Coupled solitons in rare-earth doped two-mode fiber

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Abstract: We present first ever analytical solutions for shape-preserving pulses in a Kerr nonlinear two-mode fiber doped with 3-level Λ atoms. The two modes are near-resonant with the two transitions of the atomic system. We show the existence of quasi-stable coupled bright-dark pairs if the group velocity dispersion has opposite signs at the two mode frequencies. We demonstrate the remarkable possibility allowed by the fiber dispersion for the existence of a new class of solutions for unequal coupling constants for the two modes. We present the conditions for existence and the analytical form of these solutions in presence of atomic detuning. We confirm numerically the analytical solutions for the spatio-temporal evolution of coupled solitary waves.

OCIS codes: (270.5530)Pulse propagation and solitons; (190.4370) Nonlinear optics, fibers

References and links

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1. Introduction

Coherent pulse propagation in atomic media has been one of the central issues of quantum optics since the pioneering work of McCall and Hahn [1, 2] on self induced transparency (SIT). Initial studies on SIT focused on shape preserving pulses (e.g., solitons) in resonant two-level systems [3, 4, 5]. A generalization to two pulses with frequencies tuned to the respective transitions of a 3-level Λ system led to the discovery of adiabatons [6]. These were shown to be extremely sensitive to the input conditions. A thorough investigation of two pulse propagation in three level systems demonstrating the *sech* and *tanh* pulse pairs, was carried out by Eberly and coworkers [7, 8, 9]. Efforts were made to extend the area theorem to three level systems [9]. Soliton cloning and dragging using two lasers on the two transitions of a Λ system was another interesting discovery [10]. Soliton cloning in a three level system coupled to a two-mode fiber was investigated numerically [11]. Electromagnetically induced transparency (EIT) in three level systems opened the floodgates for unprecedented control on pulse velocity, with tremendous potentials for quantum information and many other applications [12, 13]. Slowing down and storing robust objects like solitons by manipulating the dispersion in the atomic systems is now an open possibility. In a somewhat different context, mainly for the demands of long-haul communication industry, solitons in Kerr nonlinear fibers were studied extensively [14]. The existence of these solitons in a non-resonant nonlinear system depends on a fine balance between nonlinearity and dispersion. The dynamics of such solitons are governed by the nonlinear Schrödinger equation (NLS). Note that the physical origin of shape preserving pulses in this case is quite different from that of resonant nonlinearities, which is reflected by the fact that SIT-solitons are described by sine-Gordon equation. In the context of NLS-solitons there have been generalizations to two modes as well [15]. These could be the two modes of two adjacent single mode fibers or the two orthogonal modes of a birefringent fiber. With opposing character of dispersion at the two mode frequencies, the system is known to possess coupled bright-dark states [15]. Issues like control of pulse velocity in systems with non-resonant nonlinearity were also addressed [16]. Delay and advancement of long pulses were observed experimentally. Delay of pulses were shown to be controllable by means of nonlinear coupling between different frequency components in a temporally nonlocal Kerr medium [16].

Considering the fact that research has progressed so much on the above two types of nonlinearity (i.e., resonant and non-resonant) separately, there is a need to combine the expertise and extract the best from each. Perhaps the first attempt was made by Nakazawa *et al.*, who showed that both SIT and NLS solitons can coexist in a medium which has both types of nonlinearity [17, 18, 19]. Such a situation will typically correspond to a nonlinear fiber doped with, say, rare earth materials. The urgency of such research can be appreciated easily in the light of some very recent experiments involving EIT phenomena in a fiber geometry [20, 21]. The advantages of using the fiber leading to large optical depth and tighter confinement of the field, and hence

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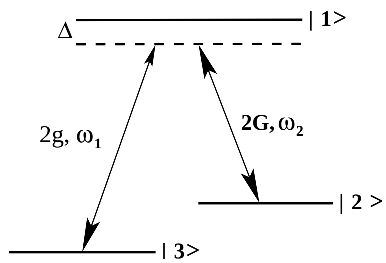

Fig. 1. The schematics of three level Λ system interacting with two fields corresponding to Rabi frequencies 2G and 2g, respectively. The single photon detuning is denoted by Δ.

power densities, are obvious. An optical depth in excess of 2000 was reported in a system where a photonic band gap fiber was used, which had the Rb atoms released by light-induced atomic desorption [21]. The tight confinement of the field allowed to have ultralow-level nonlinear optical interaction with control field powers at nano-watt regime.

Keeping in view the recent experiments and the theoretical trends, we focus on a two-mode fiber, doped with three level Λ atoms. The two modes are assumed to be near-resonant with the two relevant transitions of the Λ system (see Fig.1). To the best of our knowledge, such a system has not been probed for stable propagation of pulse pairs. We derive first ever analytical solutions for the combined effects of three level resonant nonlinearities and the non-resonant nonlinearities of the fiber. We thus considerably extend known works on solitons in three level systems and in fibers [7, 8, 15]. We show that the analytical solutions in the form of solitary waves are possible even in presence of finite detuning. We make several important observations. Fiber parameters, namely the group velocity dispersion (GVD), determines the stability aspects of the pulse pair, while their delay is governed mainly by the 3-level system. We also predict a new class of solutions with allowance for a frequency shift. These solutions are characterized by a group velocity, tunable by the GVD of the fiber. Most of the results are presented in analytical form, while the stability aspects are studied by direct integration of the propagation equations. We show that quasi-stable propagation of bright-dark pair of solitons is possible if the modes are chosen on the positive and negative sides of null group velocity dispersion. We also look at the various limiting cases in order to recover the earlier known results.

The organization of the paper is as follows. In section II we present the mathematical formulation and the analytical results. We also derive the conditions under which such solutions exist. Section III gives the results of numerical integration of coupled atom-field system, focusing on the stability aspects of the solutions of section II. Finally, in conclusions, we summarize the main results.

2. Mathematical formulation

We consider a Kerr-nonlinear two-mode fiber doped with 3-level $\Lambda-$ atoms as shown in Fig. 1. Many of the rare earth elements can well be approximated by a $\Lambda-$ system, while the two modes of the fiber could be the orthogonally polarized modes of a birefringent fiber. The two modes of the fiber, assumed to be near-resonant with the transition $|1\rangle \longleftrightarrow |2\rangle$ and $|1\rangle \longleftrightarrow |3\rangle$, respectively, are defined as

$$\vec{E}_i(z,t) = \vec{\mathscr{E}}_i(z,t)\, e^{-i(\omega_i t - k_i z)} \,+\, c.c, (i = 1,2). \tag{1}$$

Here $\vec{\mathscr{E}}_i$ is the slowly varying envelope, ω_i is the carrier frequency, and k_i is the wave number of the respective field. We use the Schrödinger formalism for the medium to describe dynamics of population and polarization of the atoms. The probability amplitudes $\mathscr{C}_i(z,t)$ of the atomic

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levels $|i\rangle$ for the Λ-system within the rotating wave approximation can be written as

$$\begin{aligned}\dot{\mathscr{C}}_1 &= -i\Delta\mathscr{C}_1 + iG\mathscr{C}_2 + ig\mathscr{C}_3 \ , \\ \dot{\mathscr{C}}_2 &= iG^*\mathscr{C}_1 \ , \\ \dot{\mathscr{C}}_3 &= ig^*\mathscr{C}_1 \ , \end{aligned} \tag{2}$$

where dot denotes $\partial/\partial t$. The Rabi frequencies $2g$ and $2G$ for the two field modes are related to the slowly varying amplitudes of $\vec{\mathscr{E}}_1$ and $\vec{\mathscr{E}}_2$ according to the relations

$$2g = \frac{2\vec{d}_{13}\cdot\vec{\mathscr{E}}_1}{\hbar}, \quad 2G = \frac{2\vec{d}_{12}\cdot\vec{\mathscr{E}}_2}{\hbar}, \tag{3}$$

where $\vec{d}_{ij}$ represents the transition dipole moment matrix element. The single photon detuning is denoted by Δ. The induced atomic polarization related to the atomic transition between levels $|1\rangle$ and $|2\rangle$ is described by

$$\vec{\mathscr{P}} = [\chi^{(1)} + \chi^{(3)}(|\mathscr{E}_1|^2 + 2|\mathscr{E}_2|^2)]\vec{\mathscr{E}}_1 \tag{4}$$

where the quantities $\chi^{(1)}$ and $\chi^{(3)}$ are known to be the linear and third-order non-linear optical susceptibilities respectively. The first term in the round bracket is responsible for self phase modulation (SPM) and the second term leads to cross-phase modulation (XPM). The polarization $\vec{\mathscr{P}}$ is a slowly varying function of both space and time coordinates because of the dependence on the slowly varying parameter $\vec{\mathscr{E}}_1$. We use the nonlinear Schrödinger's equations(NLS) to obtain the spatiotemporal evolutions of the light pulses through a doped nonlinear dispersive medium. Taking slowly varying envelop approximation and converting the equation for the Rabi frequencies of the field, we obtain

$$\frac{\partial g}{\partial z} = -i\beta_1\frac{\partial^2 g}{\partial t^2} + i\gamma_1(|g|^2 + 2|G|^2)g + i\eta_1\mathscr{C}_3^*\mathscr{C}_1 \tag{5}$$

$$\frac{\partial G}{\partial z} = -i\beta_2\frac{\partial^2 G}{\partial t^2} + i\gamma_2(|G|^2 + 2|g|^2)G + i\eta_2\mathscr{C}_2^*\mathscr{C}_1 \ , \tag{6}$$

where η_i determines the coupling to the atomic system for the i-th mode

$$\eta_i \approx \eta = \frac{4\pi\mathscr{N}\omega_i|d|^2}{c\hbar}, \ (i = 1,2). \tag{7}$$

In Eqs.(5-6), $\beta_i, i = 1,2$ represents the group velocity dispersion, $\gamma_i, i = 1,2$ denotes the Kerr nonlinearity. Note that γ_1 and γ_2 can differ because of dispersion. Let us show how the various limiting cases (earlier know results) can be recovered from our general equations. SIT solitons of McCall and Hahn [1, 2] can be recovered by setting $\beta_i = \gamma_i = G = 0$ in Eqs. (2) and (5). The SIT-NLS solitions of Nakazawa *et al.* [17] can be obtained by setting G to be zero, reducing the problem to one of single mode of the fiber interacting with a resonant two-level system. It is also clear from Eqs. (5) and (6) that one would recover the case considered by Trillo *et al* [15] by setting both the η's to be zero. We use all these limiting cases as checks of correctness of our numerical code (see section III below).

Since both the bare fiber and the bare 3-level system allow for *sech-tanh* pairs of pulses [7, 8, 15], we use the following ansatz for the solution of Eqs.(2),(5) and (6).

$$g = A\ \mathrm{sech}(Kz - \frac{t}{\sigma})e^{i(p_1 z - \Omega_1 t)} \tag{8}$$

$$G = B\ \tanh(Kz - \frac{t}{\sigma})e^{i(p_2 z - \Omega_2 t)} \tag{9}$$

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$$
\begin{aligned}
\mathscr{C}_1 &\sim e^{-i\Omega_1 t} &(10)\\
\mathscr{C}_2 &\sim e^{-i(\Omega_1-\Omega_2)t} &(11)\\
\mathscr{C}_3 &= \alpha\tanh(Kz-\frac{t}{\sigma})+(1-\alpha) &(12)
\end{aligned}
$$

In the above equations σ determines the temporal width of the pulse and $1/(K\sigma)$ gives the envelope velocity in the moving frame. The other constants are to be determined in a self-consistent fashion. The expression of $\mathscr{C}_3$ is chosen in such way that, in the remote past, all the population is in the ground state. Note that in contrast to Refs.[7, 8], we have allowed for the frequency shifts $\Omega_{1,2}$. The choice of the temporal exponential factors in Eqs. (8)-(11) is obvious and leads to the cancellation of explicit time dependence except for a problematic term $i(\Omega_1-\Omega_2)$ in $\mathscr{C}_2$ arising from Eq.(2). This term can be eliminated under the condition $\Omega_1=\Omega_2$. Henceforth, we will assume that the frequency shifts are identical for both the fiber modes, *i.e.*, $\Omega_1=\Omega_2=\Omega$. Under such constraints the expressions for the amplitudes $\mathscr{C}_1$ and $\mathscr{C}_2$ can be written as

$$
\begin{aligned}
\mathscr{C}_1 &= \left(\frac{i\alpha}{A\sigma}\right)\operatorname{sech}(Kz-\frac{t}{\sigma})e^{i(p_1z-\Omega t)} &(13)\\
\mathscr{C}_2 &= \left(-\frac{\alpha B}{A}\right)\operatorname{sech}(Kz-\frac{t}{\sigma})e^{i(p_1-p_2)z} &(14)
\end{aligned}
$$

The solutions (8), (9), (12-14) are then substituted in Eqs. (2), (5), (6) and coefficients for *Sech*, *Tanh* and *SechTanh* are then collected yielding the self-consistency relations. The Bloch part yields the following two relations

$$
\begin{aligned}
i(1-\alpha) &= \frac{\alpha(\Omega-\Delta)}{A^2\sigma} &(15)\\
A^2-B^2 &= \frac{1}{\sigma^2} &(16)
\end{aligned}
$$

The Eq. (15) gives relation between α and Δ as follows:

$$
\alpha=\frac{A^2\sigma^2}{A^2\sigma^2+i(\Delta-\Omega)\sigma}, \qquad (17)
$$

The coupled nonlinear Schrödinger part yields the following set of equations

$$
\begin{aligned}
p_1 &= \frac{|\alpha|^2\eta_1(\Delta-\Omega)}{A^4\sigma^2}+2B^2\gamma_1+\beta_1(\Omega^2-\frac{1}{\sigma^2}) &(18)\\
p_2 &= \gamma_2B^2+\beta_2\Omega^2 &(19)\\
K &= \frac{\eta_1|\alpha|^2}{\sigma A^2}+\frac{2\beta_1\Omega}{\sigma}=\frac{\eta_2|\alpha|^2}{\sigma A^2}+\frac{2\beta_2\Omega}{\sigma} &(20)\\
\frac{2\beta_1}{\gamma_1\sigma^2} &+ (A^2-2B^2)=0 &(21)\\
\frac{2\beta_2}{\gamma_2\sigma^2} &+ (2A^2-B^2)=0 &(22)
\end{aligned}
$$

In writing Eq. (18) we made use of Eq. (15). Equation(20) easily leads to the constraint

$$
\frac{|\alpha|^2}{A^2}(\eta_1-\eta_2)+2\Omega(\beta_1-\beta_2)=0. \qquad (23)
$$

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It will be shown later that for stable propagation of pulse pairs, it is essential that $\beta_1 \neq \beta_2$, and thus Ω can be evaluated using Eq. (23) as

$$\Omega = \frac{\frac{|\alpha|^2}{A^2}(\eta_2 - \eta_1)}{2(\beta_1 - \beta_2)}. \tag{24}$$

Equations (21), (22) and (16) lead to the important relation

$$3 + \frac{2\beta_1}{\gamma_1} + \frac{2\beta_2}{\gamma_2} = 0. \tag{25}$$

Equation(25) is one of the central results that should be satisfied by the nonlinearity and dispersion in the medium at the two mode frequencies. We show in the next section that this condition can be satisfied with the same or opposite signs of group velocity dispersion. One of these solutions is quasi-stable against modulational instability while the other breaks down quickly.

A close inspection of the argument of the pulse envelopes leads easily to the following relation satisfied by the group velocity v_g

$$\frac{c}{v_g} - 1 = K\sigma c = \frac{\eta_1 \sigma^2 (A\sigma)^2 c}{(A\sigma)^4 + (\Delta\sigma - \Omega\sigma)^2} + 2\beta_1 \Omega c. \tag{26}$$

It is clear from Eq. (26) that for $\eta_1 = \eta_2 = \eta$ (leading to $\Omega = 0$), large detuning (diminishing 3-level effect) leads to $v_g \rightarrow c$, which holds for bare fiber. The other extreme, namely $\Delta = 0$, reproduces the result of Eberly for perfect resonance $c/v_g = 1 + c\eta/A^2$ [7]. It should be noted that for $\eta_1 \neq \eta_2$, and hence $\Omega \neq 0$, the group velocity gets affected by the fiber GVD β.

We now demonstrate how the results of Nakazawa *et al.* [17, 18] can be recovered from our general results. This case along with few other limiting cases are discussed below in the context of testing our numerical code. We set $G = 0$ and $\eta_2 = 0,\ \eta_1 = \eta, \beta_1 = \beta$ in order to recover the coexistence of SIT-NLS solitons. This yields the following conditions

$$A^2\sigma^2 = 1, \quad 1 + \frac{2\beta}{\gamma} = 0. \tag{27}$$

The conditions (27) along with the limiting forms of the solutions for g, $\mathscr{C}_1$ and $\mathscr{C}_3$ agree well with those of Nakazawa *et al*. Further, at perfect resonance ($\Delta = 0$) for $\Omega = 0$, one has simple relations for the parameters

$$\alpha = 1, \quad K = \eta\sigma, \quad p = -\beta/\sigma^2. \tag{28}$$

3. Numerical results and discussion

In this section we present the numerical results by integrating the full set of coupled Bloch Eqs. (2) and NLS Eqs. (5) and (6) with the following scalings

$$\tau = \frac{t}{\sigma} - Kz,\ \zeta = \frac{z}{z_0},\ z_0 = \frac{\pi\sigma^2}{2\beta_2}. \tag{29}$$

We use a combination of the Runge-Kutta method and split operator method to simulate the spatiotemporal evolution of the optical pulses to delineate the effect of both resonant and non-resonant nonlinearity of the medium. We use initial conditions given by our analytical solutions to check their stability. The atomic system is assumed to be prepared in the ground state.

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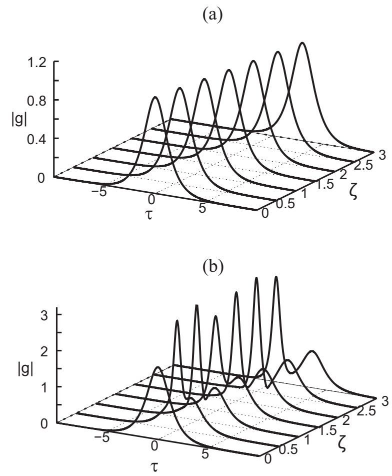

Fig. 2. (a) Propagation of the *sech* pulses with an area 2π at different propagation distances in a single mode fibre coupled with resonant nonlinearity. The Fig. 2(b) shows the break-up of a pulse with area 4π into two pulses with area 2π. The different parameters used in the numerical simulation are as follows: group velocity dispersion $\beta_1 = -0.5$ and Kerr nonlinearity $\gamma_1 = 1$.

Before we present the numerical results, a discussion of parameters leading to coupled solitons in a medium with both resonant and non-resonant nonlinearity would be in place. It was shown by the Nakazawa *et al.* that choosing a realistic system of a nonlinear fiber, doped with rare earth materials for the coexistence of SIT and NLS solitons is extremely difficult [17, 18]. The difficulty is associated with the fact that the power requirement for the N=1 solitons for the two cases differ by orders of magnitude. Hence, in their experiment the NLS soliton was suppressed by choosing the fiber mode with null GVD [19]. We do not offer any resolution of this problem and use heuristic parameter values. Although our choice of parameters is somewhat away from those of available realistic fibers and rare earth atoms, our calculations reveal clearly the intricate interplay between the resonant and non-resonant nonlinearities. The presence of the fiber is shown to lead to hitherto unknown solutions for unequal couplings. Since we present analytical expressions for most of our results, the suitability of a given fiber or atomic species can be checked with further development of fiber and materials technology. In our numerical simulations, we consider the width of the pulse $\sigma = 1$. As a consequence of our choice of parameters, most of the results are in arbitrary units and they are suppressed in the plots.

In order to ensure the correctness of our numerical code, we first studied various limiting cases. We start with the standard SIT solitons of a two-level system and verify the stable propagation of a bright soliton with area 2π, and break-up of a pulse with area 4π into two 2π solitons (results of simulation not shown). We next verify the case of nonlinear propagation of dark and bright solitons in two mode optical fibre [15] (not shown). The bright pulse can propagate without any distortion despite normal GVD, when it couples with the dark soliton with anomalous GVD through cross phase modulation. It is necessary to choose dark and bright solitons on the two sides of null group velocity dispersion for the stable solitons solutions.

As the other limiting case we studied the system investigated by Nakazawa *et el.*[18]. In Fig. 2 we have demonstrated the propagation dynamics of SIT-NLS soliton in a resonant dispersive medium in the presence of group velocity dispersion and self phase modulation. We show in Fig.2(a) the stable propagation of a 2π SIT-NLS soliton. As can be seen from Fig.2(b), in conformity with the earlier results, the input soliton with area 4π splits into two separate 2π

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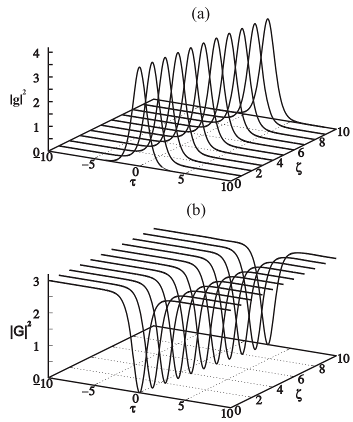

Fig. 3. Stable propagation of (a) bright and (b) dark solitons in a three level medium in presence of nonresonant nonlinearity and fiber dispersion. The different parameters are as follows: Input intensities $A^2 = 4$, $B^2 = 3$, pulse width $\sigma = 1$, Kerr coefficient $\gamma_1 = \gamma_2 = 1$, group velocity dispersion parameter $\beta_1 = 1.0$, $\beta_2 = -2.5$, coupling constant $\eta_1 = \eta_2 = 1$, single photon detuning $\Delta\sigma = 0$. Note the opposite signs of the GVD at the two mode frequencies.

solitons after traversing some distance into the medium. We next provide numerical results for the medium with competing resonant and non-resonant nonlinearities. Since distortionless propagation of the pulses is of utmost importance for any practical application, we first present the results pertaining to the stability of the pulses. In Fig. 3(a) and 3(b) we show the spatio-temporal evaluation of the bright and dark solitons for $\eta_1 = \eta_2 = 1$ and $\Delta\sigma = 0$. We have chosen the intensities of input bright and dark pulses as $A^2 = 4$ and $B^2 = 3$, respectively, such that they obey the self-consistency relation (16). Like in the case of Trillo *et al.* [15] we choose the group velocity dispersion of the bright and dark pulses above and below the null dispersion.

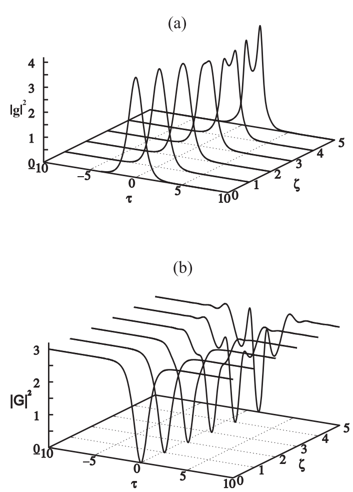

Fig. 4. Growth of instability for (a) bright and (b) dark solitons in a three level medium in presence of nonresonant nonlinearity and fiber dispersion. Parameters are the same as in Fig. 3, except that now group velocity dispersion β_1 (=-0.25) and β_2 (=-1.25) have the same sign.

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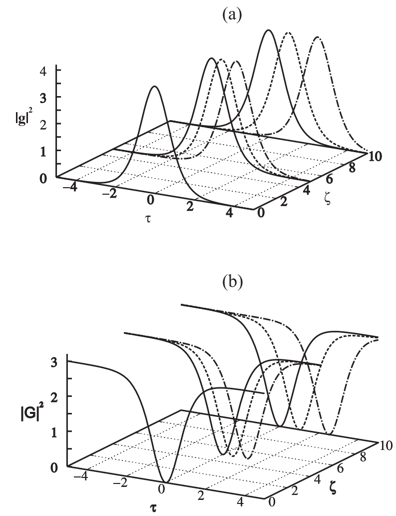

Fig. 5. Spatio-temporal evolution of the (a) bright and (b) dark solitons in (i) a two-mode fiber (solid curve), (ii) 3-level system (dashed-dot curves) and (iii) in a doped fiber (dotted curves). Cases (ii) and (iii) are plotted for $\Delta\sigma$=0 and 5, respectively. Case (ii) also corresponds to a doped fiber with $\Delta\sigma = 0$. The other parameters are as follows $A^2 = 4$, $B^2 = 3$, $\sigma = 1$, $\eta_1 = \eta_2 = 1$, $\beta_1 = 1$, $\beta_2 = -2.5$

Keeping in mind condition (25), we have used $\beta_1 = 1.0$ for the bright pulse and $\beta_2 = -2.5$ for the dark pulse, respectively. It is clear from Fig. 3(a) that the bright soliton moves without any distortion even in normal dispersion regime. This quasi-stable bright soliton propagation is possible because it couples with dark soliton via the cross phase modulation. After sufficient propagation distances, the dark soliton starts showing signs of modulations instability. In order to highlight the need for choosing opposite character of GVD for the two modes for stable bright-dark pairs, we show in Fig. 4 the case where β_1 and β_2 were chosen to have the same sign, still consistent with Eq. (25). It is clear from the figure that both the constituents of the pair disintegrate after propagating short distances.

In order to appreciate the contributions of the constituent systems separately, and also to assess the effect of detuning, we undertake a detailed comparative study of the bare sytems (fiber and 3-level atoms) with the doped fiber. The results for the spatio-temporal evolution of the bright and dark pulses are shown in Figs. 5(a) and 5(b), respectively. The solid and the dashed-dot curves give the results for the bare fiber and a resonant ($\Delta\sigma = 0$) 3-level system, respectively. In fact, the latter curves are identical with those for the fiber doped with perfectly resonant atoms. The results for finite detuning, namely $\Delta\sigma = 5$ is intermediate between these two extremes (see dotted curves in Fig.5). For very large detuning, the doped fiber results are the same as those for the bare fiber. It is thus clear that the delay aspects of the pulses in the doped fiber is derived mainly from the strong dispersion in the atomic dopants. All these conclusions are consistent with Eq.(26).

As mentioned earlier, for $\eta_1 \neq \eta_2$, the fiber GVD can modify the group delay. This is shown in Fig.6 where we have plotted $K\sigma$ $(= 1/v_g - 1/c)$ as a function of the normalized detuning $\Delta\sigma$ for $\eta_1 = 1$, and for two values of η_2, namely, $\eta_2 = 1$(solid curve) and $\eta_2 = 2$ (dashed curve). The plots are obtained by two different means, namely, (a) by direct integration and interpreting the location of the peaks as in Fig. 5(a) and (b) by using Eq. (26). Both these methods led to the same curves. The results for bare 3-level atoms with $\eta_1 = 1, \eta_2 = 2$ is also given by the solid curve. Both these cases of bare 3-level system with unequal coupling constants and a doped

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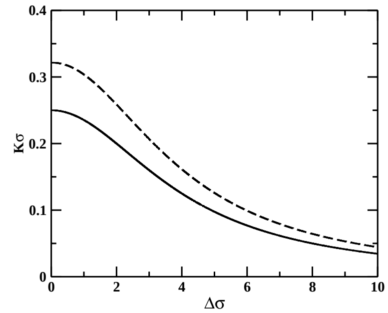

Fig. 6. $K\sigma = (v_g^{-1} - c^{-1})$ for a doped fiber as a function of normalized detuning $\Delta\sigma$ for $\eta_1 = 1$ and for two values of η_2, namely, $\eta_2 = 1$ (solid line) and $\eta_2 = 2$ (dashed line). The results for a 3-level system with $\eta_1 = 1$ and $\eta_2 = 2$ is also given by the solid curve. The other parameters are as in Fig.5.

fiber with equal coupling strength have no contribution from the rightmost term in Eq. (26). It is clear from this figure that the solutions with finite frequency shift ($\Omega \neq 0$ for $\eta_1 \neq \eta_2$) can lead to larger delays.

4. Conclusions

In conclusion, we have studied propagation of shape-preserving pulses in a two-mode fiber doped with Λ atoms. We have obtained the analytical form of the solutions, as well as the conditions for their existence with allowance for atomic detuning and frequency shifts. We have also simulated the spatio-temporal evolution of pulses by means of a combined Runge-Kutta and split-step method. We show that the stability aspects of the pulses of the doped fiber are inherited from the fiber, while the delay aspects are governed by the atomic system. Our results clearly reveal the important role of group velocity dispersion on stable propagation of these pulses. Quasi-stable propagation results in case when the two pulses are tuned at frequencies with opposing signs of GVD. The opposite case leads to a quick breakup of the pulses due to modulational instability. This is in tune with the findings of Trillo *et al.* [15]. We also reported a new class of solutions for unequal coupling strengths for the two modes. We presented a detailed study of the group delay with clear demarkation of the contributions from the fiber and the Λ atoms. Further, we demonstrated how other known results can be recovered from our general results both analytically and numerically.

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