Markov-Yukawa Transversality On Covariant Null-Plane: Baryon Form Factor And Magnetic Moments

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Abstract

The baryon-qqq vertex function governed by the Markov-Yukawa Transversality Principle ($MYTP$), is formulated via the Covariant Null-Plane Ansatz ($CNPA$) as a 3-body generalization of the corresponding $q\bar{q}$ problem, and employed to calculate the proton e.m. form factor and baryon octet magnetic moments.The e.m. coupling scheme is specified by letting the e.m. field interact by turn with the 'spectator' while the two interacting quarks fold back into the baryon. The S_3 symmetry of the matrix element is preserved in all d.o.f.'s together. The $CNPA$ formulation ensures, as in the $q\bar{q}$ case, that the loop integral is free from the Lorentz mismatch disease of covariant instantaneity (CIA), while the simple trick of 'Lorentz completion'ensures a Lorentz invariant structure. The k^{-4} scaling behaviour at large k^2 is reproduced. And with the infrared structure of the gluonic propagator attuned to spectroscopy, the charge radius of the proton comes out at $0.96fm$. The magnetic moments of the baryon octet, also in good accord with data, are expressible as $(a+b\lambda)/(2+\lambda)$, where a, b are purely geometrical numbers and λ a dynamics-dependent quantity.

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1 Introduction

There is a close connection between the physics of $q\bar{q}$ mesons and that of qqq baryons since in many ways a diquark behaves like a color 3^* antiquark. Therefore the dynamics of both systems should be alike, so that it is fair to expect that a theory which is pre-tested for one should also work for the other, except for external characteristics like the number of degrees of freedom. Some important tests for any new form of dynamics are the scaling behaviour [1] of hadronic form factors, as well as the structure of baryon magnetic moments, the latter believed to be governed by chiral $SU(3) \times SU(3)$ symmetry [2] on the one hand, and vector meson dominance (VMD) [3] on the other. The form of dynamics we wish to consider here is the Markov-Yukawa Transversality Principle ($MYTP$) [4] via the covariant null plane ansatz ($CNPA$) [5], which is applicable to all Salpeter-like equations [6]. It was recently applied to the e.m. form factor of the pion [5], showing the expected scaling behaviour ($\sim k^{-2}$). With this check on the validity of $MYTP$ under $CNPA$ conditions [5], we now wish to extend the same from the $q\bar{q}$ [5,6] to the qqq dynamics.

In Section 2 we derive the baryon-qqq vertex function under $MYTP$ on the covariant null-plane ($CNPA$), on closely parallel lines to its Covariant Instantaneity (CIA) formulation [7,8], in the same notation [8] except when new features arise. Section 3 gives the matrix element for the baryon e.m. coupling wherein the spectator interacts with the external photon, and the two interacting quarks fold back into the baryon. Section 4 gives the results for the e.m. form factor of the proton which confirm its k^{-4} behaviour at large k^2, and gives a value of the charge radius at $0.96 fm$, with the infrared part of the gluon propagator [9] attuned to baryon spectroscopy [10]. Section 5 gives the corresponding results on the magnetic moments of the baryon octet within the same framework. Section 6 concludes with a summary.

2 Derivation Of Baryon-qqq Vertex Function

To formulate the baryon-qqq vertex function under $MYTP$, we proceed as in ref.[8] for the 2- and 3-body kinematics, except for a generalization from covariant instantaneity (CIA) [7,8] to the covariant null-plane ansatz ($CNPA$) [5,6]. In this Section we consider spinless quarks [8] using the method of Green's functions, to be followed by the more realistic case of fermion quarks at the end of the Section. Now since the momentum kinematics for $q\bar{q}$ under $CNPA$ [5,6] turn out to be formally identical to those under the standard null-plane formalism [11,12] (which is easier to use), the latter notation ($\pm$) [11-12] may be profitably employed instead of the n_μ dependent notation [5,6]. Now the essential point of the $CNPA$ formalism [5,6] is that the longitudinal (z) and scalar (0) components of a 4-momentum $p_{i\mu}$ for quark $\#i$ in a hadron of mass M and 4-momentum P_μ are [5,6]:

$$p_{iz}; p_{i0} = \frac{Mp_{i+}}{P_+}; \frac{Mp_{i-}}{2P_-} \tag{2.1}$$

The former, along with the transverse components $p_{i\perp}$, obeys the 'angular condition' [13,12] which effectively defines a 3-vector $\hat{p}_i \equiv \{p_{i\perp}, p_{iz}\}$, a key ingredient for the $MYTP$ formulation to follow.

2.1 3D-4D Interlinkage for qq Green's Fns

We first derive the 3D-4D interlinkage for the qq system by the method of Green's functions, as a prototype for the qqq system to follow. The Green's function under $MYTP$ satisfies the BSE [8]:

$$(2\pi)^4 iG(q,q';P) = \frac{1}{\Delta_1\Delta_2}\int d^4q''V(\hat{q},\hat{q}'')G(q'',q';P) \tag{2.2}$$

Now define the 3D Green's function [8]

$$\hat{G}(\hat{q},\hat{q}') = \int dq_0 dq_0' G(q,q';P) \tag{2.3}$$

where the time-like components are defined a la (2.1). Integrating both sides of (2.2) gives via (2.3) the 3D BSE for a *bound* state which does not need an inhomogeneous term:

$$(2\pi)^3 D(\hat{q})\hat{G}(\hat{q},\hat{q}') = \int d^3\hat{q}''V(\hat{q},\hat{q}'')\hat{G}(\hat{q}'',\hat{q}') \tag{2.4}$$

where the 3D denominator function $D(\hat{q})$ is defined as

$$\frac{2i\pi}{D(\hat{q})} = \int \frac{dq_0}{\Delta_1\Delta_2} \tag{2.5}$$

leading (for general unequal mass kinematics) to [12]

$$D(\hat{q}) = \frac{M}{P_+}D_+(\hat{q}); \quad D_+(\hat{q}) = 2P_+[\hat{q}^2 - \frac{\lambda(M^2,m_1^2,m_2^2)}{4M^2}] \tag{2.6}$$

where (2.1) defines the 3-vector $\hat{q}$ and λ is the triangle function of its arguments. Now define the hybrid Green's functions [8]:

$$\tilde{G}(\hat{q},q') = \int dq_0 G(q,q';P); \quad \tilde{G}(q,\hat{q}') = \int dq_0' G(q,q';P) \tag{2.7}$$

Using (2.7) on the RHS of (2.2) gives

$$(2\pi)^4 iG(q,q';P) = \frac{1}{\Delta_1\Delta_2}\int d^3\hat{q}''V(\hat{q},\hat{q}'')\tilde{G}(\hat{q}'',q') \tag{2.8}$$

Integrating (2.2) w.r.t. dq_0' only, and using (2.7) again, gives

$$(2\pi)^4 i\tilde{G}(q,\hat{q}') = \frac{1}{\Delta_1\Delta_2}\int d^3\hat{q}''V(\hat{q},\hat{q}'')\hat{G}(\hat{q}'',\hat{q}') \tag{2.9}$$

Eqs.(2.9) together with the 3D equation (2.4) for $\hat{G}$ gives a connection between the hybrid $\tilde{G}$ and the 3D $\hat{G}$:

$$\tilde{G}(q,\hat{q}') = \frac{D(\hat{q})}{2i\pi\Delta_1\Delta_2}\hat{G}(\hat{q},\hat{q}') \tag{2.10}$$

Interchanging q and q' in the last equation gives the dual result

$$\tilde{G}(\hat{q},q') = \frac{D(\hat{q}')}{2i\pi\Delta_1'\Delta_2'}\hat{G}(\hat{q},\hat{q}') \tag{2.11}$$

Substituting these results in (2.8) gives the desired 3D-4D interconnection

$$G(q,q';P) = \frac{D(\hat{q})}{2i\pi\Delta_1\Delta_2}\hat{G}(\hat{q},\hat{q}';P)\frac{D(\hat{q}')}{2i\pi\Delta_1'\Delta_2'} \tag{2.12}$$

Now making spectral representations for the 4D and 3D Green's functions on both sides of eq.(2.12) in the standard manner [8], viz.,

$$G(q,q';P) = \sum_n \Phi_n(q;P)\Phi^*(q';P)/(P^2+M^2); \tag{2.13}$$

$$\hat{G}(\hat{q},\hat{q}') = \sum_n \phi_n(\hat{q})\phi_n^*(\hat{q}')/(P^2+M^2) \tag{2.14}$$

where Φ_n and ϕ_n are 4D and 3D wave functions respectively, one can directly read off from (2.12) their interconnection, valid near a bound state pole (dropping the suffix n for simplicity):

$$\Gamma(\hat{q}) \equiv \Delta_1\Delta_2\Phi(q;P) = \frac{D(\hat{q})\phi(\hat{q})}{2i\pi} \tag{2.15}$$

which tells us that the vertex function Γ under $CNPA$ is again a function of $\hat{q}$ only [8], except for its definition (2.1) under $CNPA$. This derivation is a prototype for the qqq case to follow.

2.2 3D Reduction for Scalar qqq BSE

For the qqq problem, we have a pair of internal variables which may be chosen in one of 3 distinct ways. With index #3 as basis, the pair ξ_3, η_3 may be defined as [8]

$$\sqrt{3}\xi_3 = p_1 - p_2; \quad 3\eta_3 = -2p_3 + p_1 + p_2; \quad P = p_1 + p_2 + p_3 \tag{2.16}$$

The space-like and time-like parts of ξ, η are defined as in (2.1), so that, e.g.,

$$\xi_{z3} = \frac{M\xi_+}{P_+}; \quad \xi_{03} = \frac{M\xi_-}{2P_-}$$

and similarly for η. We shall also use the $\pm$ notation in parallel with the 3-vector notation in the following.

The Green's function, after taking out an overall δ- function for the c.m. motion, may be written as $G(\xi\eta;\xi'\eta')$ at the 4D level, while the fully 3D Green's function may be defined as [8]

$$\hat{G}(\hat{\xi}\hat{\eta};\hat{\xi}'\hat{\eta}') = \int d\xi_0 d\eta_0 d\xi_0' d\eta_0' G(\xi\eta;\xi'\eta') \tag{2.17}$$

Both G and $\hat{G}$ are S_3 symmetric, since the measure $d\xi_0 d\eta_0$ is S_3-invariant. In addition, two hybrid Green's functions are [8]:

$$\tilde{G}_{3\xi}(\hat{\xi}_3\eta_3;\hat{\xi}_3'\eta_3') = \int d\xi_{30}d\xi_{30}' G(\xi\eta;\xi'\eta'); \quad \tilde{G}_{3\eta}(\xi_3\hat{\eta}_3;\xi_3'\hat{\eta}_3') = \int d\eta_{30}d\eta_{30}' G(\xi\eta;\xi'\eta'); \tag{2.18}$$

where the suffixes 3ξ,3η signify that $\tilde{G}$ is *not* S_3 symmetric since the integration now involves only *one* of the two ξ,η variables. Now the 4D qqq BSE under $MYTP$ is [8]:

$$i(2\pi)^4 G(\xi\eta;\xi'\eta') = \sum_{123}\int \frac{9d^4\xi''}{16\Delta_1\Delta_2} V(\hat{\xi}_3,\hat{\xi}_3''G(\xi_3''\eta_3;\xi_3'\eta_3') \tag{2.19}$$

where the factor 9/16=$[\sqrt{3}/2]^4$ stems from the relation $2q_{12} = \sqrt{3}\xi_3$, and the association of η_3 with ξ_3'' in the Green's function signifies that the spectator #3 remains unaffected by the interaction in the (12) pair, and so on by turns cyclically. The 3D reduction is achieved by integrating (2.19) via (2.17) which gives, as in the *CIA* derivation [8]:

$$(2\pi)^3\hat{G}(\hat{\xi}\hat{\eta};\hat{\xi}'\hat{\eta}') = \sum_{123}\frac{3\sqrt{3}}{8D_{12}}\int d^3\hat{\xi}_3'' V(\hat{\xi}_3,\hat{\xi}_3'')\hat{G}(\hat{\xi}_3''\hat{\eta}_3'';\hat{\xi}_3'\hat{\eta}_3') \tag{2.20}$$

where, as in (2.6), $D_{12} = \frac{MD_{12+}}{P_+}$, with

$$D_{12+} = 2\omega_{1\perp}^2 p_{2+} + 2\omega_{2\perp}^2 p_{1+} - 2P_{12-}p_{1+}p_{2+} \tag{2.21}$$

and $\omega_{i\perp}^2 = m_i^2 + p_{i\perp}^2$; $P_{12} = p_1 + p_2$. Now making use of the on-shellness ($\Delta_3 = 0$) of the spectator (#3), we have $P_{12-} = P_- p_{3-} = P_-\omega_{3\perp}^2/p_{3+}$, whose substitution in (2.21) gives rise to the S_3 symmetric result:

$$p_{3+}D_{12+} \equiv D_{++} = 2\sum_{123} p_{2+}p_{3+}\omega_{1\perp}^2 - 2p_{1+}p_{2+}p_{3+}P_- \tag{2.22}$$

Substitution of this result in (2.20) gives rise to the requisite 3D BSE in which the denominator function may be identified with D_{++} in an *exact* fashion. This is a much neater result than under *CIA* [7] where a corresponding denominator function could be obtained only through an approximate treatment [10]. Since this quantity will play a crucial role in this study, we recast it in terms of the ξ,η variables by first redifining it as $D_{++} \equiv P_+^2 D$, and $D \equiv D_0 + \delta D$, where

$$D_0 = \frac{1}{3}(\xi_\perp^2 + \eta_\perp^2) + \frac{1}{2}(1 - \sum_{123} m_i^2/M^2)(\xi_l^2 + \eta_l^2) + \frac{2}{3}(\sum_{123} m_i^2/3 - M^2/9) \tag{2.23}$$

with $(\xi_l, \eta_l) \equiv M(\xi_+, \eta_+)/P_+$ and

$$\delta D = \frac{\eta_l}{2M}[\eta_l^2 - 3\xi_l^2] - \frac{3}{2M^2}[\eta_l^2\xi_\perp^2 + \xi_l^2\eta_\perp^2] \tag{2.24}$$

Note that this null plane description gives different scales for the longitudinal and transverse components of the ξ,η variables.

The D-function is the driving term for the 3D eq.(2.20) which in turn is the right vehicle for spectroscopy [10], and is the source of the 3D wave function ϕ via the spectral representation (2.14). In this respect, the main role is played by the D_0 function, while δD is a correction term.

2.3 Reconstruction of 4D Vertex Function

We now indicate the steps for reconstruction of G in terms of 3D ingredients. First, the hybrid function $\tilde{G}_{3\eta}$, eq.(2.18), is expressed in terms of fully 3D quantity $\hat{G}$ exactly as in eq.(2.11) for the two-body problem:

$$\tilde{G}_{3\eta}(\xi_3\hat{\eta}_3;\xi_3'\hat{\eta}_3') = \frac{P_+D}{2i\pi p_{3+}\Delta_1\Delta_2}\hat{G}(\hat{\xi}\hat{\eta};\hat{\xi}'\hat{\eta}')\frac{P_+D'}{2i\pi p_{3+}'\Delta_1'\Delta_2'} \tag{2.25}$$

In a similar way the fully 4D G function is expressible in terms of the hybrid function $\tilde{G}_{3\xi}$ as

$$G(\xi\eta;\xi'\eta') = \sum_{123} \frac{P_+ D}{2i\pi p_{3+}\Delta_1\Delta_2} \tilde{G}_{3\xi}(\hat{\xi}_3\eta_3;\hat{\xi}'_3\eta'_3) \frac{P_+ D'}{2i\pi p'_{3+}\Delta'_1\Delta'_2} \tag{2.26}$$

At this stage we need an ansatz [8] on the $\tilde{G}_{3\xi}$ function which, as in the CIA case [8], is not determined from $MYTP$ qqq dynamics:

$$\tilde{G}_{3\xi}(\hat{\xi}_3\eta_3;\hat{\xi}'_3\eta'_3) = \hat{G}(\hat{\xi}\hat{\eta};\hat{\xi}'\hat{\eta}')F(p_3,p'_3) \tag{2.27}$$

where we have incorporated the S_3 symmetry of $\hat{G}$ and taken the balance of the p_3 (spectator) dependence in the (unknown) F function. This is subject to an explicit self-consistency check for the ansatz (2.27) which may be found by integrating both sides w.r.t. $dp_{3-}dp'_{3-}$,to give

$$\int\int \frac{M^2 dp_{3-}dp'_{3-}}{4P_-^2} F(p_3,p'_3) = 1$$

This condition is satisfied by the ansatz [8]:

$$F(p_3,p'_3) = \frac{A_3}{\Delta_3}\delta[\frac{Mp_{3-}}{2P_-} - \frac{Mp'_{3-}}{2P_-}] \tag{2.28}$$

if A_3 is determined by the equation

$$A_3 \int \frac{Mdp_{3-}}{2P_-\Delta_3} = 1$$

which gives

$$A_3 = \frac{2Mp_{3+}}{i\pi P_+}; \quad p_{3-} = \frac{\omega_{3\perp}^2}{p_{3+}}$$

After a little simplification, we have finally:

$$F(p_3,p'_3) = 4p_{3l}^2 \frac{\delta(\Delta_3)}{i\pi\Delta_3} \tag{2.29}$$

which finally defines the 4D G function in terms of $\hat{G}$ via the sequence (2.27) and (2.26). Finally the spectral representations (2.13-14) near a bound state pole give the connection between the #3 part Φ_3 of the 4D wave function and the 3D wave function ϕ:

$$\Phi_3 = \frac{2MD}{2i\pi\Delta_1\Delta_2}\phi(\xi\eta)\sqrt{\frac{\delta(\Delta_3)}{i\pi\Delta_3}} \tag{2.30}$$

whence the baryon-qqq vertex function V_3 is inferred via

$$\Phi_3 \equiv \frac{V_3}{\Delta_1\Delta_2\Delta_3}:$$

$$V_3 = \frac{MD\phi(\xi\eta)}{i\pi}\sqrt{\frac{\Delta_3\delta(\Delta_3)}{i\pi}} \tag{2.31}$$

For explanation on the appearance of the δ-function under radicals in eq.(2.31), see ref.[8] where it has been shown that this has nothing to do with any lack of connectedness in a 3-body amplitude.

2.4 Baryon-qqq Vertex with Fermion Quarks

For the more realistic case of fermion quarks, we employ the method of Gordon reduction [14-15] whose logic and advantages have been described elsewhere in the context of the $q\bar{q}$ problem [5,6]. To extend the same to the 3-body case, define a (fictitious) scalar function Φ which is related to the actual BS wave function Ψ by [15]

$$\Psi = \Pi_{123} S_{Fi}^{-1}(-p_i)\Phi(p_i p_2 p_3) \tag{2.32}$$

with an explicit indexing w.r.t. the individual quarks, which however can be subsumed in a common Dirac matrix space a la Blankenbecler et al [16], as illustrated in the next Section. The connection with sect.2.3 is now established by identifying Φ of (2.32) with the sum $\Phi_1 + \Phi_2 + \Phi_3$ where Φ_3 is given by (2.30). Therefore the form (2.31) for V_3 continues to be valid, except that its relation to Ψ is

$$\Psi = \Pi_{123} S_{Fi}(p_i)[V_1 + V_2 + V_3] \tag{2.33}$$

We end this Section with a listing of the (gaussian) structure of the 3D wave function ϕ as a solution of the fermionic counterpart of eqs.(2.20-24). Since this paper is not concerned with baryon spectroscopy (see [10] for details),we list merely the gaussian form of ϕ:

$$\phi = \exp\,[-\frac{\xi_\perp^2 + \eta_\perp^2}{2\beta_t^2} - \frac{M^2x^2 + M^2y^2}{2\beta_l^2}] \tag{2.34}$$

where the transverse and longitudinal scale parameters follow from the structure of the D function (2.23), by proceeding as in [12,10]:

$$\begin{aligned} \beta_t^4 &= \frac{8M}{81}\omega_{qq}^2/[1/4 - \frac{3C_0\omega_{qq}^2}{9M\omega_0^2}] \\ \beta_l^4 &= \frac{8M}{81}\omega_{qq}^2/[1/2 - \frac{3C_0\omega_{qq}^2}{9M\omega_0^2} - \frac{3m_q^2}{2M^2}] \end{aligned} \tag{2.35}$$

where the input parameters ω_{qq} etc are listed in [10,12]. The numerical values of the β^2 parameters for the full baryon octet in GeV^2 units are

$$\begin{aligned} \beta_t^2(N) &= 0.068; \quad \beta_l^2(N) = 0.054; \\ \beta_t^2(\Sigma) &= 0.080; \quad \beta_l^2(\Sigma) = 0.062; \\ \beta_t^2(\Lambda) &= 0.076; \quad \beta_l^2(\Lambda) = 0.061; \\ \beta_t^2(\Xi) &= 0.079; \quad \beta_l^2(\Xi) = 0.063. \end{aligned} \tag{2.36}$$

3 E.M. Coupling Of qqq Baryon

The e.m. coupling of the qqq baryon is given by fig.1 plus two more obtained by cyclic permutations of the indices. It shows that the spectator (#3) scatters against the (space-like) photon before being re-absorbed into the baryon, while the two interacting quarks $(1,2)$ fold back into the baryon.Fig.1 is in keeping with the standard additivity principle (the hallmark of the quark model) for single quark transitions. On the other hand the (complementary) diquark-photon diagram [17], which does not show a similar property, will be presumed to be *dynamically* suppressed, hence left out of further consideration in this paper.

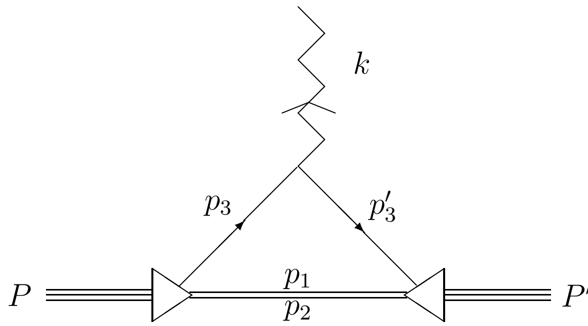

Figure 1: Triangle loop for baryon e.m. vertex

3.1 Structure of E.M. Matrix Element

The imbedding of the indices 123 in a common Dirac matrix space may be done a la ref.[16], after taking account of the S_3 symmetry a la ref.[17]. In a $[2,1]$ representation of S_3 symmetry, the two spin functions χ', χ'' are expressible as [16,17]

$$
\begin{aligned}
\mid \chi' > &= [\gamma_5 \frac{C}{\sqrt{2}}]_{\alpha\beta} \otimes U_\gamma(P) \\
\mid \chi'' > &= [i\hat{\gamma}_\mu \frac{C}{\sqrt{6}}]_{\alpha\beta} \otimes [\gamma_5 \hat{\gamma}_\mu U(P)]_\gamma
\end{aligned}
\tag{3.1}
$$

where $\hat{\gamma}_\mu$ would naively be expected to be transverse to P_μ, but one must again anticipate a problem analogous to the Lorentz mismatch problem associated with CIA [7] which gave rise to unwarranted complexities [18] in form factors, and necessitated the alternative $CNPA$ [5] formulation for the *orbital* matrix elements. In the context of *spin* matrix elements this pathology shows up as high powers of k^2 in the concerned amplitudes if $\hat{\gamma}_\mu$ is defined as $\gamma_\mu - \gamma.PP_\mu/P^2$ [7], which is equally unacceptable. On the other hand, the $CNPA$ [5] offers an alternative solution wherein the transversality is defined w.r.t. the (more universal) null-plane. The simplest possibility in this regard is to define

$$
\bar{\gamma}_\mu = \theta_{\mu\nu}\gamma_\nu; \quad \bar{\gamma}^*_\mu = \theta^*_{\mu\nu}\gamma_\nu \tag{3.2}
$$

where

$$
\begin{aligned}
\theta_{\mu\nu} &= \delta\mu\nu - n_\mu \tilde{n}_\nu = \theta_{\nu\mu}*; \\
\theta_{\mu\mu} &= 3; \quad \theta_{\mu\lambda}\theta^*_{\nu\lambda} = \theta_{\mu\nu}
\end{aligned}
\tag{3.3}
$$

These properties are consistent with those expected of a projection operator. Next, the $SU(6)$ operator for the baryon e.m. interaction is

$$
\Gamma_\mu = \sum_1^3 \gamma^{(i)}_\mu \frac{e}{2}[\lambda^{(i)}_3 + \frac{1}{\sqrt{3}}\lambda^{(i)}_8] \tag{3.4}
$$

The matrix elements of this operator must be taken between the the $SU(6)$ wave functions [17]

$$
W(P) = \frac{1}{\sqrt{2}}(\chi'\phi' + \chi''\phi'') \tag{3.5}
$$

where the isospin functions ϕ', ϕ'' have matrix elements expressible in a common (baryon) basis as [17]

$$\begin{aligned} < \phi'' \mid 1; \tau^{(i)} \mid \phi'' > &= [1; -\tau/3] \\ < \phi' \mid 1; \tau^{(i)} \mid \phi' > &= [1; \tau] \end{aligned} \tag{3.6}$$

Using (3.6), the matrix element of (3.4) between nucleon states $|N>$ is expressible as (c.f. [17]):

$$\begin{aligned} < N|\Gamma_\mu|N > &= \int d\tau V_3^* V_3 N_B^2 [\frac{e}{2} < 1 + 3\tau_z > [< \chi' \mid \gamma_\mu^{(3)} \mid \chi' > \\ &+ \frac{e}{2} < 1 - \tau_z >< \chi'' \mid \gamma_\mu^{(3)} \mid \chi'' >] \end{aligned} \tag{3.7}$$

N_B is a normalization factor to be defined further below.The two spin matrix elements are expressible in a factorized form as

$$\begin{aligned} < \chi' \mid \gamma_\mu^{(3)} \mid \chi' > &= T' \times A'_\mu \\ < \chi'' \mid \gamma_\mu^{(3)} \mid \chi'' > &= T''_{\nu\nu'} \times A''_{\mu\nu\nu'}; \end{aligned} \tag{3.8}$$

$$\begin{aligned} T' &= Tr[\tilde{S}_F(p_2) \frac{C^{-1}}{\sqrt{2}} \gamma_5 S_F(p_1) \gamma_5 \frac{C}{\sqrt{2}}] \\ T''_{\nu\nu'} &= Tr[\tilde{S}_F(p_2) \frac{C^{-1}}{\sqrt{6}} \bar{\gamma}_{\nu'} S_F(p_1) \bar{\gamma}_\nu^* \frac{C}{\sqrt{6}}]; \end{aligned} \tag{3.9}$$

$$\begin{aligned} A'_\mu &= \bar{U}(P') S_F(p'_3) i\gamma_\mu S_F(p_3) U(P) \\ A''_{\mu\nu\nu'} &= \bar{U}(P') \bar{\gamma}_{\nu'}^* \gamma_5 S_F(p'_3) i\gamma_\mu S_F(p_3) \bar{\gamma}_\nu \gamma_5 U(P) \end{aligned} \tag{3.10}$$

The symbol $\int d\tau$ in (3.8) stands for:

$$\int d\tau \equiv \int d^4 q_{12} d^4 \bar{p}_3 \tag{3.11}$$

where $\bar{p}_3 = (p_3 + p'_3)/2$.

3.2 Baryon Normalization

The evaluation of the spin matrix elements (see Appendix) reduces (3.7) to

$$< N|\Gamma_\mu|N >= \frac{e}{2} \int d\tau' W_3^* W'_3 N_B^2 [M'_\mu (1 + 3\tau_z) + M''_\mu (1 - \tau_z)] \tag{3.12}$$

where M'_μ, M''_μ are given in eqs.(A.6, A.11) respectively,and W_3 is the reduced form of V_3, eq.(2.31) as under:

$$W_3 = \frac{MD\phi(\xi\eta)}{(\pi)^{3/2}} \tag{3.13}$$

Hera (see Appendix for details) the reduced measure $d\tau'$ is given by (A.7). Repeated use of Gordon reduction (A.8) leads finally to the 'Sachs' form

$$< N|\Gamma_\mu|N >= e\bar{U}(P')[F(k^2) \frac{\bar{P}_\mu}{M} + G(k^2) \frac{\sigma_{\mu\nu} k_\nu}{2M}] U(P) \tag{3.14}$$

The baryon normalization now comes entirely from the first term in the limit of $k = 0$. To fix this quantity,instead of demanding unit charge for the *proton*, an asymmetric recipe heavily weighted against the neutron (see Eq.(3.12)), we resort to a more symmetrical treatment between the two by demanding the conservation of *baryon* number, via ω-like coupling [3],which is equivalent to the conservation of *isoscalar* charge.Thus the normalization condition boils down to

$$\frac{P_\mu}{Mi(2\pi)^4} = \frac{N_B^2}{2} \int d\tau' [M'_\mu + M''_\mu] \tag{3.15}$$

which is obtained from (3.12) after dropping the isovector parts. For later purposes we define a parameter λ:

$$\lambda = \frac{< \bar{\eta}^2 - 3\bar{\xi}^2 >}{M^2/3 + m_3^2 - \delta m^2} < 0 \tag{3.16}$$

which arises from certain terms of M''_μ, eq. (A.11), without a counterpart from M'_μ, eq.(A.6). This parameter will play a crucial role in the determination of magnetic moments (see Section 5).

4 Calculation Of The E.M. Form Factor

The charge and magnetic form factors of the baryon are given by the functions $F(k^2)$ and $G(k^2)$ in eq.(3.14) which in this model have *identical* shapes. Further,eq.(3.15) for normalization ensures that $F(0) \equiv 1$, while $G(0)$ gives directly the baryon magnetic moments in 'baryon magnetons'$(e/2M)$. The form factor problem is considered in this Section, followed by magnetic moments in the next Section. To evaluate (3.12) in a *closed* form, first write its integration measure in detail as

$$d^2q_\perp dq_+ \frac{d\bar{q}_-}{2} d^4\bar{p}_3 \delta(\Delta_3). \tag{4.1}$$

In the first step, integrate over $dq_-/2$ to give

$$\int \frac{d\bar{q}_-}{2} \frac{DD'}{\Delta_1 \Delta_2} = (2i\pi) \frac{p_{3+}}{4\bar{P}_+^2} (D + D') \tag{4.2}$$

The next step is to integrate over the factors $dp_{3-}\delta(\Delta_3)/2$ in (4.1) to give $\frac{1}{2p_{3+}}$. Combining (4.1-2), the net measure becomes:

$$d\tau_1 = (2i\pi) d^2\xi_\perp d^2\eta_\perp \frac{M^2 dx dy (D + D')}{8}; \quad x; y = \frac{\xi_+; \eta_+}{\bar{P}_+} \tag{4.3}$$

Next, define a common basis $\bar{\xi}$ and $\bar{\eta}$ as follows:

$$\eta, \eta' = \bar{\eta} \mp k/3; \quad \xi = \xi' = \bar{\xi}, \tag{4.4}$$

in terms of which the product of the 3D wave functions becomes

$$\phi\phi' = \exp[-\frac{\xi_\perp^2 + \eta_\perp^2}{\beta_t^2} - \frac{f(x, y)}{\beta_l^2}]; \tag{4.5}$$

$$2f(x,y) = \sum_{\pm}[\frac{M^2x^2}{(1\pm\hat{k}/2)^2} + \frac{M^2(y\mp\hat{k}/3)^2}{(1\pm\hat{k}/2)^2}] \quad (4.6)$$

$$x, y; \hat{k} = \frac{(\bar{\xi}_+, \bar{\eta}_+; k_+)}{\bar{P}_+}$$

Giving the translation

$$x \to x; \quad y \to y - 2\sigma_k; \quad \sigma_k \equiv \frac{\hat{k}^2/6}{1+\hat{k}^2/4} \quad (4.7)$$

the function $f(x,y)$ reduces to

$$f(x,y) = \frac{(M^2x^2 + M^2y^2)(1+\hat{k}^2/4)}{(1-\hat{k}^2/4)^2} + 2M^2\sigma_k/3 \quad (4.8)$$

The same translation to the $\bar{D} = (D+D')/2$ function, dropping odd terms, gives

$$\bar{D}(k^2) = \frac{\xi_\perp^2 + \eta_\perp^2}{3} + \frac{2}{3}m_q^2 - \frac{2}{27}M^2 + \frac{(M^2-3m_q^2)}{2}[\frac{(x^2+y^2)(1+\hat{k}^2/4)}{(1-\hat{k}^2/4)^2} + 2\sigma_k/3] + \text{“}R''; \quad (4.9)$$

$$\text{“}R'' = -\frac{3}{2}M^2y^2\frac{\sigma_k}{(1-\hat{k}^2/4)^2} + \mathcal{O}(\hat{k}^4)$$

The rest of the integration is now routine gaussian for casting the e.m. matrix element in the form (3.14). Using the basic formulae

$$\int d^2\xi_\perp d^2\eta_\perp \exp[-\frac{\xi_\perp^2+\eta_\perp^2}{\beta_t^2}] = (\pi\beta_t^2)^2; \quad (4.10)$$

$$\int MdxMdy \exp[-f(x,y)/\beta_l^2] = \pi\beta_l^2\frac{(1-\hat{k}^2/4)^2}{1+\hat{k}^2/4}\exp[-\frac{2M^2\sigma_k}{3\beta_l^2}] \quad (4.11)$$

and other allied results, all integrations are carried out explicitly, and the form factors F, G identified a la (3.14). The common form is

$$F(k^2) = \frac{(1-\hat{k}^2/4)^2}{1+\hat{k}^2/4}\exp[-\frac{2M^2\sigma_k}{3\beta_l^2}]\frac{\bar{D}(k^2)}{\bar{D}(0)} \quad (4.12)$$

4.1 Results on Proton Form Factor

Eq.(4.12) represents our final formula for the proton form factor. Before comparison with experiment [19] however, we need to invoke the principle of 'Lorentz completion' [5] to give $F(k^2)$ an explicitly Lorentz-invariant look. The trick is to consider a collinear frame so that $P_\perp = P'_\perp = 0$. From this frame it is easy to see that [5]

$$\hat{k}^2 = \frac{4k^2}{4M^2+k^2}; \quad 1-\hat{k}^2/4 = \frac{4M^2}{4M^2+k^2} \quad (4.13)$$

Substitution of (4.13) in (4.12) gives an explicitly Lorentz invariant result. Eq.(4.12) then shows that $F(k^2) \sim k^{-4}$ for large k^2, in conformity with the scaling law [1] for the baryon.

Next we consider the e.m. radius of the proton which is given by the formula

$$< R^2 >= -6\partial_{k^2} F(k^2)|_{k^2=0} \tag{4.14}$$

Expanding (4.12) up to $\mathcal{O}(k^2)$ and collecting the coefficients of the indicated derivative gives

$$< R^2 >= -\frac{6}{M^2}(-3.39) = 23.14 GeV^{-2} = (0.96 fm)^2 \tag{4.15}$$

on substitution of the β^2 values from (2.36). These results are in fair accord with the observed value of $\sim 0.90 fm$ for the proton's e.m. radius [20], considering the fact that the parameters are not adjustable but attuned to qqq spectroscopy [10].

5 Baryon Magnetic Moments

Before calculating the magnetic moments (as the coefficients of σ_μ), we first generalize the formula (3.12) for the full baryon octet, with the replacements $(1 + 3\tau_z) \rightarrow f'$ and $(1 - \tau_z) \rightarrow f''$, where the latter are listed in Table 1 below. Although we are now allowed to set $k^2 = 0$, we must take account of the i) unequal mass kinematics; and ii) the normalization (3.15) for the general baryon case. Unequal mass kinematics is ensured simply by the replacement $3 \rightarrow \sum_{123}$ with an appropriate change of S_3 basis by keeping track of the index #3 in the terms m_3 and δm^2 appearing in the quantities M'_μ and μ''.

Table I: Flavour Factors f' and f'' for Baryons

Baryon type	f'	f''
N	$\frac{e(1+3\tau_z)}{4}$	$\frac{e(\tau_z-1)(1+\lambda)}{12}$
Σ	$\frac{e(1+3T_z)}{4}$	$\frac{e(1-T_z)(1+\lambda)}{12}$
Λ	$\frac{-e}{4}$	$\frac{-e(1+\lambda)}{12}$
Ξ	$\frac{-e}{2}$	$\bar{\tau}_z \frac{-e(1+\lambda)}{6}$
$\Lambda - \Sigma$	$\frac{e\sqrt{3}}{4}$	$\frac{e\sqrt{3}(1+\lambda)}{12}$

The flavour factors in Table 1 are mostly geometrical [17], except for the (small) parameter λ which enters the expression for f'' in the second column of the table. As noted at the end of Section 3, the origin of this term may be traced to the last two terms of M''_μ, eq.(A.11), which may be regarded as dynamical corrections to $SU(6)$ that affect M''_μ but not M'_μ. The parameter λ which represents this effect, is already expressed by eq.(3.16), where the $< ... >$ sign stands for the effect of integration over the internal variables a la Section 4. The ratio λ also enters the normalization via the RHS of eq.(3.15), which contributes a factor $(2+\lambda)^{-1}$ to the magnetic moment. Collecting all these results, the baryon magnetic moments in baryon magneton(B) units are all expressible as

$$\mu_{Bm} = \frac{a + b\lambda}{2 + \lambda} \tag{5.1}$$

where a, b are geometrical numbers given in Table 2 below, along with the results (in baryon magnetons) of this calculation, experiment [19] and the Schwinger model [2].

Table 2: a, b values and magnetic moments

Baryon	a	b	μ_B	Expt[19]	Sch [2]
p	4	0	+2.710	+2.793	+2.42
n	$-8/3$	$-2/3$	-1.570	-1.913	-1.62
Λ	$-4/3$	$-1/3$	-0.741	-0.614	-0.614
$\Lambda - \Sigma$	$4/\sqrt{3}$	$1/\sqrt{3}$	+1.278	1.61	...
Σ^+	4	0	+2.407	+2.33	+2.36
Σ^-	$-8/3$	$-2/3$	-1.456	-0.89	-0.87
Ξ^0	$-8/3$	$-2/3$	-1.438	-1.236	-1.356
Ξ^-	$-4/3$	$+2/3$	-0.876	+0.75	-0.55

The results for the magnetic moments are in fair accord with experiment[19] as well as with the Schwinger model of e.m. substitution [2] in accordance with VMD [3]. The following symmetry relations, if the results are expressed in baryon magneton units, may also be noted:

$$\mu_p = \mu_{\Sigma^+}; \quad \mu_n = \mu_{\Sigma^-} = \mu_{\Xi^0} = 2\mu_\Lambda = \frac{-2}{\sqrt{3}}\mu_{\Lambda-\Sigma^0} \tag{5.2}$$

6. Summary And Conclusion

In this paper, we have attempted to extend the Markov-Yukawa Transversality Principle [4] on the covariant null-plane from the $q\bar{q}$ problem [5] to the closely related qqq system, and given an explicit construction of the corresponding baryon-qqq vertex function.As a test of this Principle, its applications have been carried out on two allied quantities viz., i) the e.m. form factor of the proton; and ii) the magnetic moments of the baryon octet. The calculation of the former has been carried out on the lines of the corresponding work on the meson e.m. form factor [5], using the method of 'Lorentz completion' for obtaining an explicitly Lorentz-invariant result. Not only is the scaling law [1] reproduced, but also a value of the proton e.m. radius obtained in fair accord with the data [20], when the infra-red part of the gluon propagator is attuned to hadron spectroscopy [10]. The application to the baryon magnetic moments reveals an interesting set of symmetry relations, eq.(5.2), when expressed in units of 'baryon magnetons'. The results also show a good pattern of accord with experiment[19] as well as with the Schwinger model [2]. These results may be regarded more by way of 'calibration' of a relatively new principle,(the $MYTP$ [4]), than any attempt at exploring newer aspects of these familiar quantities, such as the gluonic radius of the proton [21], higher order e.m. effects on the proton radius [22], strangeness effects [23], etc, for which the interested reader is referred to other publications [24].

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Appendix A: Evaluation Of Spin Matrix Elements

We indicate here the main steps for evaluating the spin matrix elements in the factorized forms (3.8-10), leading to the form (3.12). Consider first the T' and A'_μ terms. Taking the traces, T' simplifies to

$$T' = 2\frac{m_1 m_2 - p_1.p_2}{\Delta_1\Delta_2} = \frac{\Delta_1 + \Delta_2 - P_{12}^2 - \delta m^2}{\Delta_1\Delta_2} \Rightarrow \frac{-P_{12}^2 - \delta m^2}{\Delta_1\Delta_2}$$

where $P_{12} = p_1 + p_2$, and we have used the result that in the null-plane formalism, the poles in the $\Delta 1, 2$ propagators are on opposite sides of the q_- plane [12], so that the 'virtualities' $\Delta_{1,2}$ in the numerator effectively drop out. A further simplification arises from the result

$$P_{12}^2 = (\bar{P} - \bar{p}_3)^2 \Rightarrow M^2 - 2\hat{m}_3 M^2/3 + m_3^2$$

where $\hat{m}_3 \approx 1/3$ is the fraction of momentum carried by the spectator, and some odd powers in η_3 have been dropped. Thus

$$T' = \frac{M^2/3 + m_3^2}{\Delta_1\Delta_2} \tag{A.1}$$

The multiplying factor A'_μ, eq.(3.10), may be written as

$$A'_\mu = \bar{U}'(P')\frac{(m_3 - i\gamma.p'_3)i\gamma_\mu(m_3 - i\gamma.p_3)}{\Delta_3\Delta'_3}U(P)$$

For further processing, we collect together the singular factors (S.F.) involving Δ_3, Δ'_3 in A'_μ as well as the vertex functions V_3, V'_3, eq.2.31), in the form

$$S.F. \equiv \frac{\sqrt{\Delta_3\delta(\Delta_3)}\sqrt{\Delta'_3\delta(\Delta'_3)}}{\Delta_3\Delta'_3} \tag{A.2}$$

Since this singular function is non-vanishing only at a couple of points it can be bounded by making use of the inequalities

$$h.m. < g.m. < a.m.$$

in a *compensatory* manner :

$$\sqrt{\Delta_3\Delta'_3} \geq \frac{2\Delta_3\Delta'_3}{\Delta_3 + \Delta'_3};$$

$$\sqrt{\delta(\Delta_3)\delta(\Delta'_3)} \leq [\delta(\Delta_3) + \delta(\Delta'_3)]/2$$

Multiplying these two factors together and substituting in (A.2) we finally obtain the result

$$S.F. \approx \frac{\delta(\Delta_3) + \delta(\Delta'_3)}{\Delta_3 + \Delta'_3} \Rightarrow \frac{\delta(\Delta_3) - \delta(\Delta'_3)}{\Delta_3 - \Delta'_3} \tag{A.3}$$

where the last step has made repeated use of the vanishing property of the argument of a δ-function. The last expression in turn is expressible as a *derivative*:

$$\frac{\delta(\Delta_3) - \delta(\Delta'_3)}{\Delta_3 - \Delta'_3} \approx -\partial_{\bar{\Delta}_3}\delta(\bar{\Delta}_3) = -\partial_{m_3^2}\delta(\bar{\Delta}_3) \tag{A.4}$$

where $\bar{\Delta}_3 \approx m_3^2 + \bar{p}_3^2$. The trick is now to transfer the burden of differentiation from the δ-function to the rest of the integrand in the sense of integration by parts. Then the differentiation w.r.t. m_3^2 boils down to the expression

$$+\partial_{m_3^2}\bar{U}(P')(m_3 - i\gamma.p'_3)i\gamma_\mu(m_3 - i\gamma.p_3)U(P) \Rightarrow \bar{U}(P')[i\gamma_\mu(1 + \frac{M}{3m_3})U(P)$$

on making use of the Dirac equation, and dropping a small $\eta)_3$ term. Collecting all these results we have

$$A'_\mu \to \bar{U}(P')i\gamma_\mu(1+\frac{M}{3m_3})U(P)\delta(\bar{\Delta}_3) \tag{A.5}$$

This quantity is multiplied by T', eq.(A.1), and the product of (A.1) and (A.5) represents one of the dual spin-matrix elements:

$$M'_\mu = (1+\frac{M}{3m_3})\frac{M^2/3+m_3^2}{\Delta_1\Delta_2}\bar{U}(P')[\frac{\bar{P}_\mu}{M}+\sigma_\mu]U(P) \tag{A.6}$$

where the δ-function on the RHS of (A.5) is absorbed in the integration measure (3.11) which is now redefined as

$$\int d\tau' \equiv \int d^4q_{12}d^4\bar{p}_3\delta(\Delta_3) \tag{A.7}$$

and the Dirac matrix γ_μ has been be Gordon-reduced as [14]

$$i\gamma_\mu \to \frac{\bar{P}_\mu}{M}+\sigma_\mu; \quad \sigma_\mu \equiv \frac{i\sigma_{\mu\nu}k_\nu}{2M} \tag{A.8}$$

This Gordon reduction at the quark level determines the relative strengths of the charge and magnetic form factors in eq.(3.14) of text, in the Sachs convention.

In a similar way the pair T'' and A''_μ can be simplified, except for a bit heavier algebra stemming from the extra tensor indices involved in each, as well as the presence of $\bar{\gamma}_\nu$ and its covariant conjugate $\bar{\gamma}^*_\nu$ which are defined via eqs.(3.2-3). Using the properties (3.3) of the projection operators $\theta_{\mu\nu}$, it is not difficult to show that

$$T''_{\nu\nu'} = \frac{\theta^*_{\nu\lambda}\theta_{\nu'\lambda'}}{3\Delta_1\Delta_2}[(M^2/3+m_3^2)\delta_{\lambda\lambda'}+\bar{\eta}_\lambda\bar{\eta}_{\lambda'}-3\bar{\xi}_\lambda\bar{\xi}_{\lambda'}] \tag{A.9}$$

where the 'bar' symbols are as defined in (3.2). The bar symbols also include the effect of averaging over the ξ,η values for the initial and final baryons. Similarly, the quantity $A''_{\mu\nu\nu'}$ is treated exactly as in (A.5) to give

$$A''_{\mu\nu\nu'} = \theta_{\nu\rho}\theta^*_{\nu'\rho'}(1+\frac{M}{3m_3})\bar{U}(P')\gamma_{\rho'}i\gamma_\mu\gamma_\rho U(P)\delta(\bar{\Delta}_3) \tag{A.10}$$

Thus the product of (A.9) and (A.10) defines the dual quantity to M'_μ of (A.6), which on contracting over some tensor indices gives

$$M''_\mu = \frac{(1+\frac{M}{3m_3})}{\Delta_1\Delta_2}\bar{U}(P')[(\frac{\bar{P}_\mu}{M}-\frac{\sigma_\mu}{3})(M^2/3+m_3^2)-(\bar{\eta}^2/3-\bar{\xi}^2)\sigma_\mu]U(P) \tag{A.11}$$

where again the δ-function in $A''_{\mu\nu\nu'}$ of (A.10) hhas been absorbed in the new integration measure (A.7). The final formula for the spin matrix element in terms of M'_μ, M''_μ is given in eq.(3.12) of text.

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