surfaces of the movable mirrors proportional to the instantaneous photon number in the cavity. The motion of the movable mirrors induced by the radiation pressure changes the cavity's length, and alters the intensity of the cavity field, which in turn modifies the radiation pressure force itself. Thus the interaction of the cavity field with the movable mirrors through the radiation pressure is a nonlinear effect. In addition, each mirror undergoes quantum Brownian motion due to its coupling to its own independent environment at the same low temperature T. The two movable mirrors are identical with the same effective mass m, mechanical frequency ω_m and momentum decay rate γ_m, and each mirror is modeled as a quantum mechanical harmonic oscillator. We further assume that the cavity is fed with squeezed light at frequency ω_S.

In the adiabatic limit, the cavity field is a single mode with frequency ω_c [20], and we can neglect the retardation effect [21], neglect the photon creation in the cavity with moving boundaries due to the Casimir effect [22], and neglect the Doppler effect [23], thus the radiation pressure force does not depend on the velocity of the movable mirrors. Assuming the collisions of the photons on the surfaces of the movable mirrors are elastic, the momentum transferred to the mirrors per photon is $\hbar k_y - (-\hbar k_y) = 2\hbar k_y$ (see figure 1 for the direction of y), where $k_y = k\cos(\theta/2)$, k is the wave vector of the cavity field with $k=\omega_c/c$, and θ is the angle between the incident light and the reflected light at the surfaces of the movable mirrors. During the cavity round-trip time $t=2L/c$, there are $n_c\cos(\theta/2)$ photons hitting on the surfaces of the movable mirrors, so the radiation pressure force is $F = n_c\cos(\theta/2)\times 2\hbar k_y/t = n_c\hbar\frac{\omega_c}{L}\cos^2(\theta/2)$. In a reference

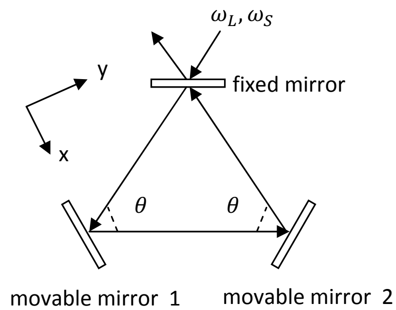

Figure 1. Sketch of the studied system. A laser with frequency ω_L and a squeezed vacuum light with frequency ω_S enter the ring cavity through the partially transmitting mirror.

frame rotating at the laser frequency, the Hamiltonian that describes the system can be written as

$$\begin{aligned} H = \hbar(\omega_c - \omega_L)n_c + \hbar g n_c \cos^2(\theta/2)(Q_1 - Q_2) + \frac{\hbar\omega_m}{2}(Q_1^2 + P_1^2) \\ + \frac{\hbar\omega_m}{2}(Q_2^2 + P_2^2) + i\hbar\varepsilon(c^\dagger - c), \end{aligned} \tag{3}$$

we have defined dimensionless position and momentum operators for the oscillators $Q_j = \sqrt{\frac{m\omega_m}{\hbar}} q_j$ and $P_j = \sqrt{\frac{1}{m\hbar\omega_m}} p_j$ (j=1,2) with $[Q_j, P_k] = i\delta_{jk}$ ($j, k = 1, 2$). Further in equation (3), $n_c = c^\dagger c$ is the number of the photons inside the cavity, c and $c^\dagger$ are the annihilation and creation operators for the cavity field with $[c, c^\dagger] = 1$. The parameter $g = \frac{\omega_c}{L}\sqrt{\frac{\hbar}{m\omega_m}}$ is the optomechanical coupling constant between the cavity field and the movable mirrors in units of s^{-1}. The different signs in front of Q_1 and Q_2 are because the radiation pressure forces exerted on the two mirrors are opposite. The parameter ε is the coupling strength of the laser to the cavity field, which is related to the input laser power $\wp$ by $\varepsilon = \sqrt{\frac{2\kappa\wp}{\hbar\omega_L}}$, where κ is the photon decay rate by leaking out of the cavity.

In the system, the cavity field is damped by photon losses via the cavity output mirror at the rate κ, and the movable mirrors are damped due to momentum losses at the same rate γ_m. Meanwhile, there are two kinds of noises affecting on the system. One is the input squeezed vacuum noise operator c_{in} with frequency $\omega_S = \omega_L + \omega_m$. It has zero mean value, and nonzero time-domain correlation functions [24]

$$\begin{aligned} \langle \delta c_{in}^\dagger(t)\delta c_{in}(t')\rangle &= N\delta(t - t'), \\ \langle \delta c_{in}(t)\delta c_{in}^\dagger(t')\rangle &= (N + 1)\delta(t - t'), \\ \langle \delta c_{in}(t)\delta c_{in}(t')\rangle &= Me^{-i\omega_m(t+t')}\delta(t - t'), \\ \langle \delta c_{in}^\dagger(t)\delta c_{in}^\dagger(t')\rangle &= M^* e^{i\omega_m(t+t')}\delta(t - t'). \end{aligned} \tag{4}$$

where $N = \sinh^2(r)$, $M = \sinh(r)\cosh(r)e^{i\varphi}$, r and φ are respectively the squeezing parameter and phase of the squeezed vacuum light. For simplicity, we choose $\varphi = 0$.

The other is quantum Brownian noises ξ_1 and ξ_2, which are from the coupling of the movable mirrors to their own environment. They are mutually independent with zero mean values and have the following correlation functions at temperature T [25]:

$$\langle \xi_j(t)\xi_k(t^{'})\rangle = \frac{\delta_{jk}}{2\pi}\frac{\gamma_m}{\omega_m}\int \omega e^{-i\omega(t-t^{'})}\left[1+\coth(\frac{\hbar\omega}{2k_B T})\right]d\omega, \tag{5}$$

where k_B is the Boltzmann constant and T is the temperature of the mirrors' environment, $j,k=1,2$.

The dynamics of the cavity field interacting with the movable mirrors can be derived by the Heisenberg equations of motion and taking into account the effect of damping and noises, which gives the quantum Langevin equations

$$\begin{aligned}
&\dot{Q}_1 = \omega_m P_1,\\
&\dot{Q}_2 = \omega_m P_2,\\
&\dot{P}_1 = -gn_c\cos^2(\theta/2) - \omega_m Q_1 - \gamma_m P_1 + \xi_1,\\
&\dot{P}_2 = gn_c\cos^2(\theta/2) - \omega_m Q_2 - \gamma_m P_2 + \xi_2,\\
&\dot{c} = -i[\omega_c - \omega_L + g\cos^2(\theta/2)(Q_1 - Q_2)]c + \varepsilon - \kappa c + \sqrt{2\kappa}c_{in},\\
&\dot{c}^{\dagger} = i[\omega_c - \omega_L + g\cos^2(\theta/2)(Q_1 - Q_2)]c^{\dagger} + \varepsilon - \kappa c^{\dagger} + \sqrt{2\kappa}c_{in}^{\dagger}.
\end{aligned} \tag{6}$$

From the second term of equation (3), we can see only the relative motion of the two movable mirrors is coupled to the cavity field via radiation pressure. On introducing the relative distance and the relative momentum of the movable mirrors by $Q_- = Q_1 - Q_2$ and $P_- = P_1 - P_2$, we find that equation (6) reduces to

$$\begin{aligned}
&\dot{Q}_- = \omega_m P_-,\\
&\dot{P}_- = -2gn_c\cos^2(\theta/2) - \omega_m Q_- - \gamma_m P_- + \xi_1 - \xi_2,\\
&\dot{c} = -i[\omega_c - \omega_L + g\cos^2(\theta/2)Q_-]c + \varepsilon - \kappa c + \sqrt{2\kappa}c_{in},\\
&\dot{c}^{\dagger} = i[\omega_c - \omega_L + g\cos^2(\theta/2)Q_-]c^{\dagger} + \varepsilon - \kappa c^{\dagger} + \sqrt{2\kappa}c_{in}^{\dagger}.
\end{aligned} \tag{7}$$

We would use standard methods of quantum optics [26] which have been adopted for discussions of quantum noise of nanomechanical mirrors [10, 25, 27, 28, 29], setting all the time derivatives in equation (7) to zero, and solving it, we obtain the steady-state mean values

$$P_-^s = 0,\, Q_-^s = -\frac{2g|c^s|^2\cos^2(\theta/2)}{\omega_m},\, c^s = \frac{\varepsilon}{\kappa + i\Delta}, \tag{8}$$

where

$$\Delta = \omega_c - \omega_L + gQ_-^s\cos^2(\theta/2) \tag{9}$$

is the effective cavity detuning, depending on Q_-^s. The Q_-^s denotes the new equilibrium relative distance between the movable mirrors. Further c^s represents the complex amplitude of the cavity field in the steady state. For a given input laser power, Q_-^s and c^s can take three distinct values, respectively. Therefore, the system displays an optical multistability [30, 31, 32], which is a nonlinear effect induced by the radiation pressure.

3. Radiation pressure and quantum fluctuations

To investigate entanglement of the two movable mirrors, we have to calculate the fluctuations in the relative momentum of the movable mirrors. This fluctuations can be calculated analytically by using the linearization approach of quantum optics [26], provided that the nonlinear effect between the cavity field and the movable mirrors is weak. We write each operator of the system as the sum of its steady-state mean value and a small fluctuation with zero mean value,

$$Q_- = Q_-^s + \delta Q_-, \quad P_- = P_-^s + \delta P_-, \quad c = c^s + \delta c. \tag{10}$$

Inserting equation (10) into equation (7), then assuming the cavity field has a very large amplitude c^s with $|c^s| \gg 1$, one can obtain a set of linear quantum Langevin equations for the fluctuation operators,

$$\begin{aligned}
&\delta \dot{Q}_- = \omega_m \delta P_-, \\
&\delta \dot{P}_- = -2g\cos^2(\theta/2)(c^{s*}\delta c + c^s \delta c^\dagger) - \omega_m \delta Q_- - \gamma_m \delta P_- + \xi_1 - \xi_2, \\
&\delta \dot{c} = -(\kappa + i\Delta)\delta c - ig\cos^2(\theta/2)c^s \delta Q_- + \sqrt{2\kappa}\delta c_{in}, \\
&\delta \dot{c}^\dagger = -(\kappa - i\Delta)\delta c^\dagger + ig\cos^2(\theta/2)c^{s*}\delta Q_- + \sqrt{2\kappa}\delta c_{in}^\dagger.
\end{aligned} \tag{11}$$

Introducing the cavity field quadratures $\delta x = \delta c + \delta c^\dagger$ and $\delta y = i(\delta c^\dagger - \delta c)$, and the input noise quadratures $\delta x_{in} = \delta c_{in} + \delta c_{in}^\dagger$ and $\delta y_{in} = i(\delta c_{in}^\dagger - \delta c_{in})$, equation (11) can be rewritten in the matrix form

$$\dot{f}(t) = Af(t) + \eta(t), \tag{12}$$

in which $f(t)$ is the column vector of the fluctuations, $\eta(t)$ is the column vector of the noise sources. Their transposes are

$$\begin{aligned}
&f(t)^T = (\delta Q_-, \delta P_-, \delta x, \delta y), \\
&\eta(t)^T = (0, \xi_1 - \xi_2, \sqrt{2\kappa}\delta x_{in}, \sqrt{2\kappa}\delta y_{in});
\end{aligned} \tag{13}$$

and the matrix A is given by

$$A = \begin{pmatrix} 0 & \omega_m & 0 & 0 \\ -\omega_m & -\gamma_m & -g\cos^2(\theta/2)(c^s + c^{s*}) & ig\cos^2(\theta/2)(c^s - c^{s*}) \\ -ig\cos^2(\theta/2)(c^s - c^{s*}) & 0 & -\kappa & \Delta \\ -g\cos^2(\theta/2)(c^s + c^{s*}) & 0 & -\Delta & -\kappa \end{pmatrix}. \tag{14}$$

The solution of equation (12) is $f(t) = M(t)f(0) + \int_0^t M(t')\eta(t-t')dt'$, where $M(t) = e^{At}$. The system is stable and reaches its steady state as $t \to \infty$ only if the real parts of all the eigenvalues of the matrix A are negative so that $M(\infty)=0$. The stability conditions for the system can be found by employing the Routh-Hurwitz criterion [33], we get

$$\begin{aligned}&\kappa\gamma_m[(\kappa^2+\Delta^2)^2+(2\kappa\gamma_m+\gamma_m^2-2\omega_m^2)(\kappa^2+\Delta^2)+\omega_m^2(4\kappa^2+\omega_m^2\\&\quad+2\kappa\gamma_m)]+2\omega_m\Delta g^2\cos^4(\theta/2)|c^s|^2(2\kappa+\gamma_m)^2>0,\\&\omega_m(\kappa^2+\Delta^2)-4\Delta g^2\cos^4(\theta/2)|c^s|^2>0.\end{aligned}\tag{15}$$

All the parameters chosen in this paper have been verified to satisfy the stability conditions (15).

Fourier transforming each operator in equation (11) by $f(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}f(\omega)e^{-i\omega t}d\omega$ and solving it in the frequency domain, the relative momentum fluctuations of the movable mirrors are given by

$$\begin{aligned}\delta P_-(\omega)=&\frac{i\omega}{d(\omega)}(2\sqrt{2\kappa}g\cos^2(\theta/2)\{[\kappa-i(\Delta+\omega)]c^{s*}\delta c_{in}(\omega)+[\kappa+i(\Delta-\omega)]\\&\times c^s\delta c_{in}^\dagger(-\omega)\}-[(\kappa-i\omega)^2+\Delta^2][\xi_1(\omega)-\xi_2(\omega)]),\end{aligned}\tag{16}$$

where $d(\omega)=-4\omega_m\Delta g^2|c^s|^2\cos^4(\theta/2)+(\omega_m^2-\omega^2-i\gamma_m\omega)[(\kappa-i\omega)^2+\Delta^2]$. Equation (16) shows $\delta P_-(\omega)$ has two contributions. The first term proportional to g originates from their interaction with the cavity field, while the second term involving $\xi_1(\omega)$ and $\xi_2(\omega)$ is from their interaction with their own environment. So the relative momentum fluctuations of the movable mirrors are now determined by radiation pressure and the thermal noise. In the case of no coupling with the cavity field ($g=0$), the movable mirrors will make Brownian motion only, $\delta P_-(\omega)=-i\omega[\xi_1(\omega)-\xi_2(\omega)]/(\omega_m^2-\omega^2-i\gamma_m\omega)$, whose mechanical susceptibility $\chi(\omega)=1/(\omega_m^2-\omega^2-i\gamma_m\omega)$ has a Lorentzian shape centered at the frequency ω_m with $2\gamma_m$ as full width at half maximum (FWHM).

The mean square fluctuations in the relative momentum of the movable mirrors are determined by

$$\langle\delta P_-(t)^2\rangle=\frac{1}{4\pi^2}\int\int_{-\infty}^{+\infty}d\omega d\Omega e^{-i(\omega+\Omega)t}\langle\delta P_-(\omega)\delta P_-(\Omega)\rangle.\tag{17}$$

To calculate the mean square fluctuations, we require the correlation functions of the noise sources in the frequency domain. Fourier transforming equations (4) and (5) gives the frequency domain correlation functions

$$\begin{aligned}&\langle\delta c_{in}^\dagger(-\omega)\delta c_{in}(\Omega)\rangle=2\pi N\delta(\omega+\Omega),\\&\langle\delta c_{in}(\omega)\delta c_{in}^\dagger(-\Omega)\rangle=2\pi(N+1)\delta(\omega+\Omega),\\&\langle\delta c_{in}(\omega)\delta c_{in}(\Omega)\rangle=2\pi M\delta(\omega+\Omega-2\omega_m),\\&\langle\delta c_{in}^\dagger(-\omega)\delta c_{in}^\dagger(-\Omega)\rangle=2\pi M^*\delta(\omega+\Omega+2\omega_m),\\&\langle\xi_j(\omega)\xi_k(\Omega)\rangle=2\pi\delta_{jk}\tfrac{\gamma_m}{\omega_m}\omega\left[1+\coth(\tfrac{\hbar\omega}{2k_BT})\right]\delta(\omega+\Omega).\end{aligned}\tag{18}$$

Upon substituting equation (16) into equation (17) and taking into account equation (18), the mean square fluctuations of equation (17) are written as

$$\langle\delta P_-(t)^2\rangle=\frac{1}{2\pi}\int_{-\infty}^{+\infty}[\omega^2A+\omega(\omega-2\omega_m)Be^{-2i\omega_mt}+\omega(\omega+2\omega_m)Ce^{2i\omega_mt}]d\omega.\tag{19}$$

where

$$
\begin{aligned}
A = & \tfrac{1}{d(\omega)d(-\omega)}(8\kappa g^2\cos^4(\theta/2)|c^s|^2\{(N+1)[\kappa^2+(\Delta+\omega)^2] \\
& +N[\kappa^2+(\Delta-\omega)^2]\}+2\gamma_m\tfrac{\omega}{\omega_m}[(\Delta^2+\kappa^2-\omega^2)^2+4\kappa^2\omega^2] \\
& \times[1+\coth(\tfrac{\hbar\omega}{2k_BT})]), \\
B = & \tfrac{8\kappa g^2\cos^4(\theta/2)c^{s*2}M}{d(\omega)d(2\omega_m-\omega)}[\kappa-i(\Delta+\omega)][\kappa-i(\Delta+2\omega_m-\omega)], \\
C = & \tfrac{8\kappa g^2\cos^4(\theta/2)c^{s2}M^*}{d(\omega)d(-2\omega_m-\omega)}[\kappa+i(\Delta-\omega)][\kappa+i(\Delta+2\omega_m+\omega)].
\end{aligned}
\tag{20}
$$

In equations (19) and (20), the term independent of g is the thermal noise contribution; while all other terms involving g are the radiation pressure contribution, including the influence of the squeezed vacuum light. Moreover, $\langle\delta P_-(t)^2\rangle$ is time-dependent, the explicit time dependence in equation (19) can be eliminated by working in the interaction picture. If we look the relative motion of the movable mirrors as a harmonic oscillator and introduce the annihilation (creation) operators b ($b^\dagger$) and $\tilde{b}$ ($\tilde{b}^\dagger$) for the oscillator in the Schrödinger and interaction picture with $[b,b^\dagger]=1$ and $[\tilde{b},\tilde{b}^\dagger]=1$. They are related by $b=\tilde{b}e^{-i\omega_m t}$ and $b^\dagger=\tilde{b}^\dagger e^{i\omega_m t}$. Then using $P_-=i(b^\dagger-b)$, and $\tilde{P}_-=i(\tilde{b}^\dagger-\tilde{b})$, we get

$$
\langle\delta\tilde{P}_-^2\rangle=\frac{1}{2\pi}\int_{-\infty}^{+\infty}[\omega^2A+\omega(\omega-2\omega_m)B+\omega(\omega+2\omega_m)C]d\omega. \tag{21}
$$

According to equation (2), the movable mirrors are said to be entangled if $\langle\delta Q_+^2\rangle$ and $\langle\delta\tilde{P}_-^2\rangle$ satisfy the inequality

$$
\langle\delta Q_+^2\rangle\langle\delta\tilde{P}_-^2\rangle<1. \tag{22}
$$

where $Q_+=Q_1+Q_2$, the total displacement of the two movable mirrors, which is not related to the radiation pressure, only determined by the thermal noise. At the temperature T, the fluctuations $\langle\delta Q_+^2\rangle$ are

$$
\langle\delta Q_+^2\rangle=0.5+\frac{1}{e^{\hbar\omega_m/(k_BT)}-1} \tag{23}
$$

Since $[Q_+,P_-]=[Q_1+Q_2,P_1-P_2]=0$, Q_+ and P_- can be simultaneously measured with infinite precision. Thus Q_+ and $\tilde{P}_-$ can also be simultaneously measured with infinite precision.

From equations (20) and (21), we find $\langle\delta\tilde{P}_-^2\rangle$ is affected by the detuning Δ, the squeezing parameter r, the laser power $\wp$, the cavity length L, the temperature of the environment T, and so on. In the following, we confine ourselves to discussing the dependence of $\langle\delta\tilde{P}_-^2\rangle$ on the squeezing parameter, the laser power, and the temperature of the environment.

4. Entanglement of the two movable mirrors

In the section, we would like to numerically evaluate the mean square fluctuations in the total displacement and the relative momentum of the movable mirrors given by equations (23) and (21) to show the entanglement of the two movable mirrors produced by feeding the squeezed vacuum light at the input mirror. To have fairly good idea of entanglement, we use the parameters of a recent experiment [18] although we are aware that the cavity geometry is different: the wavelength of the laser $\lambda = \frac{2\pi c}{\omega_L} = 1064$ nm, $L = 25$ mm, $m = 145$ ng, $\kappa = 2\pi \times 215 \times 10^3$ Hz, $\omega_m = 2\pi \times 947 \times 10^3$ Hz, the mechanical quality factor $Q' = \frac{\omega_m}{\gamma_m} = 6700$, $\theta = \pi/3$.

First we illustrate the squeezed vacuum light's effect on the entanglement between the movable mirrors. We find as $T = 41.4$ μK, the mean square fluctuations $\langle \delta Q_+^2 \rangle \approx 1$, which implies that as long as the mean square fluctuations $\langle \delta \tilde{P}_-^2 \rangle < 1$, there is an entanglement between the movable mirrors. The behavior of $\langle \delta \tilde{P}_-^2 \rangle$ at $\wp = 3.8$ mW is plotted as a function of the detuning Δ in figure 2. Different graphs correspond to different values of the squeezing of the input light. In the case of no injection of the squeezed vacuum light ($r = 0$), which means that the squeezed vacuum light is replaced by an ordinary vacuum light, we find $\langle \delta \tilde{P}_-^2 \rangle$ is always larger than unity, the minimum value of $\langle \delta \tilde{P}_-^2 \rangle$ is 1.027, obviously there is no entanglement between the movable mirrors. However, if we inject the squeezed vacuum light, it is seen that entanglement between the movable mirrors occurs, meaning that there is a quantum correlation between the movable mirrors, even through they are separated in space. We also find the movable mirrors are maximally entangled as the squeezing parameter is about $r = 1$, the corresponding minimum value of $\langle \delta \tilde{P}_-^2 \rangle$ is 0.265. So the injection of the squeezed vacuum light leads to a significant reduction of the fluctuations in the relative momentum between the movable mirrors. This is due to the fact that using the squeezed vacuum light increases the photon number in the cavity, which leads to a stronger radiation pressure acting on the movable mirrors and enhances the entanglement between the movable mirrors.

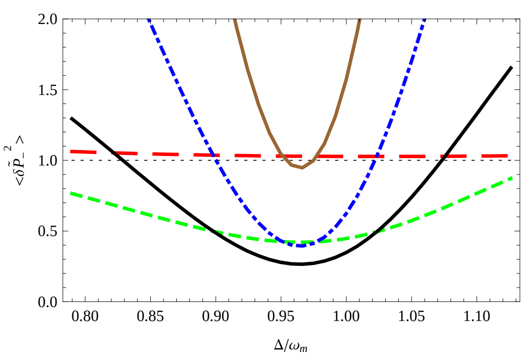

Figure 2. The mean square fluctuations $\langle \delta \tilde{P}_-^2 \rangle$ versus the detuning Δ/ω_m for different values of the squeezing of the input field. $r = 0$ (red, big dashed line), $r = 0.5$ (green, small dashed line), $r = 1$ (black, solid curve), $r = 1.5$ (blue, dotdashed curve), $r = 2$ (brown, solid curve). The minimum values of $\langle \delta \tilde{P}_-^2 \rangle$ are 1.027 (r=0), 0.420 (r=0.5), 0.265(r=1), 0.394 (r=1.5), 0.947 (r=2). The flat dotted line represents $\langle \delta \tilde{P}_-^2 \rangle$=1. Parameters: the temperature of the environment $T = 41.4$ μK, the laser power $\wp = 3.8$ mW.

Next we consider the influence of the laser power on the maximum entanglement between the movable mirrors. We fix the squeezing parameter $r = 1$, and the temperature of the environment $T = 41.4$ μK. We have already known at this temperature, $\langle\delta Q_+^2\rangle \approx 1$. Thus, if the mean square fluctuations $\langle\delta\tilde{P}_-^2\rangle < 1$, the movable mirrors become entangled. The mean square fluctuations $\langle\delta\tilde{P}_-^2\rangle$ as a function of the detuning Δ for different laser power are shown in figure 3. We find that significant entanglement occurs for a range of pumping powers.

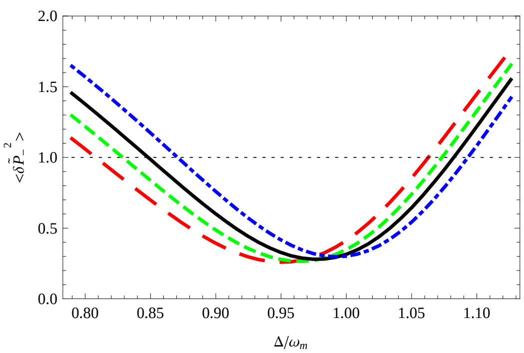

Figure 3. The mean square fluctuations $\langle\delta\tilde{P}_-^2\rangle$ versus the detuning Δ/ω_m, each curve corresponds to a different laser power. $\wp$=0.6 mW (red, big dashed curve), 3.8 mW (green, small dashed curve), 6.9 mW (black, solid curve), 10.7 mW (blue, dotdashed curve). The minimum values of $\langle\delta\tilde{P}_-^2\rangle$ are 0.259 ($\wp$=0.6 mW), 0.265 ($\wp$=3.8 mW), 0.279 ($\wp$=6.9 mW), 0.297 ($\wp$=10.7 mW). The flat dotted line represents $\langle\delta\tilde{P}_-^2\rangle$=1. Parameters: the squeezing parameter $r = 1$, the temperature of the environment $T = 41.4$ μK.

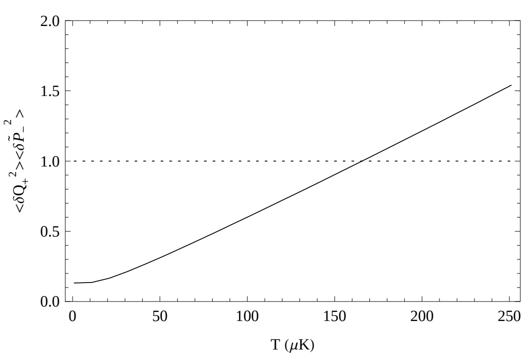

Figure 4. The value of $\langle\delta Q_+^2\rangle\langle\delta\tilde{P}_-^2\rangle$ versus the temperature of the environment T (μK). The minimum value of $\langle\delta Q_+^2\rangle\langle\delta\tilde{P}_-^2\rangle$ is 0.132 at $T = 0$ K. The flat dotted line represents $\langle\delta Q_+^2\rangle\langle\delta\tilde{P}_-^2\rangle$=1. Parameters: the squeezing parameter $r = 1$, the laser power $\wp = 3.8$ mW, the detuning $\Delta = 0.965\omega_m$.

We now show the effect of the temperature of the environment on the entanglement between the movable mirrors. We fix the squeezing parameter $r = 1$, the laser power $\wp = 3.8$ mW, and the detuning $\Delta = 0.965\omega_m$. The value of $\langle\delta Q_+^2\rangle\langle\delta\tilde{P}_-^2\rangle$ as a function of the temperature of the environment is presented in figure 4. As the temperature of the environment increases, the amount of entanglement monotonically decreases due to the thermal fluctuations. This is as expected. What is remarkable is that we find entanglement over a wide range of temperatures. As $T \geq 166$ μK, $\langle \delta Q_+^2 \rangle \langle \delta \tilde{P}_-^2 \rangle \geq 1$, the entanglement vanishes, the movable mirrors become completely separable. So decreasing the temperature of the environment can make the entanglement between the movable mirrors stronger. Note that substantial progress has been made in cooling the nanomechanical oscillators [34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44]. Further the ground state cooling using the resolved sideband regime might soon become feasible. Clearly the entanglement depends on both the quality factor of the cavity and the temperature of the environment. The optical ring cavities are expected to yield much higher quality factor: $\kappa \approx 2\pi \times 10$kHz, see for example [45], though for fixed mirrors replaced by moving mirrors, the quality factor may be deteriorated. Metheods for detection of entanglement are discussed in [7, 9]. We note here that in our case we can deduce entanglement from the knowledge of $\langle \delta \tilde{P}_-^2 \rangle$. It can be shown from equation (11) that $\langle \delta \tilde{P}_-^2 \rangle$ can be obtained from the measurement of the fluctuations in the quadrature of the output field.

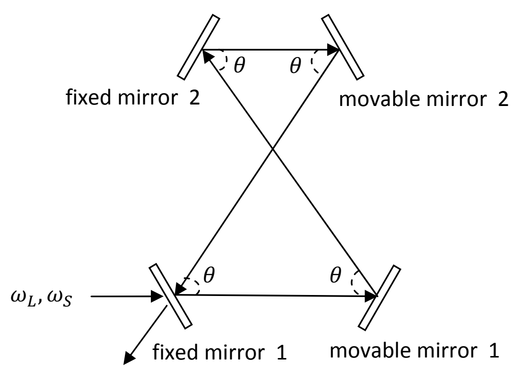

Figure 5. Sketch of 4-mirror ring cavity. A laser with frequency ω_L and squeezed vacuum light with frequency $\omega_S = \omega_L + \omega_m$ enter the ring cavity through the partially transmitting fixed mirror 1. The fixed mirror 2 and the two identical movable mirrors are perfectly reflecting.

If we use a different geometry of the ring cavity, as shown in figure 5, then we have the possibility of entangling other quadratures of the mirrors. In this case, the Hamiltonian of the system in the frame rotating at the laser frequency becomes

$$\begin{aligned} H = {} & \hbar(\omega_c - \omega_L)n_c - \hbar g n_c \cos^2(\theta/2)(Q_1 + Q_2) + \frac{\hbar\omega_m}{2}(Q_1^2 + P_1^2) \\ & + \frac{\hbar\omega_m}{2}(Q_2^2 + P_2^2) + i\hbar\varepsilon(c^\dagger - c), \end{aligned} \tag{24}$$

We note the interaction between the two movable mirrors and the cavity field depends only on the total displacement of the movable mirrors. The movable mirrors are said to be entangled if δQ_-^2 and $\delta \tilde{P}_+^2$ satisfy the inequality [7, 19]

$$\langle \delta Q_-^2 \rangle \langle \delta \tilde{P}_+^2 \rangle < 1. \tag{25}$$

where $Q_- = Q_1 - Q_2$ and $P_+ = P_1 + P_2$. The Q_- is the relative displacement of the two movable mirrors, which is not related to the radiation pressure, only determined by the thermal noise. The P_+ is the total momentum of the two movable mirrors, and depends on the radiation pressure and the thermal noise. The relation between P_+ and $\tilde{P}_+$ is the same as the relation between P_- and $\tilde{P}_-$ we defined above. Since $[Q_-, P_+] = [Q_1 - Q_2, P_1 + P_2] = 0$, Q_- and P_+ can be simultaneously measured with infinite precision. Thus Q_- and $\tilde{P}_+$ can also be simultaneously measured with infinite precision. Through calculations, we find that $\langle \delta Q_-^2 \rangle$ and $\langle \delta \tilde{P}_+^2 \rangle$ in a 4-mirror ring cavity have the same form as $\langle \delta Q_+^2 \rangle$ (equation (23)) and$\langle \delta \tilde{P}_-^2 \rangle$ (equation (21)) in a 3-mirror ring cavity, respectively. If we choose the same parameters, the same numerical results will be obtained. Therefore, using a 4-mirror ring cavity, the entanglement between two oscillators can also be obtained.

5. Conclusions

In conclusion, we have found that the injection of squeezed vacuum light and a laser can entangle the two identical movable mirrors by the radiation pressure. The result shows the maximum entanglement of the movable mirrors happens if the squeezed vacuum light with r about 1 is injected into the cavity. We also find significant entanglement over a very wide range of input laser power and temperatures of the environment.

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