Three-Quark Bethe-Salpeter Vertex Function Under Pairwise Gluon-Exchange-Like Interaction : Application To n-p Mass Difference

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Abstract

A qqq BSE formalism based on an input 4-fermion Lagrangian of 'current' u, d quarks interacting pairwise via a gluon-exchange-like propagator in its *non-perturbative* regime, is employed for the construction of a relativistic qqq-wave function under the Covariant Instantaneity Ansatz (CIA). The chiral invariance of the input Lagrangian is automatically ensured by the vector character of the gluonic propagator, while the 'constituent' masses are the low momentum limits of the dynamical mass function $m(p)$ generated by the standard mechanism of $DB\chi S$ in the solution of the Schwinger Dyson Equation (SDE). The CIA gives an exact reduction of the BSE to a 3D form which is appropriate for baryon spectroscopy, while the reconstructed 4D form identifies the hadron quark vertex function as the key ingredient for evaluating transition amplitudes via quark-loop integrals. In this paper the second stage of this 'two-tier' BSE formalism is extended from the 4D $q\bar{q}$-meson to the 4D qqq-baryon vertex reconstruction through a reversal of steps offered by the CIA structure. As a first application of this 4D qqq wave function, we evaluate the quark loop integrals for the neutron (n) - proton (p) mass difference which receives contributions from two sources : i) the strong SU(2) effect arising from the $u-d$ mass difference (4 MeV); ii) the e.m. effect of the respective quark charges. The resultant $n-p$ difference works out at 1.28 MeV (vs. 1.29 expt), with two free parameters C_0, ω_0 characterizing the infrared structure of the gluonic, which have been precalibrated from a common fit to $q\bar{q}$ and qqq spectra as well as several other observable quark loop integrals. A formal derivation, based on Green's function techniques for 3 spinless quarks, of the CIA structure of the 4D qqq-baryon vertex function as employed in the text, is given for completeness in Appendix B.

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1 Introduction: Unified BS Dynamics of 2- and 3-Quark Hadrons With 3D Kernel Support

Soon after the advent of the Faddeev theory [1], the *relativistic* 3-body problem [2] attracted instant attention as a non-trivial *dynamical* problem, as distinct from earlier "kinematical" attempts [3] at a relativistic formulation of its wave function. In this respect the relativistic 3-baryon problem had been more of academic than practical interest (until the 'pion' got involved as a key ingredient), but the situation changed qualitatively when this 3-body problem started being viewed at the quark level. Looking back after 25 years it appears that the first serious attempt in this direction was made by Feynman et al [4] who gave a unified formulation of both the $q\bar{q}$ (meson) and qqq (baryon) problems under a common dynamical framework, bringing out rather sharply an underlying duality between these two systems which in turn signifies a more basic duality between a qq diquark [5] and a $\bar{q}$ antiquark. Indeed the diquark description is quite compact and adequate for many practical purposes involving the baryon, but the more microscopic qqq description which brings out the fuller permutation (S_3) symmetry in the baryon is necessary for the actual details of a full-fledged dynamical treatment [4].

Although the FKR theory [4] marked the first step in this direction, it suffered from an inadequate treatment of the time-like d.o.f. which showed up in several ways. The latter has by itself a long history in terms of attempts at formulating the Bethe-Salpeter Equation (BSE) for $q\bar{q}$ qqq systems throughout the Seventies under the Instantaneous Approximation (IA), as has been reviewed elsewhere [6]. The normalization of the BS wave function (gaussian) has been given by Tomozawa [7] under some special assumptions which however are not general enough to be adapted to any broader dynamical BSE framework going beyond the IA.

For several years we have been involved with a certain formulation of BS-dynamics for both $q\bar{q}$ and qqq systems within a common *unified* framework (to emphasize their underlying duality), designed to address their spectroscopy on the one hand, *and* a self-consistent treatment of various quark-loop amplitudes in terms of their respective hadron-quark vertex functions on the other. The "spectroscopy" aspects are addressed through the 3D reductions of the $q\bar{q}$ BSE [8] and the qqq BSE [9], to compare with observed O(3) spectra [10], while the loop-integral aspects of transition amplitudes show up through the reconstructed vertex functions of the 4D BSE [11]. An *exact interconnection* between the two forms was achieved through the ansatz of a 3D support defined *covariantly* in the BS kernels for the $q\bar{q}$ [12] and qqq [13] systems.

The "Covariant Instaneity Ansatz" (CIA) [12] which has been at the root of this "two-tier" philosophy, and is also supported by other considerations [14] based on the Markov-Yukawa transversality condition [15], seems to offer a possible Lorentz covariant way to reconcile the apparently conflicting demands of 3D spectroscopy [10] with the 4D structure of quark-loop amplitudes. The effectiveness of the CIA in giving a concrete shape to such a "two-tier" philosophy of spectra-cum-loop integrals was summarised in a semi-review [13] in the form of appropriate BSE's for $q\bar{q}$ and qqq systems with vector-type kernels [6] with 3D support, albeit with slight modifications [6] in the respective BSE structures to facilitate greater 'manoeuvreability', in the spirit of similar efforts [16] in the past. Further, the observed spectroscopy [10] is well satisfied on both $q\bar{q}$ [8] and qqq [9] sectors with a *common set* of parameters for the respective kernels (the qq kernel has

just half the strength of the $q\bar{q}$ kernel due to color effects), so that the respective vertex functions are entirely determined within this formalism.

The other aspect of this 'two-tier' formalism concerns the crucial property of chiral symmetry *and* its dynamical breaking. The first part (chiral symmetry) is ensured without extra charge by the *vector* character of the kernel that had been present all along in this program [6,11], since the BS-kernel is a direct reflection of an effective 4-fermion term in the input Lagrangian. Indeed the vector type character of the latter lends a natural gluon exchange flavour to such a pairwise interaction among 'current' (almost massless) u, d quarks at the Lagrangian level. This structure is quite general [17], and can be adapted to the QCD requirements on the gluonic propagator involved in the pairwise interaction kernel. Of this, the perturbative part (which is well understood) is quite explicit, but the *non-perturbative* (infrared) part is not yet derivable from formal QCD premises. It can nevertheless be simulated in a sufficiently realistic manner at the phenomenological level [6,19], so as to satisfy the standard constraints of confinement as well as explicit QCD features [20] in terms of a basically 3D BSE kernel structure.

The second part, viz., dynamical breaking of chiral symmetry ($DB\chi S$) is implemented via the Nambu-Jonalasino mechanism [21] whose full-fledged form amounts to adopting the 'non-trivial' solution of the Schwinger-Dyson Equation (SDE) derived from a given input, chirally symmetric Lagrangian with current quarks. A mass function $m(p)$ [17,18] is thus generated whose low-momentum value may be identified with the bulk of the 'constituent' mass (m_q) of the u, d quarks. This accords with Politzer additivity [22], viz., $m_q = m(0) + m_c$; where m_c, the current mass, is small. This was also shown in the context of a BSE-cum-SDE treatment [23] within the CIA formalism [12]. Thus formally the BS-kernel may be regarded as a *non-perturbative* gluon propagator [23] in a BSE framework involving the dynamical/constituent mass [19, 21-24] in the quark propagator.

To recapitulate, the CIA which gives an exact interconnection between the 3D and 4D forms of the BSE, provides a unified view of 2- and 3-quark hadrons, its 3D reduction being meant for spectroscopy [8-9], and the reconstructed 4D form [12,13] for identifying the respective hadron-quark vertex functions as the key ingredients for evaluating 4D quark-loop integrals. The formalism stems from a strongly QCD-motivated Lagrangian with *current* quarks whose pairwise interaction is mediated by a gluonic propagator in its non-perturbative regime. The QCD feature of chiral symmetry is ensured by the vector nature of this interaction, while its dynamical breaking is the result of a non-trivial solution of the SDE [17,23]. Thus, unlike in conventional potential models [25], the constituent mass so generated is *not* a phenomenological artefact, but the result of a self-consistent solution of the SDE [17, 19, 23], wherein the (constituent) mass normally employed for spectroscopy [8-9] *matches* with the output dynamical mass at low momentum [23]. Thus there are only two genuine input parameters C_0, ω_0, that characterize the (phenomenological) structure of the non-perturbative gluon propagator which serves for both the 2- and 3-quark spectra in a unified fashion [8-9]. In this formalism, these two constants play a role somewhat similar to that of the (input) 'condensates' in the theory of QCD sum rules [26].

Before proceeding further, let us pause to compare this approach with other dynamical methods, e.g., chiral perturbation theory [27] which has more explicit QCD features, albeit in the perturbative regime, leading to expansions in the momenta. This is a powerful theoretical approach employing the (chiral) symmetry of QCD; its essential parameters are the current quark masses, and the method works very efficiently where its premises are logically applicable. Thus it predicts the *ground state* spectra of light quark hadrons,

including their mass splittings due to strong and e.m. breaking of SU(2), but *not* the spectra of L-excited hadrons. The latter on the other hand demand a "closed form" approach to incorporate the "soft" *off-shell* effects which in turn require a non-trivial handle on the *infrared* (non-perturbative) part of the gluonic propagator, something which the present state of the QCD art does not yet provide. Thus one needs a phenomenological input even in standard BSE-SDE approaches [18], as discussed elsewhere [23]. The chiral perturbation theory [27] also lacks this vital ingredient, as seen from the absence of form factors in its 'point' Lagrangians [27] with at most derivative terms. This shows up, e.g., through its inability to predict L-excited spectra, and finer aspects (such as convergence) of 4D quark-loop integrals which depend crucially on these "off-shell" features. *Physically* this amounts to the absence of a 'confinement scale' which governs these form factors. In other BSE-cum-SDE approaches [17-19], including the present 'two-tier' CIA formalism [12,23], this 'scale' is an integral part of the structure of the non-perturbative part of the gluon propagator [19,23], with a built-in QCD feature of chiral symmetry and its dynamical breaking through the non-trivial solution of the SDE [17,19,23]. This not only facilitates the prediction of L-excited spectra [19, 8-9] but also provides a form factor for the hadron-quark vertex function which greatly enhances its applicability to various 4D quark-loop integrals; see [12, 23-24, 28].

After this clarification on the philosophy of this two-tier BSE approach, vis-a-vis some others [26,27], we may now state the objective of the present paper: A typical application of the 4D baryon-qqq wave function reconstructed [13] from the 3D qqq BSE, as a 3-body generalization of the corresponding $q\bar{q}$-meson problem [12]. Unlike the 2-body case, however, where the steps are exactly reversible [12], such reconstruction in the 3-body case involves a loss of information on the 4D Hilbert space for a 3-body system, so that the reversal of steps is in principle not unique, and requires a 1D δ-function to fill up the information gap between 3D and 4D Hilbert space which may be directly attributed to the CIA ansatz of a 3D support to the pairwise kernel. The 2-body case just escapes this pathology as it represents a sort of degenerate situation, but the price of a 3D kernel support must show up in a reconstruction of the 4D BSE from its reduced 3D form in any ($n > 2$)-body problem [29]. A plausible 'CIA' structure for the 4D qqq wave function was suggested in [13] in a semi-intuitive fashion, but a more formal mathematical basis has since been found [29] through the use of Green's function techniques, so that the reconstructed 4D form reduces exactly to the (known) 3D form as a consistency check [29]. The final result, which is almost the same as the earlier conjecture, eq.(5.15) of [13], except for a constant that does not affect the normalization, contains a 1D δ-function corresponding to the on-shell propagation of the spectator between two successive vertex points. As explained in detail in [29], this 1D δ-function must not be confused with any signature of "non-connectedness" in the 3-body wave function [30], since the 3D form is *fully connected*. A better analogy is to the 'scattering length approximation' to the $n - p$ interaction, characterized by the appearance of a (Fermi-type) δ-function potential, in estimating the effect of chemical binding on the scattering of neutrons by a hydrogen molecule [31]. In any case the 1D δ-function appearing in this structure is entirely innocuous since it gets integrated out in any *physical* (quark loop) amplitude including the BS normalization (see Sec.2 below).

The application chosen for the baryon-qqq vertex function is to the n-p mass difference, on closely parallel lines to the meson case [28]. To racall the *physics* of the n-p mass difference, this quantity receives contributions of *opposite* signs from two distinct sources

: i) a positive one from the strong SU(2) $d-u$ mass difference;
ii) a negative one from e.m. splittings.

For the sake of completeness, Appendix B of this paper gives the main steps of the derivation [29] for the 4D structure, eq.(5.15) of [13], of the baryon-qqq vertex function by the Green's function method for 3 spinless quarks, to be employed in this paper. In Sec.2 we collect the various pieces of this quantity with the inclusion of the spin and isospin d.o.f., on the lines of [32]. Thus equipped, we outline the main steps leading to an explicit evaluation of the normalization integral, using Feynman diagrams shown in figs.1(a.b,c). A *complex* basis [9, 33, 34] for 3D momentum variables facilitates the evaluation of the resulting $3D \times 3D$ integrals, *after* the time-like momenta have been eliminated by 'pole' integrations on identical lines to the corresponding $q\bar{q}$ problem [12,23,24]. In Sec.3 we evaluate the 'shift' in the nucleon mass due to strong SU(2) breaking, by inserting a mass shift operator $-\delta m {\tau_3}^{(i)}/2$ in place of $i\hat{\gamma}_\mu e_i$ at each of the corresponding γ- vertices of figs.1(a,b,c), as shown in figs. 2.(a,b,c). Here $\delta m = 4$ MeV is the 'standard' d-u mass difference [24,28] taken as the basic input. The condensate contribution is neglected as it was found to be negligible from a similar calculation of the SU(2) mass splittings in pseudoscalar mesons [28]. Sec.4 sketches the evaluation of the e.m. contribution in accordance with the diagrams of fig.3(a,b,c), while the details of the approximations employed are collected in Appendix A. Sec.5 summarises our findings and conclusions vis-a-vis other methods.

2 Normalization of the Baryon-qqq Vertex Function

To outline the structure of the baryon-qqq vertex function from a CIA-governed BSE [12-13], we shall generally follow the notation, normalization and phase convention for the various symbols as given in [13], but adapted to the *equal* mass kinematics ($m_1 = m_2 = m_3 = m_q$). The SU(2) mass difference δm ($\approx 4MeV$) between d and u quarks will be taken into account only through a 2-point vertex $[-\delta m {\tau_3}^{(i)}/2]$ inserted in the quark propagators in figs.2 (in place of $i\gamma_\mu e_i$ for a photon), but not in the structure of the vertex function. The vertex function is written in *three* pieces in each of which *one* quark plays the role of the 'spectator' by turn. For the spin structure (not given in [13]) we employ the convention of [3] which was extended in [32] to incorporate the S_3-symmetry for the spin-cum- isospin structure in the Verde [35] notation [36]. The full 4D BS wave function Ψ reads as [13,32,33] :

$$\Psi\Delta_1\Delta_2\Delta_3 = (\Gamma_1 + \Gamma_2 + \Gamma_3) \times [\chi'\phi' + \chi"\phi"]/\sqrt{2}; \tag{2.1}$$

$$\Delta_i = {m_q}^2 + {p_i}^2; \quad (i = 1, 2, 3). \tag{2.2}$$

Here χ' and $\chi"$ are the relativistic "spin" wave functions in a 2-component mixed symmetric S_3 basis which for a **56** baryon go with the associated isospin functions ϕ' and $\phi"$ respectively. These are given by [3,17] :

$$[\chi']_{\beta\gamma;\alpha} = [(M - i\gamma.P)i\gamma_5 C/\sqrt{2}]_{\beta\gamma} \times U(P)_\alpha/(2M) \tag{2.3}$$

$$[\chi"]_{\beta\gamma;\alpha} = [(M - i\gamma.P)\gamma_\mu C/\sqrt{6}]_{\beta\gamma} \times i\gamma_5\gamma_\mu U(P)_\alpha/(2M) \tag{2.4}$$

in a spinorial basis [3,32] in which the index α refers to the 'active' quark (interacting with an external photon line, fig.1), while β, γ characterize the other two, with the further

convention that γ refers to the "spectator" in a given diagram, fig.(1). The 'hat' on γ signifies its perpendicularity to P_μ, viz., $\hat{\gamma}.P = 0$. The notations in eqs.(2.3-4) are standard, with a common Dirac basis for the entire structure, and 'C' is the charge conjugation operator for quark #3 in a 23-grouping [3,32]. P_μ is the baryon 4-momentum, $U(P)$ is its spinor representation, and $(M - i\gamma.P)/(2M)$ its energy projection operator [3,32]. Further, because of the full S_3-symmetry of the last factor in (2.1), the $(1, 2, 3)$ indices can be permuted as needed for the diagram on hand. Thus in fig.1a, $\#1(\alpha)$ interacts with the photon ; $\#2(\beta)$ is the quark which has had a 'last' qq-interaction with $\#1(\alpha)$ before emerging from the hadronic 'blob', while $\#3(\gamma)$ is the spectator [32]. In fig.1b, the roles of #1 and #2 are reversed so that, of the two 'active quarks' #1 and #2, $\#2(\alpha)$ now interacts with the photon, $\#1(\beta)$ has had the last qq- interaction with $\#2(\alpha)$, while $\#3(\gamma)$ still remains the 'spectator'. These roles are cyclically permuted, with two more such pairs of diagrams, fig.1c), to give an identical chance to each of the quarks in turn [32]. Thus there are 3 such pairs of diagrams, of which only one pair is shown. An identical consideration applies to figs.2(a,b) with $i\gamma_\mu e_i$ replaced by $(-\delta m\tau_3/2)$ consistently. The spatial vertex functions Γ_i are given for i = 3 by [13] :

$$\Gamma_3 = N_B[D_{12}\phi/2i\pi] \times \sqrt{[2\pi\delta(\Delta_3).\Delta_3]} \tag{2.5}$$

where ϕ is the full, *connected* qqq wave function in 3D form, and D_{12} is the 3D denominator function of the (12) subsystem . The second factor represents the effect of the spectator [13] whose *inverse* propagator ${D_F}^{-1}(p_3)$ *off the mass shell* is just Δ_3, eq.(2.2). The main steps leading to this unorthodox structure which has been derived recently via the techniques of Green's functions [29], are sketched for completeness in Appendix B. As already noted in Sec.1, and again explained in Appendix B, its peculiar singularity structure in the form of a "square-root" of a 1D δ-function stems from the CIA ansatz of a 3D support to the pairwise interaction kernel, but it is quite harmless as the former will appear in a *linear* form in the transition amplitude corresponding to any Feynman diagram as in figs.1-2. The complete expressions for D_{12} and ϕ are given for the equal mass case (with #3 as spectator) by [13] (see also [9]):

$$D_{12} = \Delta_{12}(M - \omega_3); \quad \Delta_{12} = 2{\omega_{12}}^2 - M^2(1 - \nu_3)^2/2 \tag{2.6}$$

$${\omega_{12}}^2 = {m_q}^2 + \hat{q}_{12}^2; \quad 2\hat{q}_{12}^\mu = \hat{p}_1^\mu - \hat{p}_2^\mu \tag{2.7}$$

$$\phi = \ e^{-(\hat{p}_1^2+\hat{p}_2^2+\hat{p}_3^2)/2\beta^2} \equiv \ e^{-\rho/3\beta^2} \tag{2.8}$$

(see further below for the definition of ρ).

$$\hat{p}_i^\mu = {p_i}^\mu + p_i.PP_\mu/M^2; \quad \hat{p}_1^\mu + \hat{p}_2^\mu + \hat{p}_3^\mu = 0 \tag{2.9}$$

$${\omega_i}^2 = {m_q}^2 + \hat{p}_i^2; \quad \nu_3 = \omega_3/M(onshell) \tag{2.10}$$

The β-parameter is defined sequentially by [8,9]:

$$\beta^4 = \frac{4}{9}M{\omega_0}^2\bar{\alpha}_s(1 - m_q/M)^2(M - < \omega >); \quad < \omega >^2 = {m_q}^2 + 3\beta^2/8 \tag{2.11}$$

$$\bar{\alpha}_s^{-1} = {\alpha_s}^{-1} - 2MC_0\frac{(1 - m_q/M)^2}{M - < \omega >}; \tag{2.12}$$

$$\frac{6\pi}{\alpha_s} = 29\ln\frac{(M- < \omega >)}{\Lambda_{QCD}}; \tag{2.13}$$

$$\Lambda_{QCD} = 200MeV; \quad \omega_0 = 158MeV; \quad C_0 = 0.29 \tag{2.14}$$

The normalization N_B, eq.(2.5), is given in accordance with the Feynman diagrams 1(a,b) by the 4D integral (c.f.[32]) :

$$\begin{aligned} iP_\mu/M &= \sum_{123}\int d^4q_{12}d^4p_3\frac{{\Gamma_3}^*\Gamma_3}{2\Delta_2\Delta_3}[< \phi'|(23)'(1)'_\mu|\phi' > +\frac{1}{3} < \phi"|(23)"_{\nu\lambda}(1)"_{\nu\lambda;\mu}|\phi" >] \\ &\quad +(1 \Leftrightarrow 2) \end{aligned} \tag{2.15}$$

where the matrix element for fig.1a is organized as a product of two spin-factors : a '23-element' expressed as a Dirac trace over the indices β, γ; and a '1-element' (with suppressed index α). The associated isospin functions ϕ are shown according to (2.1). The contribution of fig.1b is shown symbolically by $1 \Leftrightarrow 2$, while $\sum_{123}$ indicates the sum over all the 3 pairs cyclically. In representing eq.(2.12) we have dropped 'cross-terms' like ${\Gamma_i}^*\Gamma_j$, where $i \neq j$, since the presence of a $\sqrt{\delta}$-function in each Γ_i ensures that a simultaneous 'on-shell' energy conservation of $i \neq j$ spectators is not possible [32]. The various pieces of the matrix elements in (2.14) which can be read off from fig.1a in terms of the spin functions (2.3-4) are as follows:

$$(1)"_{\nu\lambda;\mu} = \bar{U}(P)S_F(p_1)i\gamma_\mu e_1 S_F(p_1)U(P) \tag{2.16}$$

$${S_F}^{-1}(p) = m_q + i\gamma.p \tag{2.17}$$

$$(1)"_{\nu\lambda;\mu} = \bar{U}(P)i\hat{\gamma}_\nu\gamma_5 S_F(p_1)i\gamma_\mu e_1 S_F(p_1)i\gamma_5\hat{\gamma}_\lambda U(P); \tag{2.18}$$

$$(23)' = Tr[C^{-1}\gamma_5(M - i\gamma.P)(m_q - i\gamma.p_2)(M - i\gamma.P)\gamma_5(m_q + i\gamma.p_3)C]/8M^2 \tag{2.19}$$

$$(23)"_{\nu\lambda} = Tr[C^{-1}\hat{\gamma}_\nu(M - i\gamma.P)(m_q - i\gamma.p_2)(M - i\gamma.P)\hat{\gamma}_\lambda(m_q + i\gamma.p_3)C]/8M^2 \tag{2.20}$$

The 'strength' e_i of the (zero-momentum) 'photon' coupling to the quark line p_i can be chosen in several ways [37]. We take here the simplest possibility, viz., $e_i = 1/3$ each. The isospin matrix element is first eliminated according to [38]:

$$< \phi'|1|\phi' > = < \phi"|1|\phi" > = 1 \tag{2.21}$$

$$< \phi'|{\tau_3}^{(1)}|\phi' > = -3 < \phi"{\tau_3}^{(1)}|\phi" > = < \tau_3 >_{(p,n)} \tag{2.22}$$

Eq.(2.20) suffices for (2.14), while (2.21) will be needed for the u-d mass difference operator $-\delta m{\tau_3}^{(1)}/2$; see Sec.3. Next, the evaluation of the traces in (2.15-18) is straightforward, after noting that (2.15-16), *after* spin-averaging, are expressible as traces. The results are

$$(23)'\theta_{\nu\lambda} = (23)"_{\nu\lambda} = (m_q + M\nu_2)(m_q + M\nu_3)\theta_{\nu\lambda} \tag{2.23}$$

$$(1)'_\mu\theta_{\nu\lambda} = (1)"_{\nu\lambda;\mu} = [2M\nu_1(m_q + M\nu_1) + \Delta_1]\theta_{\nu\lambda}P_\mu/(M{\Delta_1}^2) \tag{2.24}$$

where θ is a covariant Kronecker delta w.r.t. P_μ, viz.,

$$\theta_{\nu\lambda} \equiv \theta_{\nu\lambda} = \delta_{\nu\lambda} - P_\nu P_\lambda/P^2; \quad (P^2 = -M^2) \tag{2.25}$$

Collecting all these results and simplifying we get

$${N_B}^{-2} = \sum_{123}\int d^3\hat{p}_3\frac{(m_q + \omega_3)}{2\omega_3} \times \int d^3\hat{q}_{12}{D_{12}}^2\phi^2[e_1I_1 + e_2I_2] \tag{2.26}$$

$$2i\pi I_1 = \int M d\sigma_{12}[2M\nu_1(m_q + M\nu_1) + \Delta_1]/(M{\Delta_1}^2\Delta_2) \tag{2.27}$$

where we have "cashed" the $\delta(\Delta_3)$-function arising from $|\Gamma_3|^2$ against the time-like component of d^4p_3, and used the results

$$d^4q_{12} = d^3\hat{q}_{12}Md\sigma_{12}; \quad \nu_{1,2} = (1-\nu_3) \pm \sigma_{12} \tag{2.28}$$

The integration over $d\sigma_{12}$ involves single and double poles arising from the propagators ${\Delta_{1,2}}^{-1}$ in (2.26), while the value of ν_3 is taken 'on-shell' at ω_3/M after the $\delta(\Delta_3)$- function has been cashed. The result of a basic σ_{12}-integration is

$$\int M d\sigma_{12}{\Delta_1}^{-1}{\Delta_2}^{-1} = 2i\pi/D_{12} \tag{2.29}$$

from which others can be deduced by differentiation under *unequal* mass kinematics, or directly through a 'double pole' integration. The net result for $I_1 + I_2$, eq.(2.26), is given in eq.(2.41) below. Further, the individual terms of the summation $\sum_{123}$ in (2.25) are fixed by the values chosen for e_i (which need not be specified in advance, as they can be adapted to other conventions too [37]; see Sec.3).

The integration in (2.25) can be considerably simplified in a complex basis [9,33] defined (in momentum space) by :

$$\sqrt{2}z_i = \xi_i + i\eta_i; \quad \sqrt{2}{z_i}^* = \xi_i - \eta_i; \tag{2.30}$$

$$\sqrt{3}\xi_i = p_{1i} - p_{2i}; \quad 3\eta_i = -2p_{3i} + p_{1i} + p_{2i}; \tag{2.31}$$

where we now employ the alternative notation p_{1i} for $\hat{p}_1^{\mu}$, in view of its basically 3D content. In terms of z_i and ${z_i}^*$, the 6D integration in (2.25) is expressed as

$$d^3\hat{p}_3 d^3\hat{q}_{12} = (\sqrt{3}/2)^3 d^3\xi d^3\eta = d^3z d^3z^* \tag{2.32}$$

The further representation [9,33]

$$d^3z d^3z^* = (dz_+ d{z_-}^*)(dz_- d{z_+}^*)(dz_3 d{z_3}^*) \tag{2.33}$$

where

$$\sqrt{2}z_+ = R_1\ e^{i\theta_1}; \quad \sqrt{2}{z_-}^* = R_1\ e^{-i\theta_1} \tag{2.34}$$

$$\sqrt{2}z_- = R_2\ e^{i\theta_2}; \quad \sqrt{2}{z_+}^* = R_2\ e^{-i\theta_2} \tag{2.35}$$

$$\sqrt{2}z_3 = R_3\ e^{i\theta_3}; \quad \sqrt{2}{z_3}^* = R_3\ e^{-i\theta_3} \tag{2.36}$$

reduces the 6D integration (2.32) merely to $\pi^3 d{R_1}^2 d{R_2}^2 d{R_3}^2$, since the θ_i -variables (*not* Euler angles!) are not involved in the integrands encountered, and just sum up to $(2\pi)^3$. The positive variables R_i, (i = 1,2,3), are related to the ξ_i, η_i variables by

$$\rho \equiv R_1^2 + R_2^2 + R_3^2 = \xi^2 + \eta^2 = 2z_i{z_i}^* \tag{2.37}$$

To convert the variables ω_i that appear in the integrals (2.28) in terms of the $R_{1,2,3}$ variables is a straightforward but tedious process which can be somewhat simplified in terms of the intermediate variables $\xi^2 - \eta^2$ and $2\xi.\eta$ which form a [2,1] representation [35] of S_3-symmetry at the 'quadratic' level. Now because of the full S_3 -symmetry of the 6D integral (2.32), together with the (fortunate) circumstance of equal mass quarks in the

problem on hand, the integrand as a whole is S_3 -symmetric which permits the following simplification: Each of the quantities $\hat{p}_i^2$ and $\hat{q}_i^2$ inside (2.32) can be expanded as

$$\hat{p}_{1,2}^2 = \rho/2 + (\xi^2 - \eta^2)/4 \pm \sqrt{3}\xi/\eta/2; \quad \hat{p}_3^2 = \rho/2 - (\xi^2 - \eta^2)/2 \tag{2.38}$$

$$\hat{q}_{12}^2 = 3\xi^2/4 = \rho/2 + (\xi^2 - \eta^2)/4 \tag{2.39}$$

In all these terms the principal quantity is $\rho/2$, while the resultant effects of the mixed-symmetric corrections will show up only in the *fourth* order, etc. In the present case of equal mass kinematics it is a good approximation to neglect the latter terms, as has also been found for the qqq mass spectral results [9], so that all quantities are expressed in terms of ρ only :

$$\omega_{1,2,3} \approx \omega_{12} \approx \omega_\rho; \quad {\omega_\rho}^2 \equiv m_q^2 + \rho/2 \tag{2.40}$$

$$D_{12} \approx 2(M - \omega_\rho)[\omega_\rho^2 - (M - \omega_\rho)^2/4]. \tag{2.41}$$

The rest of the integration is expressed entirely in terms of the ρ-variable, with the resultant 6D measure given by

$$\int d^3\hat{p}_3 d^3\hat{q}_{12} F(\rho) = (\pi\sqrt{3}/2)^3 \int \rho^2 d\rho/2 F(\rho) \tag{2.42}$$

These considerations suffice for evaluating the integrals I_1 and I_2 whose resultant value is now given for $e_i = 1/3$ by :

$$D_{12}^2(I_1+I_2) = [m_q^2+m_q(M-\omega_\rho)+(M-\omega_\rho)^2/4]\times(M-\omega_\rho)^3/\omega_\rho+D_{12}(2m_q+M-\omega_\rho) \tag{2.43}$$

Substitution in (2.25) yields N_B directly. The numerical values are given collectvely at the end of Sec.4.

3 Strong SU(2) Mass Difference for the Nucleon

This calculation is on almost identical lines to Sec.2, except for the substitution $ie_1\gamma_\mu$ to $-\delta m{\tau_3}^{(1)}/2$ in figs.1(a,b) to give figs.2(a,b) which represent the effect of insertion of a 2-point vertex in a quark line. Indeed we can directly start from the counterpart of eq.(2.15) which gives the 'strong' mass shift as:

$$\begin{aligned} i\delta M_{st} &= \sum_{123} \int d^4q_{12} d^4p_3 \frac{{\Gamma_3}^*\Gamma_3}{2\Delta_2\Delta_3} \times [< \phi'|(23)'(1)'|\phi' > + \frac{1}{3} < \phi"|(23)"_{\nu\lambda}(1)"_{\nu\lambda}|\phi" >] \\ &\quad +(1 \Leftrightarrow 2) \end{aligned} \tag{3.1}$$

where we have now employed eq.(2.21) for the isospin factors, and the counterparts of (2.16) and (2.17) are respectively

$$(1)' = \bar{U}(P)S_F(p_1)[-\delta m{\tau_3}^{(1)}/2]S_F(p_1)U(P); \tag{3.2}$$

$$(1)"_{\nu\lambda} = \bar{U}(P)i\hat{\gamma}_\nu\gamma_5 S_F(p_1)[-\delta m{\tau_3}^{(1)}/2]S_F(p_1)i\gamma_5\hat{\gamma}_\lambda U(P) \tag{3.3}$$

while the definitions (2.18) and (2.19) remain unaltered. As a result, eq.(2.22) remains valid, while the counterpart of (2.23) becomes

$$(1)'\theta_{\nu\lambda} = -3(1)"_{\nu\lambda} = [2m_q(m_q + M\nu_1) - \Delta_1]\theta_{\nu\lambda}(-\delta m/2)/\Delta_1^2 \tag{3.4}$$

Carrying out the $d\sigma_{12}$-integration, the result for δM_{st} is now given by the counterpart of (2.25), viz.,

$$\delta M_{st} = 3N_B^2 \int d^3\hat{p}_3 \frac{(m_q+\omega_3)}{2\omega_3} \times \int d^3\hat{q}_{12} {D_{12}}^2 \phi^2 [J_1+J_2](-\delta m \tau_3/6) \tag{3.5}$$

in the form of an isospin operator "τ_3" for the *nucleon*, where we have represented the effect of $\sum_{123}$ by a factor of "3", and

$$\begin{aligned} D_{12}^2[J_1+J_2] &= \frac{m_q}{\omega_\rho}[(m_q+\frac{1}{2}(M-\omega_\rho))(M-\omega_\rho)[2\omega_\rho^2+m_q(M-\omega_\rho)] \\ &+(M-\omega_\rho)^2[m_q+(M-\omega_\rho)/2]^2+\Delta_{12}(m_q^2-\omega_\rho^2) \\ &+(m-\omega_\rho)(m_q+(M-\omega_\rho)/2)(\omega_\rho^2+m_q(M-\omega_\rho)/2) \\ &+\Delta_{12}^2(1-m_q(M-\omega_\rho)/\omega_\rho^2)/2] \end{aligned} \tag{3.6}$$

as the exact counterpart of (2.42) under the same approximation. It is seen from (3.5) that the *difference* $(n-p)$ is *positive*.

4 E.M. Mass Difference for the Nucleon

The diagrams for the e.m. mass difference are given by figs.3 (I,II,III) for a proton (uud) configuration to illustrate the underlying topology in accordance with the roles of the 'active' and 'spectator' quarks in turn, as explained in Sec.2. In each of these diagrams, two internal quark lines are joined by a photon line. The e.m. vertex at quark #i has the strength $e[1+3\tau_3^{(i)}]/6$ from which the isospin matrix elements of a product of *two* such factors (shown for fig.3.III) have the forms

$$<\phi';\phi"|(1+3\tau_3^{(1)})/6 \times (1+3\tau_3^{(1)})/6|\phi';\phi"> \tag{4.1}$$

for the proton (uud) cnfiguration shown in III with #3 as spectator, but in a basis (1;23) (which is consistent with the spin basis, eqs.(2.3-4)), corresponding to fig.1a, viz.[36,38]:

$$|\phi'>= u_1(u_2d_3-u_3d_2)/\sqrt{2}; \quad |\phi">= (-2d_1u_2u_3+u_1d_2u_3+u_1u_2d_3)/\sqrt{6} \tag{4.2}$$

We note in parentheses that in fig.3.III, the interchange of the two 'active' quarks #1 and #2 does *not* give a new configuration, *unlike* in figs.1 and 2; ((a) versus (b) configurations).

It is now easy to check that the matrix elements $<>'$ and $<>"$ of (4.1) are $1/9$ and $-1/9$ for the *proton* configuration. After doing the corresponding neutron case, the two results may be combined in the single operator forms [38]:

$$<.>'= (1+3\tau_3)/36; \quad <.>" = (1-5\tau_3)/36 \tag{4.3}$$

where τ_3 is the isospin operator for the nucleon as a whole [see eq.(3.5)], to be sandwiched between the neutron and proton states. The resultant isospin factor is then

$$e^2[<.>'+<.>"]/2 = e^2(1-\tau_3)/36 \Rightarrow -e^2\tau_3/36 \tag{4.4}$$

After this book-keeping on the charge factors we can drop the isospin d.o.f. $|\phi>$ from the qqq wave function and, on the basis of the equality of the $(.)'$ and $(.)"$ contributions

(2.22-23) for the spin matrix elements, it is enough to work with the $(.)'$ type to represent the full effect. Collecting these details, the net isospin contribution to the e.m. $(n-p)$ mass difference is just $e^2/18$, which (of course) comes out with the *correct* (negative) sign in the resultant e.m.contribution to the total $n-p$ difference after all the phase factors in the orbital-cum-spin space have been taken into account. The complete e.m. self energy of the nucleon (with operator τ_3), with fig.3.III as the prototype, is now given by

$$\delta M^{\gamma} = \sum_{123}[-e^2\tau_3/36]/(2\pi)^4 \int d^4p_3 d^4q_{12} d^4q_{12}{}' \Gamma_3^* \Gamma_3' k^{-2} \times [23]_{\mu}'[1]_{\mu}'/(\Delta_3\Delta_2\Delta_2') \qquad (4.5)$$

where the various momentum symbols are as shown in fig.3, with the primed quantities referring to the vertex on the right, but otherwise written in the same convention as in eqs.(2.5-10). The symbols within square brackets are analogous to (2.16-19):

$$[1]_{\mu}' = \bar{U}(P')S_F(p_1')i\gamma_{\mu}S_F(p_1)U(P); \quad (P'=P) \qquad (4.6)$$

$$[23]_{\mu}' = \frac{Tr}{8M^2}[C^{-1}\gamma_5(M-i\gamma.P')(m_q-i\gamma.p_2')i\gamma_{\mu}(m_q-i\gamma.p_2)(M-i\gamma.P)\gamma_5(m_q+i\gamma.p_3)C] \qquad (4.7)$$

And the product of (4.6) and (4.7) works out as

$$\begin{aligned} ME \equiv\ & \frac{(m_q+\omega_3)}{\Delta_1\Delta_1'}[-(\Delta_1+\Delta_1'-k^2)(\Delta_2+\Delta_2'-k^2)/4 \\ & -(\Delta_1+\Delta_1'-k^2)(m_q\omega_2+m_q\omega_2'+2\omega_2\omega_2')/2 \\ & -(\Delta_2+\Delta_2'-k^2)(m_q\omega_1+m_q\omega_1'+2\omega_1\omega_1')/2 \\ & -(\Delta_1+\Delta_2)(m_q+\omega_1')(m_q+\omega_2')/2-(\Delta_1'+\Delta_2')(m_q+\omega_1)(m_q+\omega_2)/2 \\ & -(\Delta_1'+\Delta_2)(m_q+\omega_1)(m_q+\omega_2')/2-(\Delta_1+\Delta_2')(m_q+\omega_1')(m_q+\omega_2)/2 \\ & +(m_q^2+\frac{1}{2}(P-p_3-k)^2)[(m_q+\omega_1)(m_q+\omega_2')+(m_q+\omega_1')(m_q+\omega_2) \\ & +(m_q+\omega_1)(m_q+\omega_2)+(m_q+\omega_1')(m_q+\omega_2')]] \end{aligned} \qquad (4.8)$$

Some features of this "master" expression may be noted. There is a 'natural factorization'in the variables q_{12} and q_{12}', except for the photon propagator k^{-2}, $(k=q_{12}-q_{12}')$. Further, the two blobs are connected together by the 'spectator'variable p_3 which is on the mass shell due to the presence of $\Gamma_3^*\Gamma_3'$ in eq.(4.3).

The time-like (pole) integrations over each of $d\sigma_{12}$ and σ_{12}' can be carried out *exactly* a la (2.28) and its derivatives, since the 3D vertex function $D_{12}\phi$ in Γ_3 does not involve σ_{12}, etc. After this step $\hat{q}_{12}$, $\hat{q}_{12}'$ and $\hat{p}_3$ are the 'right' 3D variables for the 'triple integration' whose essential logic may be stated as follows. The main strategy is to decouple the $\hat{q}$ and $\hat{q}'$ variables from the photon propagator k through the following device [19]:

Since k is basically space- like, it is a good approximation to replace k^{-2} by $\hat{k}^{-2}$ which equals $(\hat{q}_{12}-\hat{q}_{12}')^2$, and drop the angular correlation in the two $\hat{q}$- momenta (since the error in this neglect is zero in the first order [28]). Next we use the inequality [28]

$$(a^2+b^2)^{-1} \leq (2ab)^{-1}; \quad a \to |\hat{q}_{12}|, etc \qquad (4.9)$$

which ensures the necessary factorizability in the q-variables.In principle the corrections to this inequality can be calculated since the neglected term is approximately equal to

$-(a-b)^2/(4a^2b^2)$ which is still factorizable, but this refinement is unnecessary in view of the smallness of the e.m. effect itself. After this simplification the rest of the integration procedure is straightforward since the $\hat{q}$ and $\hat{q}'$ integrations can be done analytically, and only a 1D integration over $|p_3|$ remains for numerical evaluation. The necessary expressions are collected in Appendix A and the numerical results for all contributions are given as under.

The key parameters are the quark mass m_q and the size parameter β^2, the latter being determined dynamically through the chain of eqs.(2.11-14). As noted in Sec.1 already, the mass m_q which is usually called the 'constituent' mass, should be viewed as the sum of the (flavour independent) 'mass function' m(p) for small p, plus a small "current mass" m_c, in the spirit of Politzer additivity [22]. The mass function m(p) was generated in this BSE-cum-SDE framework through a Dynamical Chiral Symmetry Breaking mechanism in a non perturbative fashion [23]. Also from some related quark-loop calculations with $q\bar{q}$ mesons in recent times [24,28], it was found that for such 'low energy' processes the mass function m(p) is rather well approximated by m(0), so that [22], $m_q = m(0) + m_c$. Therefore the $d-u$ mass difference is the *same* at the 'constituent' or at the 'current' levels, and this is what has been denoted by δm in the text (figs.2). Its smallness compared to m_q justifies its neglect in all the functions except where it appears explicitly, viz., fig.2. We take its value at $\delta m = 4MeV$, as in related calculations [24,28], while the other quantities are predetermined from $q\bar{q}$ [8] and qqq [9] spectroscopy:

$$m_q = 265MeV; \quad \beta^2(N) = 0.052GeV^2 \tag{4.10}$$

so that there are *no free* parameters in the entire calculation. The results from Secs.2-4 are now summarized for (n-p) as :

$$N_B^{-2} = 5.5209 \times 10^{-4} GeV^{-10}; \quad [e_i = 1/3] \tag{4.11}$$

$$\delta M_{st} = +1.7134MeV; \quad \delta M^{\gamma} = -0.4396MeV. \tag{4.12}$$

Hence

$$\delta M(net) = +1.28MeV; \quad (vs.1.29MeV : Expt) \tag{4.13}$$

which is the principal result of this investigation.

5 Summary and Conclusion

This calculation fills up an important gap in the two-tier BSE formalism under 3D kernel support (CIA) for a simultaneous investigation of spectra and transition amplitudes of both $q\bar{q}$ and qqq varieties within a single unified framework [12,13]. To recapitulate the main points, the (first stage) 3D reductions of both the 2-body and 3-body BSE's had yielded good agreement with the respective spectra [8,9], with a common set of parameters $C_0 = 0.27$ and $\omega_0 = 158MeV$ characterizing the structure of the non-perturbative gluon propagator, since a third parameter, the 'constituent' mass m_q needed for spectroscopy [8,9], is essentially the dynamical mass function $m(p)$ in the low momentum limit [22, 23].

More substantial tests of the formalism are expected from the (second stage) reconstruction of the 4D hadron-quark vertex function which carries the non-perturbative off-shell information in a *closed* form. This exercise was initially confined to the meson-$q\bar{q}$ vertex function whose exact reconstruction [12] had led to several useful checks, from

4D loop integrals for hadronic and e.m. transition amplitudes [11,12], to like integrals probing the momentum dependence of the quark mass function $m(p)$ which is the 'chiral' limit ($M_\pi = 0$) [17,21,23] of the pion-quark vertex function. Indeed $m(p)$ acts as the form factor for loop integrals determining the vacuum to vacuum transitions, and is found to predict correctly several condensates, from the basic $< q\bar{q} >$ [23] to 'induced' condensates [39], *under one roof*. Further tests of the hadron-quark vertex function have come from SU(2) breaking effects like $\rho - \omega$ mixing [24] and mass splittings in pseudoscalar mesons [28], with only one additional parameter representing the $d - u$ mass difference; see also [27].

The last link in our two-tier formalism has been a reconstruction of the 4D baryon-qqq vertex function on the lines of the 2-body case [12], to facilitate the evaluation of corresponding 3-body loop integrals. This quantity was conjectured some time ago [13], but has only recently found a rigorous derivation [29] in terms of Green's function techniques, whose main steps are sketched for completeness in Appendix B. The present work represents a first application of this quantity, choosing as an example the problem of the $n - p$ mass difference. The physics of this problem is two-fold: i) the qqq vertex function which is entirely determined by the *same* dynamics as for the $q\bar{q}$ case; ii) the strong and e.m. SU(2) breaking, (figs.2 and 3), considered on identical lines to the corresponding problem of pseudoscalar SU(2) mass splittings [28]. As such we have refrained here from giving fuller references on the $d - u$ mass difference, most of which originated from Weinberg's famous paper [40], but several more references on the physics of the problem may be found in [28]. On the other hand the entire derivation shows that no free parameters are involved, so that the final figure (4.13), although a single number, must *not* be treated as an isolated quantity, but as an integral part of a much bigger package.

As a point of detail, we should also mention our neglect of the condensate contributions inserted in the internal quark lines as in figs.2, in view of a recent finding [28] that such contributions are small within the BSE framework. This may not be too surprising since, unlike in the QCD-SR method [26] where such condensate contributions are the principal source of non-perturbative effects, this is no longer the case in the present BSE treatment which was primarily designed to incorporate non-perturbative effects in the zeroeth order. (As a result, the condensate effects in the present non-perturbative scenario may well be residual).In this respect the good agreement of our net estimate of δM, eq.(4.13), with experiment (without free parameters) suggests a good support for the BS dynamics when viewed together with other related phenomena [8,9,12,23,24,39] within the same dynamical framework [12,13,23,29].

We also recapitilate the 'QCD' status of this BSE formalism [12] viv-a-vis contemporary methods like QCD sum rules [26] or chiral perturbation theory [27]. As already explained in Sec.1, the gluon exchange character of the pairwise $q\bar{q}$ or qq interactions lends them a natural chiral invariance property at the input Lagrangian level with 'current' quarks. In particular, the 'constituent' mass is *not* an input, but emerges as the low momentum limit of the dynamical mass function $m(p)$ that characterizes the quark propagators appearing in the 2- and 3- body BSE's, as a result of $DB\chi S$ [21, 17, 19, 23], since the 'current' masses of u, d quarks give only a small additive contribution [22]. The empirical aspect of the gluon propagator concerns only its *non-perturbative* regime which often requires separate parametrization even in orthodox formulations [18]. In the present formulation, its explicit parametrization with two constants C_0 and ω_0) [23] is the price for a 'closed form' representation of non-perturbative effects in the derived hadron-quark

vertex function, but the returns are rich, e.g., the description of many items that depend sensitively on the details of such form factors in the quark-loop integrals [12,23,24,28], with their structures firmly rooted in spectroscopy [8,9]. In contrast, the chiral perturbation theory [27] has a more explicit QCD content, but with greater emphasis on a perturbative treatment, as revealed by expansions in powers of small momenta and "current" masses m_c [27] for a systemic derivation of the low energy structure of the Green's function in QCD [27]. It is a powerful method, highly successful in predicting items like ground state masses *as well as* their splittings, but its lack of a closed form representation prevents an equally successful prediction of 'soft' QCD effects in enough details, such as the momentum dependence of the mass function, or of hadron-quark vertex functions in general, with other observable consequences such as failure to predict the L-excited spectra [10]. The method of QCD sum rules [26] also shares similar features such as lack of closed form representation for the form factors, and while it does predict items like the $n-p$ mass difference, the highly indirect nature of such derivations [41] brings out the parametric uncertainties involved in the simulation of soft QCD effects.

Finally we should like to comment on the principal motivation for this investigation, namely to demonstrate the practical feasibility of such realistic quark-loop calculations for the relativistic 3-quark problem with a full-fledged (BS) dynamical framework whose basic parameters are linked all the way to spectroscopy. The present calculation indeed suggests that not only quark-loops involving mesons [12,23,24,28] but even those involving the (less trivial) qqq baryon are amenable to a similar degree of dynamical sophistication without excessive efforts, so that it makes sense to speak of an effective "4-fermion coupling" for both $q\bar{q}$ and qq pairs within a common parametric framework. This is somewhat remiscient of Bethe's "second principle" theory, originally suggested at the two-nucleon level of nuclear forces, now reinterpreted at the quark level, with a simple extension to include the antiquark in the dynamical description. (This extension would not make sense at the NN level, since the NN and $N\bar{N}$ forces are very different from each other). Indeed such a dynamics had been strongly suggested (with concrete examples) in a perspective review not too long ago [42], but it seemed to have gone largely by default, as evidenced by a strong tendency in the contemporary literature to continue to rely on "ad-hoc form factors" [43] to simulate the vertex functions, instead of generating them dynamically. Hopefully, some efforts in this direction have been recently in evidence [44], using the Nambu Jonalasino model [21] of contact 4-fermion interactions. It is to be hoped that Bethe's "second principle" perspective will be upheld by such investigations, at least until such time as a fully satisfactory solution to QCD is forthcoming.

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Appendix A: Evaluation of the Integral (4.5)

The master expression (4.8) after being substituted in the full e.m. self energy contribution (4.5) is integrated over each $d\sigma_{12}$ and $d\sigma_{12}{}'$. The final result is

$$\delta M^{\gamma} \;=\; \sum_{123} \frac{2e^2}{9}\tau_3 \int \frac{(m_q+\omega_3)}{2\omega_3}\frac{d^3\hat{p}_3}{4\pi}\frac{d^3\hat{q}_{12}}{4\pi}\frac{d^3\hat{q}'_{12}}{4\pi}\frac{1}{2\hat{q}_{12}2\hat{q}'_{12}} \times$$

$$F(\hat{q}_{12},\hat{q}'_{12},\hat{p}_3)exp(-\frac{2}{3}[\hat{q}^2_{12}+\hat{q}'^2_{12}+3\hat{p}^2_3]/\beta^2) \tag{A.1}$$

where

$$\begin{aligned}
&F(\hat{q}_{12},\hat{q}'_{12},\hat{p}_3) = \\
&(m_q+\omega_{12})(m_q+\omega_{12}{}')k^2+(M\omega_3-\frac{1}{2}(M^2-m_q^2))[(m_q+\omega_{12})^2 \\
&+(m_q+\omega_{12}{}')^2+(m_q+\omega_{12})(m_q+\omega_{12}{}')-(m_q+\omega_{12})^2D_{12}{}'/2\omega_{12}{}' \\
&-(m_q+\omega_{12}{}')^2D_{12}/2\omega_{12}-(m_q+\omega_{12})(m_q+\omega_{12}{}')[D_{12}/2\omega_{12} \\
&+D_{12}{}'/2\omega_{12}{}']k^2[(m_q+2\omega_{12})(m_q+2\omega_{12}{}')-m_q^2]/2 \\
&-[(m_q+2\omega_{12})(m_q+2\omega_{12}{}')-m_q^2][D_{12}/2\omega_{12}+D_{12}{}'/2\omega_{12}{}']/2 \\
&-\frac{1}{8}\frac{D_{12}D_{12}{}'}{\omega_{12}\omega_{12}{}'}+k^4-k^2[D_{12}/\omega_{12}+D_{12}{}'/\omega_{12}{}']
\end{aligned} \tag{A.2}$$

Using eq.(4.9), the integration over $\hat{q}_{12}$ and $\hat{q}'_{12}$ can be done independently of each other, and thus can be written in a compact notation as follows

$$\delta M^\gamma = \sum_{123}(+\frac{2}{9}e^2\tau_3)\int \hat{p}_3^2 d\hat{p}_3 \frac{(m_q+\omega_3)}{2\omega_3}F_1\ e^{[-\hat{p}_3^2/\beta^2]} \tag{A.3}$$

where

$$\begin{aligned}
F_1 \;=\;& J_{11}J_{11}+[M\omega_3-\frac{1}{2}M^2+\frac{1}{2}m_q^2](2J_{20}J_{00}+2J_{10}J_{10})-2J_{20}I_{00} \\
&-2J_{10}I_{10}+J_{11}{}'J_{11}{}'/2-J_{01}J_{01}m_q^2/2-I_{10}{}'J_{10}{}' \\
&+m_q^2I_{00}J_{00}-I_{00}I_{00}/2-J_{02}J_{02}/4+I_{01}I_{01}
\end{aligned} \tag{A.4}$$

and with $(n=0,1,2; m=0,1,2)$,

$$J_{nm};I_{nm}=2^{-1/2}\int \hat{q}_{12}d\hat{q}_{12}\ e^{(-\frac{2}{3}\hat{q}^2_{12}/\beta^2)}[\sqrt{2}\hat{q}_{12}]^m(m_q+\omega_{12})^n[1;\frac{1}{2}D_{12}/\omega_{12}]; \tag{A.5}$$

$$J_{nm}{}';I_{nm}{}'=2^{-1/2}\int \hat{q}_{12}d\hat{q}_{12}\ e^{(-\frac{2}{3}\hat{q}^2_{12}/\beta^2)}[\sqrt{2}\hat{q}_{12}]^m(m_q+2\omega_{12})^n[1;\frac{1}{2}D_{12}/\omega_{12}]; \tag{A.6}$$

Appendix B: Derivation of the qqq Vertex Structure, Eq.(2.5)

B.1: Method of Green's Functions

We outline here some essential steps leading to a formal derivation of eq.(2.5) which was written down in a semi-intuitive fashion in [13]. To that end we shall employ the method of Green's functions for 2- and 3- particle scattering *near the bound state pole*, since the inhomogeneous terms are not relevant for our purposes. For simplicity we shall consider identical *spinless* bosons, with pairwise BS kernels under CIA conditions [12], first for the 2-body case for calibration, and then for the 3-body system.

B.2: Two-Quark Green's Function

Apart from some results already goven in the text, we shall use the notation and phase conventions of [12,13] for the various quantities (momenta, propagators, etc). The 4D qq Green's function $G(p_1p_2;{p_1}'{p_2}')$ near a *bound* state satisfies a 4D BSE without the inhomogeneous term, viz. [12,13],

$$i(2\pi)^4 G(p_1p_2;{p_1}'{p_2}') = {\Delta_1}^{-1}{\Delta_2}^{-1}\int d{p_1}''d{p_2}'' K(p_1p_2;{p_1}''{p_2}'')G({p_1}''{p_2}'';{p_1}'{p_2}') \quad \text{(B.2.1)}$$

where

$$\Delta_1 = {p_1}^2 + {m_q}^2, \quad \text{(B.2.2)}$$

and m_q is the mass of each quark. Now using the relative 4- momentum $q = (p_1 - p_2)/2$ and total 4-momentum $P = p_1 + p_2$ (similarly for the other sets), and removing a δ-function for overall 4-momentum conservation, from each of the G- and K- functions, eq.(B.2.1) reduces to the simpler form

$$i(2\pi)^4 G(q.q') = {\Delta_1}^{-1}{\Delta_2}^{-1}\int d\hat{q}'' M d\sigma'' K(\hat{q},\hat{q}'')G(q'',q') \quad \text{(B.2.3)}$$

where $\hat{q}_\mu = q_\mu - \sigma P_\mu$, with $\sigma = (q.P)/P^2$, is effectively 3D in content (being orthogonal to P_μ). Here we have incorporated the ansatz of a 3D support for the kernel K (independent of σ and σ'), and broken up the 4D measure dq'' arising from (2.1) into the product $d\hat{q}''Md\sigma''$ of a 3D and a 1D measure respectively. We have also suppressed the 4-momentum P_μ label, with ($P^2 = -M^2$), in the notation for $G(q.q')$.

Now define the fully 3D Green's function $\hat{G}(\hat{q},\hat{q}')$ as [29]

$$\hat{G}(\hat{q},\hat{q}') = \int\int M^2 d\sigma d\sigma' G(q,q') \quad \text{(B.2.4)}$$

and two (hybrid) 3D-4D Green's functions $\tilde{G}(\hat{q},q')$, $\tilde{G}(q,\hat{q}')$ as

$$\tilde{G}(\hat{q},q') = \int M d\sigma G(q,q'); \tilde{G}(q,\hat{q}') = \int M d\sigma' G(q,q'); \quad \text{(B.2.5)}$$

Next, use (B.2.5) in (B.2.3) to give

$$i(2\pi)^4 \tilde{G}(q,\hat{q}') = {\Delta_1}^{-1}{\Delta_2}^{-1}\int dq'' K(\hat{q},\hat{q}'')\tilde{G}(q'',\hat{q}') \quad \text{(B.2.6)}$$

Now integrate both sides of (B.2.3) w.r.t. $Md\sigma$ and use the result [12]

$$\int M d\sigma {\Delta_1}^{-1}{\Delta_2}^{-1} = 2\pi i D^{-1}(\hat{q}); \quad D(\hat{q}) = 4\hat{\omega}(\hat{\omega}^2 - M^2/4); \quad \hat{\omega}^2 = {m_q}^2 + \hat{q}^2 \quad \text{(B.2.7)}$$

to give a 3D BSE w.r.t. the variable $\hat{q}$, while keeping the other variable q' in a 4D form:

$$(2\pi)^3 \tilde{G}(\hat{q},q') = D^{-1}\int d\hat{q}'' K(\hat{q},\hat{q}'')\tilde{G}(\hat{q}'',q') \quad \text{(B.2.8)}$$

Now a comparison of (B.2.3) with (B.2.8) gives the desired connection between the full 4D G-function and the hybrid $\tilde{G}(\hat{q},q')$-function:

$$2\pi i G(q,q') = D(\hat{q}){\Delta_1}^{-1}{\Delta_2}^{-1}\tilde{G}(\hat{q},q') \quad \text{(B.2.9)}$$

Again, the symmetry of the left hand side of (B.2.9) w.r.t. q and q' allows us to write the right hand side with the roles of q and q' interchanged. This gives the dual form

$$2\pi i G(q,q') = D(\hat{q}')\Delta_1{}'^{-1}\Delta_2{}'^{-1}\tilde{G}(q,\hat{q}') \tag{B.2.10}$$

which on integrating both sides w.r.t. $Md\sigma$ gives

$$2\pi i \tilde{G}(\hat{q},q') = D(\hat{q}')\Delta_1{}'^{-1}\Delta_2{}'^{-1}\hat{G}(\hat{q},\hat{q}'). \tag{B.2.11}$$

Substitution of (B.2.11) in (B.2.9) then gives the symmetrical form

$$(2\pi i)^2 G(q,q') = D(\hat{q})\Delta_1{}^{-1}\Delta_2{}^{-1}\hat{G}(\hat{q},\hat{q}')D(\hat{q}')\Delta_1{}'^{-1}\Delta_2{}'^{-1} \tag{B.2.12}$$

Finally, integrating both sides of (B.2.8) w.r.t. $Md\sigma'$, we obtain a fully reduced 3D BSE for the 3D Green's function:

$$(2\pi)^3\hat{G}(\hat{q},\hat{q}') = D^{-1}(\hat{q}\int d\hat{q}''K(\hat{q},\hat{q}'')\hat{G}(\hat{q}'',\hat{q}') \tag{B.2.13}$$

Eq.(B.2.12) which is valid near the bound state pole (since the inhomogeneous term has been dropped for simplicity) expresses the desired connection between the 3D and 4D forms of the Green's functions; and eq(B.2.13) is the determining equation for the 3D form. A spectral analysis can now be made for either of the 3D or 4D Green's functions in the standard manner, viz.,

$$G(q,q') = \sum_n \Phi_n(q;P)\Phi_n^*(q';P)/(P^2+M^2) \tag{B.2.14}$$

where Φ is the 4D BS wave function. A similar expansion holds for the 3D G-function $\hat{G}$ in terms of $\phi_n(\hat{q})$. Substituting these expansions in (B.2.12), one immediately sees the connection between the 3D and 4D wave functions in the form:

$$2\pi i\Phi(q,P) = \Delta_1{}^{-1}\Delta_2{}^{-1}D(\hat{q})\phi(\hat{q}) \tag{B.2.15}$$

whence the BS vertex function becomes $\Gamma = D\times\phi/(2\pi i)$ as found in [12]. We shall make free use of these results, taken as qq subsystems, for our study of the qqq G-functions in Sections 3 and 4.

B.3: 3D Reduction of the BSE for 3-Quark G-function

As in the two-body case, and in an obvious notation for various 4-momenta (without the Greek suffixes), we consider the most general Green's function $G(p_1p_2p_3;p_1{}'p_2{}'p_3{}')$ for 3-quark scattering *near the bound state pole* (for simplicity) which allows us to drop the various inhomogeneous terms from the beginning. Again we take out an overall delta function $\delta(p_1+p_2+p_3-P)$ from the G-function and work with *two* internal 4-momenta for each of the initial and final states defined as follows [13]:

$$\sqrt{3}\xi_3 = p_1 - p_2\ ;\quad 3\eta_3 = -2p_3 + p_1 + p_2 \tag{B.3.1}$$

$$P = p_1+p_2+p_3 = p_1{}' + p_2{}' + p_3{}' \tag{B.3.2}$$

and two other sets ξ_1, η_1 and ξ_2, η_2 defined by cyclic permutations from (B.3.1). Further, as we shall consider pairwise kernels with 3D support, we define the effectively 3D momenta $\hat{p}_i$, as well as the three (cyclic) sets of internal momenta $\hat{\xi}_i, \hat{\eta}_i$, (i = 1,2,3) by [13]:

$$\hat{p}_i = p_i - \nu_i P \ ; \quad \hat{\xi}_i = \xi_i - s_i P \ ; \quad \hat{\eta}_i - t_i P \tag{B.3.3}$$

$$\nu_i = (P.p_i)/P^2 \ ; \quad s_i = (P.\xi_i)/P^2 \ ; \quad t_i = (P.\eta_i)/P^2 \tag{B.3.4}$$

$$\sqrt{3}s_3 = \nu_1 - \nu_2 \ ; \quad 3t_3 = -2\nu_3 + \nu_1 + \nu_2 \ \ (+\text{cyclicpermutations}) \tag{B.3.5}$$

The space-like momenta $\hat{p}_i$ and the time-like ones ν_i satisfy [13]

$$\hat{p}_1 + \hat{p}_2 + \hat{p}_3 = 0 \ ; \quad \nu_1 + \nu_2 + \nu_3 = 1 \tag{B.3.6}$$

Strictly speaking, in the spirit of covariant instantaneity, we should have taken the relative 3D momenta $\hat{\xi}, \hat{\eta}$ to be in the instantaneous frames of the concerned pairs, i.e., w.r.t. the rest frames of $P_{ij} = p_i + p_j$; however the difference between the rest frames of P and P_{ij} is small and calculable [13], while the use of a common 3-body rest frame ($P = 0$) lends considerable simplicity and elegance to the formalism.

We may now use the foregoing considerations to write down the BSE for the 6-point Green's function in terms of relative momenta, on closely parallel lines to the 2-body case. To that end note that the 2-body relative momenta are $q_{ij} = (p_i - p_j)/2 = \sqrt{3}\xi_k/2$, where (ijk) are cyclic permutations of (123). Then for the reduced qqq Green's function, when the *last* interaction was in the (ij) pair, we may use the notation $G(\xi_k\eta_k; \xi_k{}'\eta_k{}')$, together with 'hat' notations on these 4-momenta when the corresponding time-like components are integrated out. Further, since the pair ξ_k, η_k is *permutation invariant* as a whole, we may choose to drop the index notation from the complete G-function to emphasize this symmetry as and when needed. The G-function for the qqq system satisfies, in the neighbourhood of the bound state pole, the following (homogeneous) 4D BSE for pairwise qq kernels with 3D support:

$$i(2\pi)^4 G(\xi\eta; \xi'\eta') = \sum_{123} {\Delta_1}^{-1}{\Delta_2}^{-1} \int d\hat{q}''_{12} M d\sigma_{12}{}'' K(\hat{q}_{12}, \hat{q}''_{12}) G(\xi_3{}''\eta_3{}''; \xi_3{}'\eta_3{}') \tag{B.3.7}$$

where we have employed a mixed notation (q_{12} versus ξ_3) to stress the two-body nature of the interaction with one spectator at a time, in a normalization directly comparable with eq.(B.2.3) for the corresponding two-body problem. Note also the connections

$$\sigma_{12} = \sqrt{3}s_3/2; \quad \hat{q}_{12} = \sqrt{3}\hat{\xi}_3/2; \quad \hat{\eta}_3 = -\hat{p}_3, \quad etc \tag{B.3.8}$$

The next task is to reduce the 4D BSE (B.3.7) to a fully 3D form through a sequence of integrations w.r.t. the time-like momenta s_i, t_i applied to the different terms on the right hand side, *provided both* variables are simultaneously permuted. We now define the following fully 3D as well as mixed (hybrid) 3D-4D G-functions according as one or more of the time-like ξ, η variables are integrated out:

$$\hat{G}(\hat{\xi}\hat{\eta}; \hat{\xi}'\hat{\eta}') = \int\int\int\int ds dt ds' dt' G(\xi\eta; \xi'\eta') \tag{B.3.9}$$

which is S_3-symmetric.

$$\tilde{G}_{3\eta}(\xi\hat{\eta}; \xi'\hat{\eta}') = \int\int dt_3 dt_3{}' G(\xi\eta; \xi'\eta'); \tag{B.3.10}$$

$$\tilde{G}_{3\xi}(\hat{\xi}\eta;\hat{\xi}'\eta') = \int\int ds_3 ds_3' G(\xi\eta;\xi'\eta'); \tag{B.3.11}$$

The last two equations are however *not* symmetric w.r.t. the permutation group S_3, since both the variables ξ, η are not simultaneously transformed; this fact has been indicated in eqs.(B.3.10-11) by the suffix "3" on the corresponding (hybrid) $\tilde{G}$-functions, to emphasize that the 'asymmetry' is w.r.t. the index "3". We shall term such quantities "S_3-indexed", to distinguish them from S_3-symmetric quantities as in eq.(B.3.9). The full 3D BSE for the $\hat{G}$-function is obtained by integrating out both sides of (B.3.7) w.r.t. the *st*-pair variables $ds_i ds_j' dt_i dt_j'$ (giving rise to an S_3-symmetric quantity), and using (B.3.9) together with (B.3.8) as follows:

$$(2\pi)^3 \hat{G}(\hat{\xi}\hat{\eta};\hat{\xi}'\hat{\eta}') = \sum_{123} D^{-1}(\hat{q}_{12}) \int d\hat{q}_{12}'' K(\hat{q}_{12},\hat{q}_{12}'') \hat{G}(\hat{\xi}''\hat{\eta}'';\hat{\xi}'\hat{\eta}') \tag{B.3.12}$$

This integral equation for $\hat{G}$ which is the 3-body counterpart of (B.2.13) for a qq system in the neighbourhood of the bound state pole, is the desired 3D BSE for the qqq system in a *fully connected* form, i.e., free from delta functions. Now using a spectral decomposition for $\hat{G}$

$$\hat{G}(\hat{\xi}\hat{\eta};\hat{\xi}'\hat{\eta}') = \sum_n \phi_n(\hat{\xi}\hat{\eta};P)\phi_n^*(\hat{\xi}'\hat{\eta}';P)/(P^2+M^2) \tag{B.3.13}$$

on both sides of (B.3.12) and equating the residues near a given pole $P^2 = -M^2$, gives the desired equation for the 3D wave function ϕ for the bound state in the connected form:

$$(2\pi)^3 \phi(\hat{\xi}\hat{\eta};P) = \sum_{123} D^{-1}(\hat{q}_{12}) \int d\hat{q}_{12}'' K(\hat{q}_{12},\hat{q}_{12}'') \phi(\hat{\xi}''\hat{\eta}'';P) \tag{B.3.14}$$

Now the S_3-symmetry of ϕ in the $(\hat{\xi}_i, \hat{\eta}_i)$ pair is a very useful result for both the solution of (B.3.14) *and* for the reconstruction of the 4D BS wave function in terms of the 3D wave function (B.3.14), as is done in the subsection below.

B.4: Reconstruction of the 4D BS Wave Function

We now attempt to *re-express* the 4D G-function given by (B.3.7) in terms of the 3D $\hat{G}$-function given by (B.3.12), as the qqq counterpart of the qq results (B.2.12-13). To that end we adapt the result (B.2.12) to the hybrid Green's function of the (12) subsystem given by $\tilde{G}_{3\eta}$, eq.(B.3.10), in which the 3-momenta $\hat{\eta}_3, \hat{\eta}_3'$ play a parametric role reflecting the spectator status of quark #3, while the *active* roles are played by $q_{12}, {q_{12}}' = \sqrt{3}(\xi_3, {\xi_3}')/2$, for which the analysis of subsec.B.2 applies directly. This gives

$$(2\pi i)^2 \tilde{G}_{3\eta}(\xi_3\hat{\eta}_3;{\xi_3}'\hat{\eta}_3') = D(\hat{q}_{12}){\Delta_1}^{-1}{\Delta_2}^{-1}\hat{G}(\hat{\xi}_3\hat{\eta}_3;\hat{\xi}_3'\hat{\eta}_3')D(\hat{q}_{12}'){\Delta_1}'^{-1}{\Delta_2}'^{-1} \tag{B.4.1}$$

where on the right hand side, the 'hatted' G-function has full S_3-symmetry, although (for purposes of book-keeping) we have not shown this fact explicitly by deleting the suffix '3' from its arguments. A second relation of this kind may be obtained from (B.3.7) by noting that the 3 terms on its right hand side may be expressed in terms of the hybrid $\tilde{G}_{3\xi}$ functions vide their definitions (B.3.11), together with the 2-body interconnection between $(\xi_3, {\xi_3}')$ and $(\hat{\xi}_3, \hat{\xi}_3')$ expressed once again via (B.4.1), but without the 'hats' on

η_3 and $\eta_3{}'$. This gives

$$\begin{aligned}(\sqrt{3}\pi i)^2 G(\xi_3\eta_3;\xi_3{}'\eta_3{}') &= (\sqrt{3}\pi i)^2 G(\xi\eta;\xi'\eta') \\ &= \sum_{123}\Delta_1{}^{-1}\Delta_2{}^{-1}(\pi i\sqrt{3})\int d\hat{q}_{12}''Md\sigma_{12}{}''K(\hat{q}_{12},\hat{q}_{12}'')G(\xi_3{}''\eta_3{}'';\xi_3{}'\eta_3{}') \\ &= \sum_{123} D(\hat{q}_{12})\Delta_1{}^{-1}\Delta_2{}^{-1}\tilde{G}_{3\xi}(\hat{\xi}_3\eta_3;\hat{\xi}_3'\eta_3{}')\Delta_1{}'^{-1}\Delta_2{}'^{-1} \qquad \text{(B.4.2)}\end{aligned}$$

where the second form exploits the symmetry between ξ,η and ξ',η'.

At this stage, unlike the 2-body case, the reconstruction of the 4D Green's function is *not yet* complete for the 3-body case, as eq.(B.4.2) clearly shows. This is due to the *truncation* of Hilbert space implied in the ansatz of 3D support to the pairwise BSE kernel K which, while facilitating a 4D to 3D BSE reduction without extra charge, does *not* have the *complete* information to permit the *reverse* transition (3D to 4D) without additional assumptions; see [29] for details. The physical reasons for the 3D ansatz for the BSE kernel have been discussed in detail elsewhere [23,29], vis-a-vis contemporary approaches. Here we look upon this "inverse" problem as a purely *mathematical* one.

We must now look for a suitable ansatz for the quantity $\tilde{G}_{3\xi}$ on the right hand side of (B.4.2) in terms of *known* quantities, so that the reconstructed 4D G-function satisfies the 3D equation (B.3.12) exactly, as a "check-point" for the entire exercise. We therefore seek a structure of the form

$$\tilde{G}_{3\xi}(\hat{\xi}_3\eta_3;\hat{\xi}_3'\eta_3{}') = \hat{G}(\hat{\xi}_3\hat{\eta}_3;\hat{\xi}_3'\hat{\eta}_3')\times F(p_3,p_3{}') \qquad \text{(B.4.3)}$$

where the unknown function F must involve only the momentum of the spectator quark #3. A part of the $\eta_3,\eta_3{}'$ dependence has been absorbed in the $\hat{G}$ function on the right, so as to satisfy the requirements of S_3-symmetry for this 3D quantity [29].

As to the remaining factor F, it is necessary to choose its form in a careful manner so as to conform to the conservation of 4-momentum for the *free* propagation of the spectator between two neighbouring vertices, consistently with the symmetry between p_3 and $p_3{}'$. A possible choice consistent with these conditions is the form (see [29] for details):

$$F(p_3,p_3{}') = C_3\Delta_3{}^{-1}\delta(\nu_3-\nu_3{}') \qquad \text{(B.4.4)}$$

Here $\Delta_3{}^{-1}$ represents the "free" propagation of quark #3 between successive vertices, while C_3 represents some residual effects which may at most depend on the 3-momentum $\hat{p}_3$, but must satisfy the main constraint that the 3D BSE, (B.3.12), be *explicitly* satisfied.

To check the self-consistency of the ansatz (B.4.4), integrate both sides of (B.4.2) w.r.t. $ds_3ds_3{}'dt_3dt_3{}'$ to recover the 3D S_3-invariant $\hat{G}$-function on the left hand side. Next, in the first form on the right hand side, integrate w.r.t. $ds_3ds_3{}'$ on the G-function which alone involves these variables. This yields the quantity $\tilde{G}_{3\xi}$. At this stage, employ the ansatz (B.4.4) to integrate over $dt_3dt_3{}'$. Consistency with the 3D BSE, eq.(B.3.12), now demands

$$C_3\int\int d\nu_3d\nu_3{}'\Delta_3{}^{-1}\delta(\nu_3-\nu_3{}') = 1;(since dt = d\nu) \qquad \text{(B.4.5)}$$

The 1D integration w.r.t. $d\nu_3$ may be evaluated as a contour integral over the propagator Δ^{-1} , which gives the pole at $\nu_3=\hat{\omega}_3/M$, (see below for its definition). Evaluating the residue then gives

$$C_3 = i\pi/(M\hat{\omega}_3);\quad \hat{\omega}_3^2 = m_q{}^2+\hat{p}_3^2 \qquad \text{(B.4.6)}$$

which will reproduce the 3D BSE, eq.(B.3.12), *exactly*! Substitution of (B.4.4) in the second form of (B.4.2) finally gives the desired 3-body generalization of (B.2.12) in the form

$$3G(\xi\eta;\xi'\eta') = \sum_{123} D(\hat{q}_{12})\Delta_{1F}\Delta_{2F}D(\hat{q}'_{12})\Delta_{1F}{}'\Delta_{2F}{}'\hat{G}(\hat{\xi}_3\hat{\eta}_3;\hat{\xi}'_3\hat{\eta}'_3)[\Delta_{3F}/(M\pi\hat{\omega}_3)] \qquad \text{(B.4.7)}$$

where for each index, $\Delta_F = -i\Delta^{-1}$ is the Feynman propagator.

To find the effect of the ansatz (B.4.4) on the 4D BS *wave function* $\Phi(\xi\eta;P)$, we do a spectral reduction like (B.3.13) for the 4D Green's function G on the left hand side of (B.4.2). Equating the residues on both sides gives the desired 4D-3D connection between Φ and ϕ:

$$\Phi(\xi\eta;P) = \sum_{123} D(\hat{q}_{12})\Delta_1{}^{-1}\Delta_2{}^{-1}\phi(\hat{\xi}\hat{\eta};P) \times \sqrt{\frac{\delta(\nu_3 - \hat{\omega}_3/M)}{M\hat{\omega}_3\Delta_3}} \qquad \text{(B.4.8)}$$

From (B.4.8) and eq.(2.1) of the text, we infer the structure of the baryon-qqq vertex function V_3 as given in eq.(2.5) of the text. For a detailed discussion of the significance of this result, vis-a-vis contemporary approaches, see [29].

Figure Captions

Fig.1: Diagrams for BS normalization of Baryon-qqq vertex function. 1(a) shows quark #1 emitting a zero momentum photon ($k = 0$); its last qq interaction was with #2, while #3 is the spectator. 1(b) is the same diagram with the roles of #1 and #2 interchanged. 1(c) denotes schematically two more such pairs of diagrams obtained with cyclical permutations of the indices (123) in pairs. The 4-momenta on the quark lines are shown as used in the text.

Fig.2: Diagrams for the two-point interactions of the quark lines with the mass shift operator $-\delta m\tau_3{}^{(1)}/2$ in place of the photon in fig.1, but otherwise with identical topological correspondence of figs.2(a,b,c) to figs.1(a,b,c).

Fig.3: Diagrams for the e.m. self-energy of the uud (proton) configuration. 3(III) is shown in detail with full momentum markings as employed in the text, and corresponds to quark #3 as the spectator, while the quark lines #1 and #2 are joined by a transverse photon line. Similarly 3(I) and 3(II) correspond to #1 and #2 respectively as spectators in turn. Note that, unlike in fig.1 and fig.2, the interchange of #1 and #2 in fig.3(III) does not give a new configuration.

Note : Due to lack of adequate software for a proper 'DVI' rendering of the three figures above, it has not been found possible to include the same in this file. The inconvenience on this account is regretted. However, any interested reader will find it quite easy to reconstruct the three "quark loop" figures (in each of which there are *three* internal quark lines between two baryon vertex blobs) on the basis of the information supplied in the captions above. As a further guidance, the interested reader may refer to ref. [28] which contains similar figures with two quark lines each between the corresponding vertex blobs of the pseudoscalar mesons concerned.

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