Strong-driving-assisted multipartite entanglement in cavity QED

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We propose a method of generating multipartite entanglement by considering the interaction of a system of N two-level atoms in a cavity of high quality factor with a strong classical driving field. It is shown that, with a judicious choice of the cavity detuning and the applied coherent field detuning, vacuum Rabi coupling produces a large number of important multipartite entangled states. It is even possible to produce entangled states involving different cavity modes. Tuning of parameters also permits us to switch from Jaynes-Cummings to anti-Jaynes-Cummings like interaction.

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Two or more quantum systems are entangled when it is impossible to describe their physical properties by means of a direct product of their respective density operators. Entanglement is a natural consequence of linearity of Hilbert spaces and its controlled generation and measurement have been intensively pursued [1]. In a *Gedankenexperiment* after Schrödinger [2], using the properties of quantum theory, the states of an alive and a dead cat are correlated with two microscopic states of a decaying atomic nucleus. The absence of these unusual states in our everyday experience led to intense debate about the validity of quantum theory in macroscopic systems. In recent years, great effort has been put into preparation of the so-called Schrödinger cat states in the laboratory [3,4], where the extreme cat states have been reduced to mesoscopic quantum states with classical counterparts, the so-called coherent states. Decoherence processes, their rate being increased with the size of the system, have been claimed to be the mechanism inhibiting the manifestation of entanglement in macroscopic objects. In this sense, realizing bigger Schrödinger cat states in the laboratory will let us test decoherence by monitoring their decay [3,5]. Meanwhile, multipartite entangled systems are also considered since they are interesting in connection with quantum information [6,7].

Cavity QED, where atoms interact with a quantized electromagnetic field inside a cavity, have already proved to be a useful tool for testing fundamental quantum properties [7,8]. Here, we demonstrate how a large number of multipartite entangled states can be generated by adding a strong coherent field to the system [9,10]. The coherent drive affords great flexibility in generating entangled states since it provides freedom in choosing the detuning and strength of the field. Our method enables us to entangle different atoms, cavity modes and atoms and cavity modes. Furthermore, it is demonstrated how the external drive has an effect similar to that of a Ramsey field before and after the cavity [11], if we work in dressed-state basis. It is noteworthy that purely anti-Jaynes-Cummings like interaction is demonstrated.

The interaction of a two-level atom with a single mode of the electromagnetic field, described by the Jaynes-Cummings model [12], is one of the simplest and most fundamental quantum systems. It is typically realized in cavity QED experiments [7,8] in different frequency regimes and configurations. Here, we consider the interaction of a single mode (frequency ω) in a cavity of high quality factor with a spatially narrow bunch of N two-level atoms (transition frequency ω_o), driven additionally by an external classical field (frequency ω_L). The associated Hamiltonian reads

$$H = \hbar\omega_o\sum_{j=1}^{N}\sigma_j{}^\dagger\sigma_j + \hbar\omega a^\dagger a + \hbar\Omega\sum_{j=1}^{N}(e^{-i\omega_L t}\sigma_j{}^\dagger + e^{i\omega_L t}\sigma_j) + \hbar g\sum_{j=1}^{N}(\sigma_j{}^\dagger a + \sigma_j a^\dagger), \qquad (1)$$

where $\sigma_j = |g_j\rangle\langle e_j|$ and $\sigma_j{}^\dagger = |e_j\rangle\langle g_j|$ are the spin-flip operators down and up, respectively, associated with the upper level $|e_j\rangle$ and lower level $|g_j\rangle$ of atom j; a and $a^\dagger$ are the annihilation and creation operators associated with the intracavity photon field; g and Ω, both chosen to be real, are the coupling constants of the interaction of each atom with the cavity mode and with the driving field, respectively. A more realistic model of the cavity mode would include its interaction with a dissipative environment, finite Q, and a thermal bath, finite temperature. Here, we are interested in the strong coupling regime, $g > \kappa$, where dissipation can be neglected.

The description of the system of Eq. (1) is changed to a reference frame rotating with the driving field frequency

$$H^L = \hbar\Delta\sum_{j=1}^{N}\sigma_j{}^\dagger\sigma_j + \hbar\delta a^\dagger a + \hbar\Omega\sum_{j=1}^{N}(\sigma_j{}^\dagger + \sigma_j) + \hbar g\sum_{j=1}^{N}(\sigma_j{}^\dagger a + \sigma_j a^\dagger), \qquad (2)$$

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with $\Delta = \omega_o - \omega_L$ and $\delta = \omega - \omega_L$. For the sake of simplicity, we set $\Delta = 0$ hereafter. We define

$$
\begin{aligned}
H^L &= H_o^L + H_{int}^L, \\
H_o^L &= \hbar\delta a^\dagger a + \hbar\Omega \sum_{j=1}^{N} ({\sigma_j}^\dagger + \sigma_j), \\
H_{int}^L &= \hbar g \sum_{j=1}^{N} ({\sigma_j}^\dagger a + \sigma_j a^\dagger).
\end{aligned} \tag{3}
$$

The Hamiltonian H^L in the interaction picture yields

$$
\begin{aligned}
H^I = \frac{\hbar g}{2} \sum_{j=1}^{N} \Big(&|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j| + e^{2i\Omega t}|+_j\rangle\langle -_j| \\
&- e^{-2i\Omega t}|-_j\rangle\langle +_j| \Big) a e^{-i\delta t} + \text{H. c.},
\end{aligned} \tag{4}
$$

where the dressed states $|\pm_j\rangle = (|g_j\rangle \pm |e_j\rangle)/\sqrt{2}$ are eigenstates of $(\sigma_x)_j = \sigma_j^\dagger + \sigma_j$ with eigenvalues ± 1, respectively. In the strong driving regime $\Omega \gg \{g, \delta\}$, we can realize a rotating-wave approximation and eliminate from Eq. (4) the terms that oscillate with high frequencies

$$
\begin{aligned}
H_{eff} &= \frac{\hbar g}{2} \sum_{j=1}^{N} \Big(|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j| \Big)(a e^{-i\delta t} + a^\dagger e^{i\delta t}) \\
&= \frac{\hbar g}{2} \sum_{j=1}^{N} (\sigma_j^\dagger + \sigma_j)(a e^{-i\delta t} + a^\dagger e^{i\delta t}).
\end{aligned} \tag{5}
$$

A noteworthy feature of Eq. (5), more evident if $\delta = 0$ and $N = 1$ are chosen, is the simultaneous realization of Jaynes-Cummings (JC) and anti-Jaynes-Cummings (AJC) interaction, appearing naturally in trapped ions [13] but not in the context of cavity QED.

Some examples of the possible applications of the interaction described in Eq. (5) are given. If at $t = 0$, $N = 1$, the 1-atom-field state is $|g\rangle|0\rangle = (|+\rangle + |-\rangle)|0\rangle/\sqrt{2}$, the evolved state after a time t will be

$$
\frac{1}{\sqrt{2}}(|+\rangle|\alpha\rangle + |-\rangle|-\alpha\rangle), \tag{6}
$$

with $\alpha = g(e^{i\delta t} - 1)/2\delta$. The microscopic-mesoscopic entangled state of Eq. (6) is usually called the Schrödinger cat state. Clearly, for the simpler case $\delta = 0$, we have $\alpha = -igt/2$, which shows fast resonant generation of Schrödinger cat states as compared with dispersive methods [3]. Rewriting Eq. (6) in the Schrödinger picture,

$$
\begin{aligned}
\frac{1}{2}\Big[&|g\rangle (e^{-i\Omega t}|\alpha e^{-i\omega t}\rangle + e^{i\Omega t}|-\alpha e^{-i\omega t}\rangle) \\
&+ e^{-i\omega_o t}|e\rangle (e^{-i\Omega t}|\alpha e^{-i\omega t}\rangle - e^{i\Omega t}|-\alpha e^{-i\omega t}\rangle) \Big],
\end{aligned} \tag{7}
$$

shows that measurement of the atomic state will produce the so-called even or odd coherent states in the cavity field, depending on whether $|g\rangle$ or $|e\rangle$ was found, respectively. Throughout this work, we specify when the final states are written in the Schrödinger picture, as in Eq. (7), to illustrate the unusual way in which phases appear after the conveniently realized transformations. If at $t = 0$, $N = 2$, the 2-atom-field state is $|g_1 g_2\rangle \otimes |0\rangle$, the evolved state after a time t will be

$$
\frac{1}{2}\Big[|\phi_1\rangle|2\alpha\rangle + |\phi_2\rangle|-2\alpha\rangle + (|\phi_3\rangle + |\phi_4\rangle)|0\rangle \Big], \tag{8}
$$

where $|\phi_i\rangle$ are the eigenstates of the atomic operator $\sum_{j=1}^{2}(\sigma_j^\dagger + \sigma_j)$ with eigenvalues $\gamma_{1,2} = \pm 2$ and $\gamma_{3,4} = 0$. The state of Eq. (8) is a bigger and more elaborate microscopic-mesoscopic 2-atom-field entangled state. Measuring the two atoms in the state $|g_1 g_2\rangle$ will produce, in the Schrödinger picture, the field state

$$
\mathcal{N}\Big(e^{-2i\Omega t}|2\alpha e^{-i\omega t}\rangle + e^{2i\Omega t}|-2\alpha e^{-i\omega t}\rangle + 2|0\rangle \Big), \tag{9}
$$

which is a triple mesoscopic field superposition state with not only an "alive" or "dead" cat, but also and primarily an "absent" one. The effective interaction of Eq. (5) lets us create more sophisticated and bigger microscopic-mesoscopic N-atom-field entangled states and field superposition states, which will not be described here. Some of these new states [13], in fact, present larger quantum interference regions when displayed in phase-space, showing distinctly their quantum nature.

When $\delta = \pm 2\Omega$ and $|\delta| \gg g$, Eq. (4) turns into

$$
\begin{aligned}
H_{JC}^{(+)} &= \frac{\hbar g}{2} \sum_{j=1}^{N} \Big(|+_j\rangle\langle -_j|\, a + |-_j\rangle\langle +_j|\, a^\dagger \Big), \\
H_{AJC}^{(-)} &= \frac{\hbar g}{2} \sum_{j=1}^{N} \Big(|-_j\rangle\langle +_j|\, a + |+_j\rangle\langle -_j|\, a^\dagger \Big),
\end{aligned} \tag{10}
$$

which represent, in the $|\pm_j\rangle$ atomic dressed basis, an effective implementation of a Jaynes-Cummings or an anti-Jaynes-Cummings interaction. As is known, the anti-JC interaction does not appear naturally in the context of cavity QED, where the JC model discards the so-called nonconserving energy terms, corresponding to exciting (deexciting) the internal atomic state while creating (annihilating) an intracavity photon. Another noteworthy feature of the interactions described in Eqs. (10) is that they produce, in a controlled manner, absorption and emission of an intracavity photon while preserving the energy mean value of the atomic state. Clearly, the apparent imbalance in the energy stems from the external driving field, which, surprisingly, is intense enough to be considered classical. Furthermore, let us imagine the situation where the atom is initially in the ground state $|g\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$ and performs a JC interaction in the atomic dressed basis $|\pm\rangle$, following any of the Hamiltonians described in Eqs. (10). Then, by measuring the atom at the end of the interaction in the atomic bare basis $\{|g\rangle, |e\rangle\}$, our procedure is equivalent to a conventional JC evolution with two Ramsey zones, one before and one after the atom-cavity interaction. This result makes Ramsey zones unnecessary and may open new possibilities for phase-sensitive measurements [14] in closed cavities, as is discussed later.

Let us now concentrate on the possibilities of the studied scheme in the case of interaction of a bunch of N atoms with two quasiresonant normal modes in the cavity, always assisted by a strong external driving field. The associated Hamiltonian, as in (2), with $\Delta = 0$, reads

$$H^{ab} = \hbar\delta_a a^\dagger a + \hbar\delta_b b^\dagger b + \hbar\Omega\sum_{j=1}^{N}({\sigma_j}^\dagger + \sigma_j) + \hbar g_a \sum_{j=1}^{N}({\sigma_j}^\dagger a + \sigma_j a^\dagger) + \hbar g_b \sum_{j=1}^{N}({\sigma_j}^\dagger b + \sigma_j b^\dagger), \quad (11)$$

where $\{a^\dagger, b^\dagger\}$ and $\{a, b\}$ are the creation and annihilation operators associated with the two cavity modes, while $\delta_a = \omega_a - \omega_L$ and $\delta_b = \omega_b - \omega_L$. We now define

$$H^{ab} = H_o^{ab} + H_{int}^{ab} \quad (12)$$

with

$$H_o^{ab} = \hbar\delta_a a^\dagger a + \hbar\delta_b b^\dagger b + \hbar\Omega\sum_{j=1}^{N}({\sigma_j}^\dagger + \sigma_j),$$

$$H_{int}^{ab} = \hbar g_a \sum_{j=1}^{N}({\sigma_j}^\dagger a + \sigma_j a^\dagger) + \hbar g_b \sum_{j=1}^{N}({\sigma_j}^\dagger b + \sigma_j b^\dagger). \quad (13)$$

In the interaction picture, defined by the formal separation in Eq. (13), the Hamiltonian changes to

$$\tilde{H}^{ab} = \frac{\hbar}{2}\sum_{j=1}^{N}\Big(|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j| + e^{2i\Omega t}|+_j\rangle\langle -_j| - e^{-2i\Omega t}|-_j\rangle\langle +_j|\Big)(g_a a e^{-i\delta_a t} + g_b b e^{-i\delta_b t}) + \mathrm{H.c.} \quad (14)$$

In the strong driving limit, $\Omega \gg \{g, \delta_a, \delta_b\}$, we obtain

$$H_{eff}^{ab} = \frac{\hbar}{2}\sum_{j=1}^{N}(\sigma_x)_j[g_a(a e^{-i\delta_a t} + a^\dagger e^{i\delta_a t}) + g_b(b e^{-i\delta_b t} + b^\dagger e^{i\delta_b t})]. \quad (15)$$

This Hamiltonian will produce states with features similar to those produced by the one of Eq. (5), only that now the displacement will act simultaneously on each of the two cavity modes. If at $t = 0$, with $N = 1$ and $\delta_a = \delta_b = 0$, the initial atom-field state is $|g\rangle|0\rangle|0\rangle = (|+\rangle + |-\rangle)|0\rangle|0\rangle/\sqrt{2}$, the evolved state at time t will be

$$\frac{1}{\sqrt{2}}\Big(|+\rangle|\alpha\rangle|\beta\rangle + |-\rangle|-\alpha\rangle|-\beta\rangle\Big), \quad (16)$$

with $\alpha = g_a t/2$ and $\beta = g_b t/2$. If $g_a = g_b$, we would have $\alpha = \beta$. Equation (16) describes an elaborate tripartite entangled state involving one microscopic and two mesoscopic systems. If we measure the atomic state in the bare basis $\{|g\rangle, |e\rangle\}$, we will find the field in the so-called entangled coherent states [15]

$$\mathcal{N}_{ab}^{\pm}\big(e^{-i\Omega t}|\alpha e^{-i\omega t}\rangle|\beta e^{-i\omega t}\rangle \pm e^{i\Omega t}|-\alpha e^{-i\omega t}\rangle|-\beta e^{-i\omega t}\rangle\big), \quad (17)$$

respectively. These states have recently been proposed as an important tool in theory and experiments relating to the field of quantum information [16]. Similar states were proposed recently [17], entangling the vacuum and a coherent state in two different cavities.

As before, Eq. (14) is taken in the limit where $\delta_a = \delta_b = \pm 2\Omega$. The resulting Hamiltonians are

$$H_{ab}^{(+)} = \frac{\hbar g}{2}\sum_{j=1}^{N}\Big[|+_j\rangle\langle -_j|\,(a+b) + |-_j\rangle\langle +_j|\,(a^\dagger + b^\dagger)\Big],$$

$$H_{ab}^{(-)} = \frac{\hbar g}{2}\sum_{j=1}^{N}\Big[|-_j\rangle\langle +_j|\,(a+b) + |+_j\rangle\langle -_j|\,(a^\dagger + b^\dagger)\Big], \quad (18)$$

with $g_a = g_b$. As will be seen, these Hamiltonians are able to produce other kinds of interesting nonclassical states. For example, if at $t = 0$, and with $N = 1$, the atom-field state is $|+\rangle|0\rangle|0\rangle$, then, after a time interval t of the $H_{ab}^{(+)}$, the atom-field state will be

$$\cos(\sqrt{2}gt)|+\rangle|0\rangle|0\rangle - i\sin(\sqrt{2}gt)|-\rangle\frac{(|0\rangle|1\rangle + |1\rangle|0\rangle)}{\sqrt{2}}. \quad (19)$$

It is easy to show that if the interaction time $\tau = \sqrt{2}\pi/4g$ is taken, the final state will be

$$\frac{1}{\sqrt{2}}(|0\rangle|1\rangle + |1\rangle|0\rangle). \quad (20)$$

This state is a maximally entangled state between the two cavity modes in the $\{|0\rangle, |1\rangle\}$ subspace. This entangled state was recently produced in cavity QED with a sequence of differently tuned interactions of a single atom with two cavity modes [18]. Equation (14) may also accept the limit $\delta_a = -\delta_b = \pm 2\Omega$, combining JC evolution in the one mode and anti-JC evolution in the other one, producing states that are beyond our present scope. Note that a number of other interesting situations can arise by choosing nonzero detuning between the external field and the atomic transition frequency ($\Delta \neq 0$).

So far, all previous results are suitable for implementation in the microwave and optical regimes in cavity QED experiments, with atoms flying through the cavities or conveniently trapped inside them. The microwave regime in the strong coupling limit, involving high-Q cavities and long-living Rydberg atomic levels, is more adequate when considering the production and measurement of the proposed nonclassical states. The optical regime, even if enjoying more restricted strong coupling conditions, may profit from the newly designed interactions, in particular Eqs. (5) and (10), in an experimental setup that comfortably incorporates external driving [19]. Open cavities, in both regimes, offer the best possibilities of experimental realization, insofar as direct illumination of the atoms with an external classical field is concerned. Nevertheless, in the case of closed microwave cavities, an external field directly coupled to a second normal cavity mode could be used. Moreover, the external field could be coupled to the same cavity mode, profiting from a known equivalence with the present scheme [20]. This alternative would turn the interactions described in Eqs. (10) into a practical realization of a closed high-Q cavity plus two Ramsey zones (without them), thus combining the advantages of these two powerful tools. In the case of an open cavity, one can envisage adding another microwave cavity, always transversal to the crossing atoms, which would keep them continuously driven, satisfying the requirements of the proposed scheme. Spatially narrow bunches of atoms sent through microwave cavities have already been implemented in the laboratory [21,22].

Entanglement involving mesoscopic states is known to be very sensitive to decoherence. Nevertheless, the interplay between a continuously pumped entangled state, as the ones discussed in this work, and decoherence processes might lead to a nontrivial dynamics towards an atom-field steady-state. These considerations are under current research and will be published elsewhere.

The question of possible implementation of these ideas in the context of trapped ions is relevant [10]. A careful study of Eq. (1) shows that this interaction could be implemented in a simple way, by means of simultaneous carrier and red sideband excitation, as currently achieved in the laboratory. The condition of strong driving for the carrier could be easily satisfied and the red sideband excitation would require the Lamb-Dicke regime.

It has been shown that an additional driving field in the usual cavity QED experiments, and also in trapped ion systems, can be an important tool for generating multipartite nonclassical states. We expect that accessibility to these broader class of entangled states, from which we presented a reduced number, offers a deeper insight and stimulates the active field of multipartite entanglement.

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