

RESONANCES IN SMALL FERMI SYSTEMS

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ABSTRACT

The systematics of the size dependence of the resonant response of small metal particles and nuclei to incident electromagnetic radiation is studied. The known radius⁻¹ variation of the full width at half maximum (FWHM) in matrix-embedded metal particles is qualitatively accounted for by a quantum calculation of the response within a simple model. In free clusters, the behaviour is more complicated, possibly because of thermal excitation of surface modes. For nuclei, the FWHM shows strong shell-structure-linked oscillations across the periodic table. Focussing on the lower envelope of the oscillations (magic nuclei), the downward trend of the FWHM is consistent with the radius⁻¹ variation. A schematic theoretical description of the systematics in nuclei is presented. If the FWHMs are scaled by the respective Fermi energies and the inverse radii by the Fermi wave vectors, the data sets for matrix-embedded metal particles and nuclei become comparable in magnitude.

1. Introduction

The resonant response of small Fermi systems to electromagnetic radiation has been receiving a good deal of attention in various branches of physics. By ‘small’ we mean a system whose dimensions are smaller than, or at most comparable to, the mean free path of the fermions and also the wavelength of the incident radiation. Two sets of physical systems which satisfy these criteria are small metal particles and clusters on the one hand, and atomic nuclei on the other. In this contribution, we shall compare the systematics of resonances in these two sets of systems, namely the Mie resonance in metal particles and the giant dipole resonance (GDR) in nuclei, with varying numbers of fermions.

Certain points of resemblance between the electronic properties of metal particles and the properties of nuclei have been discussed earlier, see, *e.g.*, Sugano.¹ Electrons in metal particles and nucleons in nuclei both constitute finite Fermi systems at temperatures much less than the respective Fermi energies – a fact which cuts across the very different scales of length, mass and energy in the two systems. Metal particles, like nuclei (and unlike atoms), exhibit saturation, or constancy of particle density, with increasing size. Also, shell structure in energy levels – long familiar in nuclear physics – manifests itself in the relative abundances, polarizabilities and ionization potentials of metal clusters as well,² although the large magic numbers in the two cases differ because of the different strengths of the spin-orbit force.

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Here we focus on the analogy between the response of metal particles and nuclei to electromagnetic radiation. In metal particles, the Mie resonance involves displacing the conduction electron cloud with respect to the background of positive ions, and there are electromagnetic restoring forces.^{3,4,5} In a nuclear giant dipole resonance, protons are displaced with respect to neutrons and strong interactions provide the restoring force.^{6,7}

In Section 2, we discuss the Mie resonance in metal particles with an emphasis on size dependence of the resonance width. We also present there a calculation of the width within the framework of linear response theory. In Section 3, we discuss cross section systematics of GDR in nuclei and the various contributions to its width. In Section 4, we point out similarities and differences between the systematics in metal particles and nuclei. Conclusions are given in Section 5.

Some of the results discussed in this paper have appeared elsewhere.⁸

2. The Mie Resonance in Small Metal Particles

The absorption spectrum of small metal particles shows a strong resonance in the optical region. Studied by G. Mie in the early part of the century, the origin of this resonance can be understood from the following simple argument,³ in the limit that the wavelength λ of the radiation far exceeds the radius R of the particle. Recalling that the field that penetrates a sphere of dielectric constant ε in a uniform external electric field E_o is $3E_o/(2 + \varepsilon)$, we see that there is resonant penetration at the frequency ω_0 at which $\varepsilon(\omega_0) = -2$. Using the free electron gas expression $\varepsilon(\omega) = 1 - \omega_p^2/\omega^2$, the resonance frequency is given by $\omega_0 = \omega_p/\sqrt{3}$, where $\omega_p = \sqrt{4\pi n e^2/m}$ is the plasma frequency, n being the electron number density. Thus, when electromagnetic radiation of frequency ω_0 falls on a metal particle with radius $R \ll \lambda$, there is a resonant response.

The systematics of the Mie resonance in the regime $R \ll \lambda$ have been studied experimentally for two types of samples: (i) Metal particles embedded in various matrices such as glass and the inert element solids. In such samples, particle agglomeration can pose a problem, but it is sometimes possible to achieve mutual isolation of particles. However, a spread of sizes cannot be avoided, a typical spread of radius being $\sim 20\%$. (ii) Free metal clusters in a beam, separated size-wise by mass spectroscopy. Measurements of the photoresponse are done while the particles are in beam, and often quite hot, though it is difficult to pin down the temperature reliably.

In Section 2.1 below, we briefly summarize the results of experiments on type (i) samples, and outline a linear response RPA calculation of the width. An inverse-size dependence is obtained, and the proportionality constant is calculated, correcting a long-standing error. Experiments on type (ii) samples are discussed in Section 2.2. This is followed by a discussion of physical effects which may contribute to the observed differences of behaviour of samples of types (i) and (ii), in particular the role of surface fluctuations at finite temperature.

2.1. Matrix-held Particles: 1/R Law for the FWHM

A fairly large range of sizes has been investigated in experiments on samples of type (i); for a review, see Ref. 9. It is found that ω_0 does not vary very strongly with size — less than 10% as the size is changed from $\sim 10\text{\AA}$ to $\sim 100\text{\AA}$. This is not surprising, as ω_p and thus ω_0 depends primarily only on the electron density. The sign of the shift of ω_0 (towards the red or blue end of the spectrum) has been a subject of debate. But more dramatic is the effect of size variation on the full width at half maximum (FWHM), which changes several-fold. In fact, experimental results reveal a systematic dependence of Γ_{av} (FWHM averaged over the spread of sizes in a sample) on R :

$$\Gamma_{av} = K \frac{\hbar v_F}{R} + \Gamma_{\infty}, \quad (1)$$

where v_F is the Fermi velocity, K is a constant of order unity and Γ_{∞} is the width in the bulk medium. Equation (1) describes the variation of the linewidth of Ag particles in a variety of host matrices. The constant K depends on the matrix⁹; it goes down by a factor ~ 3 as the matrix is changed from glass to an inert element solid like Ar and Ne , presumably due to differences in the potential at the outer surface of each metal particle.

Calculation of the photoabsorption by small metallic spheres was first attempted by Kawabata and Kubo¹⁰ within a simple model; an error in their calculation was corrected in Ref. (11). Below we briefly recapitulate the principal steps of the calculation. Suppose a volume fraction α of identical, small spherical particles with dielectric constant $\varepsilon \equiv \varepsilon_1 - i\varepsilon_2$ is well dispersed in a medium with real dielectric constant ε_0 . The absorption coefficient is given by^{10,11}

$$\gamma = \frac{18\pi\alpha\varepsilon_0^{3/2}}{\lambda} \frac{\varepsilon_2}{(2\varepsilon_0 + \varepsilon_1)^2 + \varepsilon_2^2}, \quad (2)$$

provided that $\lambda \gg R$. The Mie resonance occurs when $2\varepsilon_0 + \varepsilon_1 = 0$. The lineshape is approximately a Lorentzian centered at the resonance frequency ω_0 .

The random phase approximation (RPA) for the system involves calculating the dielectric constant using a noninteracting electron gas model.¹⁰ The confining spherical potential well is taken to be infinitely deep. Within linear response theory, $\varepsilon_2(\omega)$ can be expressed in terms of the one-electron energies E_i and eigenfunctions $|i\rangle$:

$$\varepsilon_2(\omega) = \frac{4\pi^2\hbar}{\omega^3 V} \sum_{i,j} \frac{f(E_i)(1-f(E_j))}{E_j - E_i} |\langle j|\dot{J}_z|i\rangle|^2 \delta(\hbar\omega - E_j + E_i). \quad (3)$$

Here V is the volume of the spherical well, \dot{J}_z is the rate of change of the current operator and $f(E_i)$ is the occupation number of state i .

In a hard-walled sphere of radius R , each eigenfunction is a product of a spherical harmonic $Y_{\ell M}(\Omega)$ and a radial eigenfunction \mathcal{R}_{nl} which involves the spherical Bessel

function j_ℓ of order ℓ . The one-electron energy is $E_{n\ell} = \hbar^2 k_{n\ell}^2 / 2m$ where $(k_{n\ell}R)$ is the location of the n 'th zero of j_ℓ .

At temperature $T = 0$, the Fermi function $f(E_i)$ is replaced by the step function $\theta(\mu - E_i)$ where μ is the chemical potential. On evaluating the matrix element in Eq. (3), we obtain

$$\varepsilon_2(\omega) = \frac{16\pi e^2}{m^2 R^5 \omega^4} \sum_{\substack{n_1 \ell_1 \\ n_2 \ell_2}} \frac{1}{2} (\ell_1 \delta_{\ell_1, \ell_2+1} + \ell_2 \delta_{\ell_2, \ell_1+1}) E_{n_1 \ell_1} E_{n_2 \ell_2} \theta(\mu - E_{n_1 \ell_1}) \theta(E_{n_2 \ell_2} - \mu) \delta(\hbar\omega + E_{n_1 \ell_1} - E_{n_2 \ell_2}). \quad (4)$$

In the limit $k_F R \rightarrow \infty$ where $k_F \equiv \sqrt{2\mu m / \hbar^2}$ is the Fermi wave vector, the sums over (n_1, ℓ_1) and (n_2, ℓ_2) in Eq. (4) may be replaced by integrals. Though several authors had earlier obtained this equation, they had not evaluated the integral correctly.^{10,12} The correct procedure involves using the Debye expansion for large order Bessel functions.¹³ $\varepsilon_2(\omega)$ can be evaluated in closed form¹¹:

$$\varepsilon_2(\omega) = \frac{4}{\pi} \frac{e^2}{\hbar\omega R} \frac{1}{\nu^3} G(\nu), \quad (5)$$

where $\nu \equiv \hbar\omega/\mu$ and $G(\nu) = g(\nu) - g(-\nu)$ with

$$g(\nu) = \frac{(1+\nu)^{\frac{3}{2}}}{3} + \frac{\nu^2(1+\nu)^{\frac{1}{2}}}{4} - \frac{\nu^2(2+\nu)}{8} \log \left| \frac{\sqrt{1+\nu} + 1}{\sqrt{1+\nu} - 1} \right|. \quad (6)$$

In the limit of large radii, the real part $\varepsilon_1(\omega)$ of the dielectric constant is approximately $\varepsilon_1(\omega) = 1 - \omega_p^2/\omega^2$. The absorption as a function of frequency is given by Eqs. (2), (5) and (6), and the FWHM Γ of the Mie resonance is given by $2\varepsilon_2/|\partial\varepsilon_1/\partial\omega|$ evaluated at ω_0 . The result is

$$\Gamma = \frac{3}{4} \frac{G(\nu_0)}{\nu_0} \frac{v_F}{R}, \quad (7)$$

where $\nu_0 \equiv \hbar\omega_0/\mu$. In the limit $\nu \rightarrow 0$, we see from Eq. (6) that the ratio $G(\nu)/\nu$ approaches unity; the ratio is less than unity for nonzero values of ν .

Clearly, the model used is very simple in many respects. Though it captures the crucial finite-size aspect and predicts a $1/R$ dependence of the width, the value of the slope is sensitive to a number of physical effects which have not been included. As mentioned above, the experimentally determined value of the slope is sensitive to the outside matrix, suggesting that outer surface effects are important, and the approximation of an infinite square well may be too drastic. The correction coming from a large but finite depth V_0 of the well has recently been computed¹⁴ to leading order in μ/V_0 . Another extension that has been considered¹⁵ is to account for finite temperatures in leading order in T/μ . Both effects ($V_0 \neq \infty, T \neq 0$) lead to a further broadening of the line. Finally, we mention the review,⁴ which discusses the interesting possibility that the dielectric constant itself may vary in space, close to

the surface of the metal particle. This has strong effect on the charge density near the surface, and thus on the resonant response.

2.2. Resonances in Free Clusters

Experiments on samples of type (ii) have been performed on alkali clusters with between two and hundreds of conduction electrons.^{16,17,18} The data for smaller clusters¹⁶ indicate that there is a strong response over a relatively narrow frequency interval in the case of magic numbers, and over a much broader frequency range in cases which fall between magic numbers. In the latter case, the spectrum shows splittings, which can be interpreted in terms of shape deformations. For instance, a triaxial ellipsoid exhibits three different resonance conditions and frequencies, corresponding to different internal fields along the three axes.⁵ Even if we confine our attention to magic clusters, however, there is no evidence for a $\Gamma \sim 1/R$ law of the type seen in embedded samples. Nor is the apparent conflict resolved by recent in-beam experiments on larger free clusters of K and Li .

Thermal excitation of surface fluctuations,¹⁹ and consequent broadening of the resonance line, is a physical effect which may at least partially account for the observed differences of behaviour. This effect is expected to be absent in the embedded samples, as the surface of the metal particle is constrained not to move by the surrounding matrix. A controlled change of temperature (from $\sim 4^\circ K$ to $\sim 300^\circ K$) produces virtually no change in the pattern of the resonant response.²⁰ By contrast, the response of type (ii) samples (in-beam clusters) shows a marked dependence on the temperature T . A possible reason is that since the clusters are in free flight, their shape is free to fluctuate. Thermal excitation of such surface-shape fluctuations is a possibility, given the relatively high temperatures of the clusters, which reflect their process of formation. An experimental complication is that clusters of widely different sizes are usually prepared at different temperatures, making it difficult to disentangle effects coming from variations of R and T .

A simple argument yields the size dependence of Γ that would result from thermally excited shape fluctuations, if the temperature were held constant. Consider deformations of shape around a sphere, and suppose the shape changes occur on a time scale much longer than the probe time. Let δ be a dimensionless measure of the deformation. Assuming the volume is constant, the increase of surface area is proportional to δ^2 , and the associated increase of surface energy is $E = a\sigma R^2\delta^2$ where σ is the surface tension and a is a constant of order unity. At high enough T , equipartition is valid, implying $\langle E \rangle = k_B T/2$, so that

$$\delta_{RMS} \equiv \langle \delta^2 \rangle^{1/2} \sim \sqrt{\frac{k_B T}{\sigma}} \frac{1}{R}. \quad (8)$$

Since a value δ of the deformation causes a splitting of the Mie resonance line into lines separated by $\omega_0\delta$, the width $\Gamma_{surf\ fluc}$ resulting from such surface fluctuation

contributions is proportional to δ_{RMS} :

$$\Gamma_{surffluc} = c\omega_0 \sqrt{\frac{k_B T}{\sigma}} \frac{1}{R}, \quad (9)$$

where c is a constant. It is interesting that an inverse- R dependence shows up again, although the origin is quite different from the quantum size effect discussed in Section 2.1. Part of the reason that this $1/R$ dependence has not been seen in experiments may be that the temperatures used to prepare clusters of different sizes vary appreciably.

3. Giant Dipole Resonances in Nuclei

3.1. Cross Section Systematics

Photo-neutron cross sections have been measured in a large number of nuclei; for a compilation of the data, see Ref. (21). For spherical nuclei, the peak frequency ω_0 is known⁷ to exhibit a systematic empirical dependence on the mass number A :

$$\omega_0 = 31.2A^{-1/3} + 20.6A^{-1/6} \text{ MeV}. \quad (10)$$

We shall discuss this dependence (*vis-à-vis* the weak variation of ω_0 with A in metal particles) in Section 4.1. The FWHM provides a simple, single characterization of the resonance spectrum, and has been used earlier to extract global trends with varying A , for heavy nuclei. For instance, Bergère²² and Snover²³ have shown plots of the FWHM in the regions $A > 90$ and $166 > A > 63$, respectively, including nuclei whose resonance spectra exhibit split peaks. These plots show that the FWHM exhibits systematic oscillations in the ranges studied, with local minima near spherical, near-magic nuclei.

We wanted to see whether the systematics observed earlier for the FWHM of heavy nuclei^{22,23} persisted in lighter nuclei as well. Accordingly, we examined the FWHM in about 120 nuclei ranging from ${}^3\text{He}$ to ${}^{239}\text{Pu}$, using primarily the cross-section data compiled in Ref. (21). (We re-examined the heavier nuclei in order to have a uniform procedure for all A .) For those cases where the data follows a curve with a single peak, it was straightforward to determine the FWHM. For cases with two or more (closely overlapping) peaks, we found the FWHM by drawing a smooth curve with a single maximum through the data points, trying to ensure that the areas under the smooth curve and the experimental data were nearly equal. Those nuclei where the data seem incomplete (${}^3\text{H}$, ${}^{19}\text{F}$) or have too much structure (${}^{14}\text{C}$, ${}^{18}\text{O}$, ${}^{24,26}\text{Mg}$) were ignored. Results are displayed in Fig. 1. For light nuclei (see inset in Fig. 1), we also estimated the errors in the FWHMs, arising from (a) the existence of more than one data set in some cases, and (b) the inherent uncertainty in extracting the FWHM by our procedure. In the range $A > 90$, our values agree well with those of Bergère.²²

Examination of the results in Fig. 1 for light nuclei ($A < 50$) (see the inset) shows that the FWHM continues to display local minima at, by and large, the magic numbers. The rapid oscillations of the FWHM versus A are due to the relative crowding in of magic numbers for small A . With the sole exception of ^{28}Si , all the minima occur at or near the magic numbers. (Although ^{28}Si is not a magic nucleus, it displays behaviour similar to a magic nucleus in at least one other context: the plot of nuclear electric quadrupole moment vs. Z or N passes through a zero near ^{28}Si , indicating a prolate to oblate transition.²⁴) Conversely, each magic number has a corresponding minimum, with the possible exception of $N = 40$ ($A = 72$), where there is a hint of a local minimum, but the data does not allow us to draw a firm conclusion. In any case, 40 is known to be a weak magic number.

Fig. 1. Full width at half maximum Γ of the total photoneutron cross section data, Ref. (21), versus the nucleon number A . Note the systematic modulations, with minima at the proton (Z) or neutron (N) magic numbers. The curve is drawn as a guide to the eye. Dashed lines indicate regions of sparse or nonexistent data. The inset shows the region $A \leq 45$ in greater detail.

The systematic oscillations in the region $A < 50$ in Fig. 1 are statistically significant. As is evident from the inset, the error in the FWHM is less than 1 MeV in almost all cases, and is generally much smaller. The amplitude of oscillations, on the other hand, is at least 5-6 MeV (*e.g.*, ${}^3\text{He}$ to ${}^4\text{He}$, or ${}^{40}\text{Ca}$ to ${}^{45}\text{Sc}$), and is sometimes as large as 14 MeV (*e.g.*, ${}^9\text{Be}$ to ${}^{14}\text{N}$). The oscillations are as systematic and as pronounced as those for large A , the only difference being that there are fewer points per oscillation.

That the photo-response of a nucleus even as light as He can be thought of in the same terms as that of heavier nuclei may seem surprising, but the very fact that the FWHMs for light nuclei fit in well with the systematics across the periodic table provides an *a posteriori* justification for the use of the FWHM even for $A < 50$.

An interesting feature of Fig. 1 is the overall downward trend of the oscillatory curve, evident if, for instance, we focus on points in the lower envelope of the curve. These points correspond mostly to spherical, magic nuclei. The observed downward trend is further discussed in Section 4.2.

In the rest of this section we present schematic theoretical considerations aimed at understanding the empirical systematics, in contrast to more customary detailed theoretical studies of the width for individual nuclei.

3.2. Resonance Widths — Overall Trend

One may distinguish between two types of contributions to the FWHM. Firstly, there is the intrinsic width of the resonance which comes from the finite lifetime of the collective mode, and which is present in all cases. The intrinsic width itself receives contributions from a variety of physical mechanisms to be discussed below. This is the only contribution to the FWHM in spherical nuclei. Secondly, in nonspherical nuclei, static deformations in shape can lead to two distinct resonance frequencies, corresponding to a splitting of the line. In such cases, the FWHM receives additional contributions.

The intrinsic width Γ_i can be written as the sum of three terms (see, *e.g.*, Ref. (7)).

$$\Gamma_i = \Delta\Gamma + \Gamma^\uparrow + \Gamma^\downarrow, \quad (11)$$

reflecting contributions from distinct physical effects. The fragmentation width $\Delta\Gamma$ corresponds to the fact that the collective ($1p - 1h$) state which is the doorway state for the GDR, is not a single state, but is in most cases already appreciably fragmented. This effect (mean-field damping or one-body friction) is the finite nucleus analogue of Landau damping in a bulk medium. It occurs due to the scattering of the nucleons from the ‘wall’ or ‘surface’ of the self-consistent mean field potential. This is the physical effect which was accounted for by the quantum calculation of the resonance width of a metal particle, discussed in Section 2.1 above. The second term Γ^\uparrow is the escape or decay width corresponding to the direct coupling of the ($1p - 1h$) doorway state to the continuum, giving rise to its decay into a free nucleon and an ($A - 1$)

nucleus. Finally, the spreading width Γ^\downarrow is due to the coupling of the $(1p - 1h)$ doorway state to more complicated $(2p - 2h)$ states of the nucleus, the transition occurring on account of genuine two-body effects (collisional damping or two-body friction).

Let us see how each contribution to Eq. (11) is expected to vary with radius R . Our arguments are schematic, and aimed at establishing the general, systematic trend with size.

The R -dependence of the first two terms may be estimated using a simple argument based on estimating the frequency of collisions with the surface. Such an argument has been used successfully²⁰ to estimate $\Delta\Gamma$ in metal particles; the estimate agrees with the result of the quantum calculation presented in Section 2.1. The idea is that individual fermions moving with Fermi velocity v_F hit the wall with mean time $\sim R/v_F$; the inverse time $\sim v_F/R$ then determines the contribution to the width arising from wall effects — both for $\Delta\Gamma$ and Γ^\uparrow . The subject of one-body dissipation has also been discussed in the nuclear physics literature.²⁵ The spreading width Γ^\downarrow , on the other hand, arises from two-particle collisions. We expect Γ^\downarrow to vary smoothly with energy and nuclear size for magic cases, since the collisional mean free path Λ shows similar smooth variations (see, *e.g.*, Ref. (26)). The average time between two collisions is $\sim \Lambda/v_F$ and hence Γ^\downarrow is expected to be $\sim v_F/\Lambda$. In the limit $R \rightarrow \infty$, this is the only contribution.

The total intrinsic width is thus expected to be of the same form as Eq. (1) with Γ_∞ being the spreading width in the $R \rightarrow \infty$ limit.

3.3. Resonance Widths — Shell Effects

In Section 3.2, we discussed only the monotonic variation of the FWHM that obtains for spherical nuclei. As we see from Fig. 1, when we consider *all* nuclei, superimposed on this monotonic variation, there are striking and strong shell-structure-linked oscillations in the FWHM. We discuss the origin of these oscillations in two broad representative regions, namely $150 < A < 190$ and $80 < A < 150$.

In the range $150 < A < 190$, Dietrich and Berman²¹ have fitted two-component Lorentz curves to the photoneutron cross section data. This indicates a splitting of the line, due to deformation of the nucleus. We denote the two resonance energies by ω_{01} and ω_{02} , with $\omega_{01} < \omega_{02}$, and the corresponding widths by Γ_1 and Γ_2 . For a spheroidal deformation, Eq. (10) leads us to expect that ω_{01} and ω_{02} correspond to oscillations along the semimajor and semiminor axes respectively. Equation (1) then implies that $\Gamma_1 < \Gamma_2$. This is indeed found to be true, for $150 < A < 190$, for the values of the widths tabulated in Ref. (21). In fact, with the exceptions of ^{55}Mn and ^{63}Cu , this is true for all the nuclei listed there. This provides additional evidence for the overall decrease of the intrinsic width with increasing radius. The increase of the FWHM away from the spherical cases can be ascribed, at least partially, to the fact that deformations produce a splitting of the line, and also cause $\Gamma_2 > \Gamma(> \Gamma_1)$,

where Γ would be the width if the nucleus were undeformed. Since deformations of the shape follow shell-structure systematics, with smallest deformations close to the magic numbers, so does the FWHM.²⁷

In the region $80 < A < 150$, Dietrich and Berman²¹ have fitted one-component Lorentz curves to the photoneutron cross section data (with the exceptions of ¹²⁷*I* and ¹⁴⁸*Nd*). As is clear from Fig. 1, oscillations of the FWHM versus A in this region are as prominent as those for $150 < A < 190$. Thus, even when the line is unsplit, the width of the best-fit Lorentzian oscillates as a function of A , with minima at the magic numbers. This indicates that the intrinsic width Γ_i can itself show shell-structure-linked oscillations. Bergère²² has correlated the FWHM in this region with the ratio $E(4^+)/E(2^+)$, which characterizes the ‘softness’ of nuclei. Here $E(J^+)$ is the energy of the first J^+ state in the nuclear spectrum.

4. Similarities and Differences between Nuclei and Metal Particles

4.1. Resonance Frequency

The size dependence of the natural frequency of vibration ω_0 can be deduced by using a simple classical picture of the collective mode.

In the metal particle, the restoring force arises from the electric field produced by layers of opposite charges on diametrically opposite sides, and acts on each of the A conduction electrons in the particle. The oscillator frequency ω_0 is given by the square root of the ratio of the total restoring force per unit displacement to the mass involved. Since both force and mass are proportional to A , the frequency ω_0 is roughly size independent.

In the nucleus, on the other hand, the restoring force arises from short-range strong interactions amongst nucleons. In a hydrodynamic description, it is modelled by the surface or volume symmetry energy terms in the semiempirical mass formulas. In the Goldhaber-Teller model,²⁸ the collective state corresponds to the motion of the proton cloud through the neutron cloud without mutual distortion. The restoring force is proportional to $A^{2/3}$ and the mass parameter is proportional to A . Hence, $\omega_0 \sim A^{-1/6}$. In the Steinwedel-Jensen model,²⁹ on the other hand, the relative proton-neutron density changes in such a way as to maintain constant overall density throughout. The restoring force per unit mass is proportional to R^{-2} , and hence $\omega_0 \sim A^{-1/3}$. (See Eq. (10).)

4.2. Resonance Width

We wanted to see if Eq. (1), which holds for embedded metal particles, also describes the downward trend of Γ in nuclei with increasing A , evident in Fig. 1. A similar $1/R$ dependence has been discussed earlier³⁰ for nuclei in the range $A > 50$.

On dividing across by the Fermi energy ϵ_F , we see that Eq. (1) predicts that Γ_{av}/ϵ_F is a linear function of $(k_F R)^{-1}$, where k_F is the Fermi wave vector. Interestingly, on using these dimensionless scaled variables, we can directly compare the Ag -particle and nuclear data (Fig. 2) which in absolute terms differ by six orders of magnitude

Fig. 2. Γ/ϵ_F versus $(k_F \tilde{R})^{-1}$ for embedded metal particles and nuclei. The dashed and dotted lines are best fits for Ag/Ar (+, Refs. (9,31)) and Ag/Ne Ref. (9). The nuclei shown here are singly (\bullet) or doubly (\circ) magic nuclei from the lower envelope of the oscillating curve in Fig. 1. The solid straight line is the best fit to this data set. The dot-dashed line indicates the ‘average’ trend of the oscillatory curve in Fig. 1. Note the similarities between the scaled nuclear and particle data despite the fact that the two data sets differ by six orders of magnitude in energy and five orders of magnitude in length. For $A \geq 90$, not all error bars are shown; nuclei in the same cluster have roughly similar error bars.

in energy and five orders of magnitude in length. We used the values $\epsilon_F = 38$ MeV and $k_F = 1.36 \text{ fm}^{-1}$ for nuclei, and $\epsilon_F = 5.49 \text{ eV}$ and $k_F = 1.20A^{-1}$ for *Ag* particles. We have chosen to plot data for *Ag* particles in argon and neon matrices as interactions with surrounding inert gas atoms are likely to be minimal, and a large range of sizes has been studied for *Ag/Ar*.^{9,31} We have used RMS radii \tilde{R} , as these are well determined for nuclei; for *Ag* particles, we took \tilde{R} to be given by $\sqrt{3/5}$ times the quoted radii. Since Γ oscillates as a function of size in nuclei, and we are interested in displaying the overall downward trend, we have replotted points corresponding to singly or doubly magic nuclei from the lower envelope of the curve in Fig. 1; the line marked ‘magic’ is the best fit line through these points. Thus these points are consistent with a linear dependence on $(k_F\tilde{R})^{-1}$, though other monotonic variations with \tilde{R} cannot be ruled out. We also examined the average downward trend of the oscillatory curve in Fig. 1, and found that it could also be fit to a linear dependence. The slope of the average line (marked ‘average’ in Fig. 2) is larger than that of the solid line, and is comparable to the slopes of the dashed and dotted lines. Thus (1) holds to a good approximation for nuclei also. In particular, it is interesting to see how well the doubly magic nuclei ${}^4\text{He}$, ${}^{16}\text{O}$, ${}^{40}\text{Ca}$, ${}^{90}\text{Zr}$, and ${}^{208}\text{Pb}$ follow a straight line.

Of course, there are also some differences between nuclei and metal particles. Consider the limit $R \rightarrow \infty$, corresponding to nuclear matter or the bulk metal. From Fig. 2, we see that if the straight lines are extrapolated towards $(k_F\tilde{R})^{-1} = 0$, the resulting intercept on the Γ/ϵ_F axis is much larger for nuclei than for metal particles. This is presumably because $k_F\Lambda$ is much smaller in nuclear matter²⁶ than in bulk metals at room temperature³² reflecting the greater effect of collisions in the former case. As a result, the limiting contribution Γ_∞ constitutes a substantial fraction of the total width for spherical nuclei (for ${}^{208}\text{Pb}$ it is about 50%), while for metal particles of comparable A , the contribution Γ_∞ is a much smaller fraction of the width.

5. Conclusion

We conclude by recapitulating the main points of this paper.

The $1/R$ law (Eq. (1)) describing the observed broadening of the width of the Mie resonance in embedded metal particles is captured by a simple model of electrons in a spherical well. The quantum mechanical response can be computed analytically when the well is infinitely deep, but the model is too simple to correctly predict experimental values of the slope, which are sensitive to the surrounding medium. Smaller metal clusters in free flight exhibit a more complicated behaviour, perhaps because of the thermal excitation of surface fluctuations. At high temperatures, the resulting broadening is proportional to \sqrt{T}/R .

In nuclei, the FWHM of the total photoneutron cross section shows shell-structure-linked oscillations as a function of A even for $3 < A < 50$. Disregarding oscillations, for instance by focusing on magic nuclei, the FWHM generally decreases with

increasing A approximately as $A^{-1/3} \sim 1/R$. While a complete theory has not been presented here, we have given a schematic theoretical description which allows one to understand at least the principal trends.

Striking similarities are seen when the FWHMs for nuclei are compared with photoabsorption FWHMs in embedded metal particles, after proper rescaling of the energies and lengths (Fig. (2)).

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After this paper was written, we received a copy of the monograph by Bertsch and Broglia³³ which deals with dipolar oscillations in metal clusters and nuclei, and bears on many of the points discussed in this paper.

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