THE THREE-BODY PROBLEM: THIRD GENERATION AND BEYOND?

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IT may be wondered why such an unorthodox title should at all constitute a theme for an article on a physical subject. What is, after all, so special about the "three-body problem"? The reason is partly mathematical, but mainly historical which reveals the peculiar role the three-body problem has played throughout the development of physics, especially since the advent of quantum mechanics. This article is intended to sketch a birds eye view of this underlying infrastructure and its pivotal role through the successive stages of unfolding of the mysteries of atomic, nuclear and subnuclear physics.

A. *The Atomic Three-Body Problem*

From a purely mathematical point of view, the first thought that comes to mind in the context of a three-body problem is its insolubility, whether it is the classical gravitational problem of the Sun-Earth-Moon, or the quantum electromagnetic problem of the neutral helium atom. For, only the corresponding two-body problems (Sun-Earth or hydrogen-atom) admit of "exact" solutions in terms of the mathematical tools developed through the last three centuries. Any problem with more than two "bodies" within a certain physical system necessarily entails approximation methods, albeit with arbitrary numerical precision. This circumstance nevertheless represents a big gap in formal mathematical technology despite the amazing developments of the 3 preceding centuries. This is, of course, on the explicit understanding that the *mathematical framework* within which our physical laws have been given formal expression over the centuries is *the correct one*. For it is conceivable that if the same physical laws had been clothed in a totally different kind of mathematics, perhaps the difference between the exactly soluble two-body problem and the 'insoluble' three-body problem would not have been so sharp.

Granted the inverse square law for electrical and gravitational forces, the sharpness of a formal distinction between solubility and insolubility lies more at the abstract philosophical level than at the practical (numerical) level of physical interest. From the latter point of view, powerful approximation methods should in principle suffice for the three-body problem involving planetory or atomic systems. Of the two such basic methods, (i) perturbative and (ii) variational, the former has all along been a standard tool for the calculation of planetory motion, while the latter found its first non-trivial application in the He-atom problem. In both these cases, the theoretical bases of both gravitation and quantum electrodynamics (QED) are so well founded that a study of the properties of atomic and planetary system consisting of three (or more) bodies is *not* considered likely to "shake" their (ultimate) theoretical foundations. And indeed, upto the sixties, it had almost appeared that atomic physics had reached a state of self satisfied complacency when the (then available) ground and low-lying states of various atoms had seemed to be fully amenable to the variational and/or perturbative methods not only for the 'bulk' effects but also the 'fine-structure' effects due to the higher order QED corrections (Lamb shift, anomalous m-m, etc). Against such a background, any special role for the atomic three-body problem in the sense of providing a *fresh probe* into the theoretical foundations of QED seemed difficult to justify: It looked like just one more system, *albeit* an important one, whose properties would "fall in line" with the accepted framework of QED.

However, this oversimplified perspective on the atomic three-body problem has changed significantly since the sixties. Thanks to certain spectacular experimental developments, especially those of synchrotron radiation and vacuum ultraviolet techniques, the detection of much higher atomic excitations than merely the low lying ones of pre-1960 has opened up new theoretical challenges for understanding hitherto unexplored areas of atomic physics. Indeed, there are now qualitatively *new* atomic phenomena, such as autoionization, intermediate-energy electron-atom scattering, three-body break-up reactions, which have brought to the fore fresh "atomic" motivations for the *physics* of *three-body* systems whose characteristic flavour simply could not have been brought out by the basically two-electron physics of the pre-1960 days. To a lesser extent, three-body motivations in atomic and molecular domains have also come from the vastly improved experimental physics of the upper atmosphere over the past two decades. These phenomena have proved to be an ideal testing ground for certain characteristic features of the atomic three-body problem, incorporating new dynamical degrees of freedom and corresponding quantum numbers. They have lent a new richness to the *dimensions* of comparison between theory and experiment at the *atomic* level, hitherto unavailable through two-body physics. This new physics has gathered momentum through the (independent) development of powerful mathematical tools (far beyond the perturbative and variational), to match the full-fledged rigour of the three-body problem.

In short the three-body problem has brought a new dimension of excitement to bear on the atomic domain by opening up fresh vistas of comparison between theory and experiment. And yet, in fairness to the overall physics perspective, it should perhaps be emphasized that, significant as these developments are at the atomic level, they are nevertheless expected to further "*confirm*", *not* "shake", the theoretical foundations of QED, in as much as no *observable* discrepancy with the latter is likely to emerge from the atomic three-body physics. This contrasts with the situation in the *nuclear* domain where, as will be seen from the following, the three-body problem has had an even bigger role to play, holding in effect, a *major key* to the very understanding of *strong interactions*—the nuclear counterpart of QED for atomic interactions. It should also be recorded in fairness to history that the *theoretical* renaissence in the atomic three-body problem (including the development of mathematical tools), owes its origin to the corresponding *nuclear* problem where the theoretical game was the first to start.

B. *The Nuclear Three-Body Problem*

The nuclear-three-body-problem which comes logically at the next deeper level of hierarchy of forces, however, assumes a totally different perspective far beyond the formal (rather superficial) analogy of the nuclear H_3 and He_3 to atomic helium. A qualitative difference arises from the fact that at the nuclear level the effective N-N force is still too empirical, after almost four decades of investigation, compared to its Coulombic partner at the atomic level. This circumstance had once provoked a Churchillian comment to the effect that in no other field had so much effort been directed towards so little end. Bethe had put the state of the art at the nuclear level, more succinctly as a "second-principle-theory" (compared to the first-principle-theory at the atomic level), wherein various nuclear properties and scattering processes must be deduced from an empirical N-N potential. As a result, there is considerable scope for "adjustment" of the potential in response to the compulsions of "agreement with observations". Unfortunately this philosophy had been so much in vogue for so many years that till very recently, the be-all and end-all of (theoretical) nuclear physics used to be attempts to find "bigger and better" potentials for successively improved (parametric) fits to the data. Indeed for a long time (at least up to the early sixties) there had hardly seemed to be any physically worthwhile motivation for such excercises until the nuclear-three-body problem showed the way.

What was the perspective on physics which the nuclear-three-body-problem tried to provide us? Now that the dust has somewhat settled after the hectic activities through the sixties and seventies, it is possible to answer this question more objectively without necessarily following the details of the historical trend. The answer lies basically in that, while the two-body problem carries information about the interaction mainly at a large separation, the three-body-problem incorporates a rich degree of information corresponding to small inter-particle distances as well. Now such information about, say, the atomic three-body system is merely of academic interest since there never had existed any doubt on the basic soundness of the e.m. interaction anyway. On the other hand, such information is almost vital at the nuclear level for testing the quality of the input N-N interaction, since the two-body problem which merely checks the long distance part of this interaction is not sensitive enough to discriminate among a large number of candidates. In other words, the "inside-out" philosophy characterizing the e.m. interaction for atomic systems is no longer operative at the nuclear level, and must be replaced by an "outside-in" strategy wherein the empirical N-N interaction must be continually sharpened in response to the finer and finer observational features of nuclear systems beyond the relatively insensitive two-body problem. In particular, the nuclear 3-body problem (and few-body-problems in general) provides powerful constraints on the (parametric) structure of the N-N interaction unavailable from mere two-body data.

Role of "Exact" Wave Functions

How does one give shape to such a philosophy in concrete terms? Since the wave function is the repository of the physical information on any system, the main issue ultimately boils down to the determination of the wave function for the system concerned. This is more easily said than done, for the wave function must be determined not in a "casual" (variational) manner, but in a *causal*, deductive fashion which should directly reflect the precise quantitative nature of the input N-N interaction. This is a tall order which cannot be implemented with any arbitrary shape of the N-N input: One would need a reasonably simple form for the latter for a practicable three-body programme with an "exact" wave function.

Initially, the so-called "separable potential" seemed to fit the bill rather ideally. Since it had been known to solve the two-body problem exactly and give a realistic description of several sensitive two-body data, it came in very handy and indeed was found to reduce the three-body to an effective two-body level. This was a great practical simplification in the context of the (then available) computer technology. Though a general theoretical formulation of the three-body problem in terms of two-body "reaction matrices" (an observational derivative of the "potentials") had been simultaneously available, it was soon found to reduce to the effective separable potential description for a practical implementation of the new philosophy of doing three-body physics with *exact* wave functions. This movement became very strong in the sixties, and as a three-body calculational programme, soon registered several notable success through comparison with experimental data.

Meson-Exchange Potentials

However, by the late sixties it was time to realise that the separable potential, despite its practical successes, could not be the final answer to the quest for N-N potentials which must eventually be based on field-theoretic premises, and had better be "local" in character so as to be amenable to an Yukawa-type interpretation based on exchanges of suitably heavy quanta. This was *not* a new idea, and had indeed been known since the forties but no one had "dared" to give serious thought to its implementation for nearly 3 decades. Now, with the proven success of the exact-wave-function philosophy, albeit with separable potentials, and with the rapid strides in computer technology in the sixties, the "separable potential" logically gave way to the separable approximation to various "local" potentials, of which the long-range part (pion-exchange) was quite unambiguous. However their short-range aspects still needed effective parametrization, despite the availability of a variety of vector mesons (ρ, ω, ϕ) as theoretical candidates for heavy quanta exchanges. These approaches, while logically sound, did not register success commensurate with the (numerical) efforts employed. The programme has continued through the seventies almost on an industrial scale, but the lessons of physics which had been learnt by the mid-sixties have remained unchanged: The exact three-body problem offers a very rich probe into the structure of the two-nucleon interaction whose choice in turn must be decided on independent (preferably field-theoretical) grounds. In retrospect the separable potential seems unwittingly to have played a crucial historical role in conveying this (otherwise obvious) message.

C. *The Quark-Three-Body Problem*

The sixties saw yet another major development—perhaps the biggest one of the century *viz*., the revolutionary proposal concerning the quark constitutions of baryons (qqq) and mesons $q\bar{q}$. (Since in many ways an antiquark ($\bar{q}$) is equivalent to a diquark (qq) system, the two ideas are in some sense dual to each other). The significance of the quark idea to the three-body problem goes far beyond the simple arithmetic of quark constitution of hadrons, and reveals the growing importance of the three-body problem as one traverses inwards from atomic to nuclear to subnuclear. Indeed, we have seen that already from the first (atomic) stage to the second (nuclear), the three-body system has grown from a crucial dynamical system rich with new exotic phenomena, to an even more active ingredient controlling the inner dynamics of nuclear interactions. Now at the subnuclear level, the three-quark system is the lowest unit, short of which, quarks do not have a physical existence! At this level, the physics is controlled by an entirely different scenario: there is a hidden quantum number (color) which has a few highly indirect (nevertheless genuine) observational signatures, but which must be held together or confined by a minimum of three quarks (and/or quark-antiquark pair). The understanding of the confining (gluonic) forces is still highly empirical—only their short-range aspects are somewhat understood—but it nevertheless brings out the vital role of the three-body structure which has by now become an essential ingredient of the underlying physics!

The quark-level three-body physics, coming as it did during the peak years of nuclear three-body activity, did enjoy most of the technical facilities (angular momentum reduction, classification of quantum numbers, etc.) then being developed at the nuclear level, but there had to be important differences. *First*, because of the confinement requirement, the three-quark problem is necessarily a bound-state problem, and the simplest form of q-q interaction to fit in with that scenario is the harmonic interaction which solves not only the three-body problem but also the N-body problem with equal ease. In more recent times, the form of the confining interaction has been gravitating toward the linear form (on the basis of guage theories), necessitating a more conscious use of standard three-body techniques (hyperspherical coordinates, etc.). "Lattice" theories which are also getting quite popular in the context of confinement are however, characterized by rather different ("plaquette") techniques, having little in common with Schrödinger or Bethe-Salpeter languages for more orthodox three-body systems.

Quark-Perspective on Meson Exchanges

The development of quark-level dynamics has also led to a reappraisal of what should properly constitute a more fundamental approach to the problem of nuclear-interactions. Thus while the original effective forms of N-N interaction did not lose much ground (there was not much to lose in view of their "second principle nature any way), the meson-exchange potentials, which had hitherto been acquiring the "illusory status" of a more fundamental approach to the N-N interaction, started losing their "Rebecca spell" in the face of the quark-onslaught. For, in the (microscopic) background of the $q\bar{q}$ picture, the elementarity of the meson-exchanges could no longer be defended, and had to give way (sooner or later) to a more composite picture based on quark and/or anti-quark exchanges. But this loss of meson supremacy has not come without a fight. The pion-exchange potential has still held its ground firmly and only the short range part based on the heavy meson (ρ, ω, ϕ) exchanges which had already been in trouble, seems to be partly giving way to the $q\bar{q}$ picture: At short distances, the elementarity of the meson and its $q\bar{q}$ compositeness seem to be playing a complementary role in the N-N potential picture, much like the wave-particle duality in quantum mechanics. The role of color as a hidden variable has further confused the picture, so that an early resolution of this duality problem still seems to be a distant goal.

To summarize, the quark picture has, among other things, given a new orientation to the three-body problem as a fundamental physical unit which must be studied in its own right, and has brought to sharp focus the inadequacy of the naive concept of meson-exchange potentials, with the realization that mesons are no longer elementary entities. The photon is perhaps the only particle which has so far managed to preserve its 'elementary' status in the face of the quark challenge.

D. *The Fourth Generation?*

Quarks have never been (nor will be) "seen" explicitly, and yet they have come to stay in a big way. So far they remain the foremost contenders to elementarity, in the company of the photon, the gluon and perhaps the so-called "leptons" (electron, muon, neutrino, etc.). However, with the rapid proliferation of quark varieties (flavours) on the one hand (with each flavour coming in three colours), and of lepton types on the other, even this total number of so-called "elementary" entities has reached alarming proportions. So, while "grand unification theories" on the basis of this existing set of fundamental building blocks are being developed on an industrial scale, a parallel movement has been quietly under way with a different kind of goal: to explore the substructures of quarks and leptons! A deeper motivation for such an exercise (apart from considerations of economy of description) comes from the observation of a remarkable similarity of successive quark and lepton generations as indicated below:

$e\ \mu\ \tau;\ \nu_e\ \ \nu_\mu\ \nu_\tau$

$u\ C\ t(?);\ d\ s\ b$

This similarity has led to the belief that both quarks and leptons are probably composites of common subconstituents, and that their successive generations are mere manifestations of some suitable quantum numbers arising out of the composite description. These ideas are still in their infancy and, so far, no general consensus has emerged for giving a unique mathematical formulation as a common denominator to most contemporary points of view. The only common denominator in this regard is that if such substructures should exist at all, they must necessarily be fermions and hence constitute a three-body-problem at the minimum (since the alternative of a one-body problem would be a trivial paraphrase of the original leptons and quarks themselves. And this now brings us to the fourth *generation* three-body problem with a far more active dynamical role for the underlying triplet at this (deepest?) level than we have so far been used to, even in terms of the third generation quark level. The future is full of promise for the three-body-problem.