

Parametrization of the quark mixing matrix involving its eigenvalues

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Abstract

A parametrization of the 3×3 Cabibbo-Kobayashi-Maskawa matrix, V , is presented in which the parameters are the eigenvalues and the components of its eigenvectors. In this parametrization, the small departure of the experimentally determined V from being moduli symmetric (i.e. $|V_{ij}| = |V_{ji}|$) is controlled by the small difference between two of the eigenvalues. In case, any two eigenvalues are equal, one obtains a moduli symmetric V depending on only three parameters. Our parametrization gives very good fits to the available data including CP-violation. Our value of $\sin 2\beta \approx 0.7$ and other parameters associated with the ‘unitarity triangle’ $V_{11}V_{13}^* + V_{21}V_{23}^*V_{31}V_{33}^* = 0$ are in good agreement with data and other analyses.

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1 Introduction

Flavor mixing of the quarks, in the Standard Model, is understood through the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Since the first [1] explicit parametrization of the CKM matrix, V , for three generations, many different parametrizations have been suggested [2, 3]. In all these cases, the mixing matrix V is parametrized in terms of four parameters, three angles and a phase. However, other approaches to the parametrization are possible and available [4, 5, 6].

Any 3×3 unitary matrix V can be expressed in terms of its eigenvalues $E_i = \exp(i\alpha_i)$, $i = 1, 2, 3$ as follows,

$$V = W\hat{V}W^\dagger, \quad (1)$$

where $\hat{V} = \text{diag}(E_1, E_2, E_3)$ and the diagonalizing matrix W is unitary. In reference [4], arguments were presented that due to re-phasing freedom, the eigenvalues can be chosen at will and there they were fixed to be the three roots of unity, so that V depended on the four parameters needed to specify W . For confrontation of data, W was chosen to depend on only two parameters which resulted in a symmetric V (i.e., $V_{ij} = V_{ji}$) and which gave a reasonable fit to the data available at that time.

In the approach of reference [6], the CKM matrix was parametrized as

$$V(\theta) = \cos \theta I + i \sin \theta U. \quad (2)$$

The parameter θ determines the relative importance of the trivial part, I *vis a vis* the non-trivial part U . The hermitian and unitary U (independent of θ) depends on two real positive parameters [7]. Since $U = U^\dagger$, V is moduli symmetric (i.e. $|V_{ij}| = |V_{ji}|$). Such a matrix can always be made symmetric by rephasing [7, 8] and in general has only three parameters. Such a V gave a good fit to the available data [3] though its predictions for ρ, η and $\sin 2\beta$ were on the larger side compared to the recently available data [3].

In this paper we consider a parametrization of V based on Eq(1) for general eigenvalues, (that is as explicit parameters), even though it is clear that they have no physical significance. As we shall see, such a parametrization exhibits different features of the mixing matrix not accessible otherwise. In particular, we also are motivated to have the eigenvalues as explicit parameters because there may be an underlying connection between the eigenvalues of the mixing matrix in the quark and lepton sectors. The first hint of this came from the application of the approach of Eq(2) to the neutrino mixing matrix V_ν [9]. Writing

$$V_\nu(\theta_\nu) = \cos \theta_\nu I + i \sin \theta_\nu U_\nu, \quad (3)$$

one finds that the maximal mixing of ν_μ and ν_τ (indicated by the atmospheric neutrino data [9]) requires $\theta_\nu = \pi/4$. The fit to the CKM data gave $\theta = \pi/4!$. The remarkable equality $\theta = \theta_\nu = \pi/4$ suggests an underlying quark-lepton symmetry in this approach, even though the full V and V_ν are very different. The really interesting point is to realize that in these parametrizations the parameters θ and θ_ν completely determine the eigenvalues of V and V_ν respectively! In fact, the actual eigenvalues of V in Eq(2) are $exp(i\theta)$, $exp(-i\theta)$ and $exp(-i\theta)$ while the two real parameters in U determine the corresponding eigenvectors. This applies *mutatis mutandis* to V_ν . In general, if two eigenvalues are equal it follows that V can depend on at most three parameters and can be made symmetric (see Section II below and [8]).

By considering the general case, when the three eigenvalues are different, we can obtain a V which is ‘asymmetric’ (i.e. $|V_{ij}| \neq |V_{ji}|$). Experimentally, this asymmetry is quite small and in fact the major part of V is indeed given by the parametrization of the form Eq(2). In the ‘eigenvalue parametrization’ which we consider the small asymmetry is contributed by the small difference between two eigenvalues. By confronting this and other possible ways of parametrizing the CKM matrix one can hope to obtain a better understanding of the nature and structure of the quark mixings.

In Section II, Eq(1) is considered in detail and the general notation, formulae and their consequences are given. In Section III we consider the confrontation of the eigenvalue parametrization with data for simplified choices of W . Numerical results for the fits are presented in Section IV. Finally we conclude with a brief summary and remarks in Section V.

2 General eigenvalue parametrization of V

Our starting point is Eq(1). We can write it as

$$V = \sum_{k=1}^3 E_k N_k, \quad (4)$$

where N_k are the ‘projectors’ for V . They satisfy $\sum_{k=1}^3 N_k = I$, $N_k = N_k^\dagger$, $N_k N_{k'} = N_k \delta_{kk'}$ and $(N_k)_{lm} = W_{mk}^* W_{lk}$, $l, k = 1, 2, 3$ where W_{lk} are matrix elements of the matrix W . We can choose the overall phase of V , in general, so that on eliminating N_3 , we have

$$V = I + F_1 N_1 + F_2 N_2, \quad (5)$$

where

$$F_i = (E_i - 1) = (\exp(i\alpha_i) - 1), i = 1, 2. \quad (6)$$

The choice $\alpha_3 = 0$ or $E_3 = 1$ has been made in Eqs(5, 6). The columns 1 to 3 of W are the orthonormal eigenvectors of V for eigenvalues E_1 to E_3 . Consequently we can write the hermitian projection matrices as

$$N_k = \begin{pmatrix} c_k \\ b_k \\ a_k \end{pmatrix} \otimes (c_k^*, b_k^*, a_k^*) = \begin{pmatrix} |c_k|^2 & c_k b_k^* & c_k a_k^* \\ b_k c_k^* & |b_k|^2 & b_k a_k^* \\ a_k c_k^* & a_k b_k^* & |a_k|^2 \end{pmatrix}, \quad (7)$$

where we have introduced the compact notation $(W_{1k}, W_{2k}, W_{3k}) = (c_k, b_k, a_k)$ for $k = 1, 2, 3$. Our parametrization is based on Eqs(5 – 7). It is the generalization of Eq(2) to which Eq(5) reduces when two eigenvalues of V are equal.

In Eq(5), V depends on two eigenvalues E_1 and E_2 or equivalently on the real parameters α_1 and α_2 . The matrices N_1 and N_2 seemingly depend on six complex numbers a_k, b_k, c_k , $k = 1, 2$. However, by simple rephasing we can make N_1 real, that is, take a_1, b_1 and c_1 to be real and positive. Further, we can choose one (non-zero) component of the eigenvector for E_2 to be real. We choose c_2 to be real. The unitarity of W (or the orthonormality of the eigenvectors) gives us

$$a_1 a_2^* + b_1 b_2^* + c_1 c_2^* = 0, \quad (8)$$

$$|a_1|^2 + |b_1|^2 + |c_1|^2 = 1, \quad (9)$$

$$|a_2|^2 + |b_2|^2 + |c_2|^2 = 1. \quad (10)$$

These equations show that each N_1 and N_2 depends on two real parameters or W depends on four real parameters. These can, for example, be taken to be $|a_1|, |b_1|$ and $|a_2|, |b_2|$. Note that Eq(8) will determine the real and imaginary parts of a_2 and b_2 (c_1, c_2 being real). Thus, V in Eq(5) depends seemingly on six parameters, α_1 and α_2 which determine the eigenvalues plus the four in N_1 and N_2 which determine the corresponding eigenvectors.

Before confronting data, we discuss some general consequences of Eq(5) pertaining to the asymmetry of V . For a unitary matrix the departure from moduli symmetry (i.e. $|V_{ij}| = |V_{ji}|$) is conveniently given by the formula

$$\begin{aligned} \Delta(V) &\equiv |V_{12}|^2 - |V_{21}|^2 = |V_{23}|^2 - |V_{32}|^2 = |V_{31}|^2 - |V_{13}|^2 \\ &= -16 \sin\left(\frac{\alpha_1 - \alpha_2}{2}\right) \sin\left(\frac{\alpha_1 - \alpha_3}{2}\right) \sin\left(\frac{\alpha_2 - \alpha_3}{2}\right) J(W), \end{aligned} \quad (11)$$

where

$$J(W) = \text{Im}(W_{11} W_{21}^* W_{12}^* W_{22}) = \text{Im}(c_1 b_1^* c_2^* b_2), \quad (12)$$

is the Jarlskog [10] invariant for the matrix W in our notation.

This formula (see also [8]) determines the conditions when V is moduli symmetric.

(i) If $J(W) = 0$ then $|V_{ij}| = |V_{ji}|$ even though all the eigenvalues are different. In our choice of parameters above, since a_1, b_1, c_1 and c_2 are real, it is imperative that a_2 and b_2 have an imaginary part so that $J(W) \neq 0$ and Eq(5) would give an asymmetric V .

(ii) If any two eigenvalues of V are equal then $|V_{ij}| = |V_{ji}|$ even though $J(W) \neq 0$. If we take $E_2 = E_3 = 1$ i.e. $\alpha_2 = \alpha_3 = 0$ and put $\alpha_1 = 2\theta$ then with a little manipulation Eq(2) can be written as $V(\theta) \exp(i\theta) = I + (F_1 - 1)N_1$ where the parameters a, b, c of reference[6] are written as $a = -i|a_1| \exp[i(\phi_{b_1} - \phi_{c_1})]$, $b = i|b_1| \exp[i(\phi_{a_1} - \phi_{c_1})]$, $c = -i|c_1| \exp[i(\phi_{a_1} - \phi_{b_1})]$.

The main point is that whenever two eigenvalues are equal we can write the unitary matrix V as $V = I + \lambda N$ where $\lambda = 2i \sin \theta \exp(i\theta)$ and $N = N^\dagger = N^2$ [11].

It is very interesting that from the eigenvalue parametrization approach, the parametrization of Eq(2, 3) turns out to be a particular case of Eq(5) even though the original motivation for Eq(2, 3) was quite different. Furthermore, the interesting parameters θ and θ_ν turn out to be eigenvalues of the two mixing matrices V and V_ν respectively.

In the next section we go on to confront Eq(5 – 7) with available data which shows that there is a small asymmetry in V , that is $\Delta(V) \neq 0$ though small.

3 Numerical Results

Experiments can only determine $|V_{ij}|$ for us. Since V is unitary, four independent moduli are sufficient to determine all the nine $|V_{ij}|$. This implies that we can have many different parametrizations of the complete complex matrix V as long as they give the same $|V_{ij}|$ in agreement with experiments.

The Particle Data Group gives experimentally determined ranges for $|V_{ij}|$. One can convert these ranges into a central value with errors and use these for fitting. This procedure has a draw back that the unitarity constraints in the moduli are not exact. Instead we use the ‘standard’ parametrization (Eq(11.3), Section 11 in [3]) to fit the moduli. Accordingly, we take [3], $s_{12} = 0.2229 \pm 0.0022$, $s_{23} = 0.0412 \pm 0.0020$ and $s_{13} = 0.0036 \pm 0.0007$ with $\delta_{13} = 59^\circ \pm 13^\circ = (1.02 \pm 0.22)$ radians. This gives the moduli matrix $V_{mod} = (|V_{ij}|)$ to be

$$V_{mod} = \begin{pmatrix} 0.974835 \pm 0.000503 & 0.222899 \pm 0.002199 & 0.0036 \pm 0.0007 \\ 0.222786 \pm 0.002198 & 0.973996 \pm 0.000509 & 0.0411997 \pm 0.001999 \\ 0.00793254 \pm 0.000877 & 0.0405888 \pm 0.0019569 & 0.999144 \pm 0.0000825 \end{pmatrix}. \quad (13)$$

We confront the central values to determine the parameters in our parametrization. The advantage of using Eq(13) is that the unitarity constraints on $|V_{ij}|$ are satisfied.

As noted earlier, our parametrization in Eq(5) has six real parameters while the data gives only four independent inputs, namely four $|V_{ij}|$'s. There are two obvious ways to reduce the parameters.

[A] Keep the two eigenvalues of V (i.e. α_1 and α_2) as parameters and choose a W with only two parameters. That is, the corresponding eigenvectors are determined by only two real numbers. We explore this possibility here. We will see that a small α_2 (implying the eigenvalue $E_2 \approx 1 - i\alpha_2$ is near $E_3 = 1$) controls the asymmetry in V .

[B] The other way is to arbitrarily choose the eigenvalues in advance (e.g. in [4]) and then determine the parameters which determine the eigenvectors of V . We will consider this briefly for sake of comparison.

Type A fits

In these fits the W used has only two parameters which determine the eigenvectors of V . To obtain such a W we consider a general four parameter W and reduce the parameters to two guided by notions of simplicity.

Case(i) We start with a W parametrized by three angles $\theta_{12}, \theta_{13}, \theta_{23}$ and a phase δ_{13} as in Eq(11.3), Section 11 of [3]. If one requires that W be moduli symmetric i.e. $|W_{ij}| = |W_{ji}|$ then one needs two conditions, namely $\theta_{13} = \theta_{23}$ and $-2 \cos \delta_{13} = \cot \theta_{12} \tan \theta_{23} \sin \theta_{23}$. This results in a two parameter W which we refer to as W_P . Using W_P and the two parameters α_1 and α_2 from the eigenvalues we make a four parameter fit to the $|V_{ij}|$ given in V_{mod} in Eq(13). One obtains an excellent fit, the numerical values of the parameters are given in Table I. Note that in this case only c_1 and c_2 are real but $a_i, b_i, i = 1, 2$ are complex. Consequently both the matrices N_1 and N_2 are complex though given in terms of two real parameters.

Case(ii) In this case, we consider the parametrization of W a la Kobayashi-Maskawa ([1] or Eq(11.4) in Section 11 of [3]). Here W becomes moduli symmetric if one simply takes $\theta_2 = \theta_3$, so to reduce the parameters to two we make the choice, $\delta = \pi - \theta_1$. This gives $\cos \delta = -\cos \theta_1$ and $\sin \delta = \sin \theta_1$. This two parameter W , denoted by W_{KM} is quite simple

$$W_{KM} = \begin{pmatrix} C_1 & -S_1 C_2 & -S_1 S_2 \\ S_1 C_2 & C_1 - i S_2^2 S_1 & i S_1 C_2 S_2 \\ S_1 S_2 & i S_1 C_2 S_2 & C_1 - i C_2^2 S_1 \end{pmatrix}, \quad (14)$$

where $C_i \equiv \cos \theta_i, S_i \equiv \sin \theta_i, i = 1, 2$. This case, has the feature that

Re $a_2 = 0$. The numerical values of the parameters $a_1 = S_1 S_2$, $a_2 = i S_1 C_2 S_2$ etc are given in Table I. Note that except for a_2 and b_2 others are real, so that N_1 is real but N_2 is not. Again the fit to $|V_{ij}|$ is excellent and the value of the parameters in the two fits differ only slightly.

The main contribution to the CP-violation parameter $J(V) = \text{Im}(V_{11} V_{22} V_{12}^* V_{21}^*)$ comes from the first two terms of Eq(5) which give a moduli symmetric V (the limit α_2 or $F_2 \rightarrow 0$). In fact, for the complete V , $J(V) = 2.744 \times 10^{-5}$ while it becomes equal to 2.87×10^{-5} when $\alpha_2 = 0$. The small value of $\alpha_2 = -0.106365$ radians (compared to $\alpha_1 = 1.88053$ radians) gives the small asymmetry, $\Delta(V) = 5 \times 10^{-5}$.

Type B fits

In these fits one specifies or chooses the eigenvalue parameters α_1 and α_2 and then determines the four parameters in W from the data. These can be chosen to be $|a_i|, |b_i|, i = 1, 2$. These determine the imaginary parts of a_2 and b_2 through Eq(8) since c_2 is real. For comparison with above results, we choose $\alpha_1 = -\alpha_2 = 120^\circ$ following reference[4]. The choice of α_1 and α_2 can not be completely arbitrary because of the numerical values of $|V_{ij}|$ determined experimentally. For example, the choice[4] $\text{Tr}V = 0$ implies inequalities like $||V_{22}| - |V_{33}|| \leq |V_{11}| \leq |V_{22}| + |V_{33}|$ etc.. These are satisfied by the data but will not be valid for every unitary matrix. The numerical results are given in Table I. The fit is as good as Type A fit but there is no obvious reason why $\Delta(V)$ is so small because the contribution of $F_1 N_1$ and $F_2 N_2$ are of the same order. Note that $|F_1| = |F_2|$ and $|(N_1)_{ij}| \approx |(N_2)_{ij}|$. The phases of $(N_k)_{ij}$ conspire in some way to give a small asymmetry.

In conclusion, we consider the Type A fits to be more meaningful as they display the structure of V , that it is mainly moduli symmetric with a small parameter monitoring the small asymmetry. As can be seen from Table I the parameters a_i, b_i, c_i for the type A case are very similar, particularly their moduli. In fact, $|a_2| \approx |a_1|$, $|b_2| \approx |c_1|$ and $|c_2| \approx |b_1|$, so that, the matrix elements of N_1 and N_2 are comparable. However, their coefficients $F_1 = (-1.3048 + 0.952415 i)$ and $F_2 = (-0.00565138 - 0.106164 i)$ are very different. Since $|F_1| \approx 1.6$ and $|F_2| \approx 0.1$, the $I + F_1 N_1$ part gives the major contribution to V while the much smaller $F_2 N_2$ contributes to give a small asymmetry.

4 Predictions for the parameters of the unitarity triangle

The unitarity constraint

$$V_{11}V_{13}^* + V_{21}V_{23}^* + V_{31}V_{33}^* = 0, \quad (15)$$

can be written as $z_1 + z_2 + z_3 = 0$ where $z_i = V_{i1}V_{i3}^*$, $i = 1, 2, 3$. The angles of this triangle, in standard notation, are $\alpha = \arg(-z_3/z_1)$, $\beta = \arg(-z_2/z_3)$ and $\gamma = \arg(-z_1/z_2)$. These can be determined directly from our fits and are given in Table II. In addition, the values of ρ and η (defined as $-z_1/z_2 = \rho + i\eta$) are also given. They are connected to the angles through

$$\sin \alpha = \frac{\sin \beta}{\sqrt{\rho^2 + \eta^2}} = \frac{\sin \gamma}{\sqrt{(1 - \rho)^2 + \eta^2}}; \quad \tan \gamma = \eta/\rho. \quad (16)$$

The ρ and η defined here and Eq(16) are valid for any exact parametrization of the CKM matrix.

Since we fit $|V_{ij}|$ given in Eq(13), that is, the inputs for all the three fits are the same, we obtain the same values for the angles, ρ and η in all the cases, These are given in column 1, Table II. These values are to be compared to those obtained by the Particle Data Group [3], namely, $\beta = 24^\circ \pm 4^\circ$, $\gamma = 59^\circ \pm 13^\circ$ and $\bar{\rho} = 0.22 \pm 0.10$ and $\bar{\eta} = 0.35 \pm 0.05$. There is a very minute numerical difference between (ρ, η) and $(\bar{\rho}, \bar{\eta})$. For the definition of the latter see reference [12]. The agreement between their and our values is quite satisfactory. For comparison, the second column of Table II, gives the values obtained when $\alpha_2 = 0$, that is, when two eigenvalues are equal, $E_2 = E_3 = 1$ and V is symmetric. As one can see the values of η and particularly ρ are higher. Also, there is a change in the values of the angles by about $10 - 20^\circ$. This change can be seen more clearly in the values of $\sin 2\beta$ in the two cases. For the symmetric case, $\sin 2\beta = 0.9532$ compared to $0.6988 \approx 0.7$ for the asymmetric (or actual) V . These are to be compared to the measured [13] value $\sin 2\beta = 0.78 \pm 0.08$. Our value 0.7 is in reasonable agreement. Experiments are in progress both at Belle and BaBar for a better value of $\sin 2\beta$ and to measure $\sin 2\alpha$. We should have a clearer picture in a couple of years.

5 Summary and final remarks

In the parametrization considered here the parameters directly determine the mathematical structure of the CKM matrix V , namely its eigenvalues E_i

and its eigenvectors. Such a parametrization brings out the fact that experimentally V is practically moduli symmetric, the small asymmetry observed is due to two eigenvalues being very close to each other. It is intriguing to note that smallness of the asymmetry measured by $\Delta(V) = 5.00 \times 10^{-5}$ is of the same order as $J(V) = 2.74 \times 10^{-5}$, even though, in general, they are not related.

Finally, extension of this approach to the lepton sector in the near future would be of much interest. Furthermore our approach can be easily extended for the case of four or more generations.

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- [11] This is obvious from the ‘Projector Representation’ (PR) in Eq(4) or Eq(5). In fact the PR for the $n \times n$ case tells us that V is moduli symmetric and depends on n real parameters if it has only two independent eigenvalues. For ‘asymmetry’ it should have at least three or more different eigenvalues. For example, for $n = 4$, V is moduli symmetric for the cases $E_1 \neq E_2 = E_3 = E_4$ and $E_1 = E_2 \neq E_3 = E_4$ but can be asymmetric $E_1 \neq E_2 \neq E_3 = E_4$ or if all four eigenvalues are different.
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	Type A Case (i)	Type A Case(ii)	Type B
α_1	107.748°	107.746°	120°
α_2	-6.09428°	-6.09424°	-120°
c_1	0.134232	0.134234	0.129796
b_1	-0.99062 - 0.0000876785 i	0.99062	0.99126
a_1	0.025322 - 0.00343008 i	0.0255536	-0.0235971
c_2	0.99062	-0.99062	0.991539
b_2	0.134232 - 0.000647057 i	0.134234 - 0.00065895 i	-0.129753 + 0.0000376876 i
a_2	-0.00343008 - 0.0253136 i	0.0255451 i	0.00333381 + 0.00158317 i

Table I : Parameters in V (Eq(5)) determined by fitting the central values of $|V_{12}|$, $|V_{21}|$, $|V_{13}|/|V_{23}|$ in Eq(13) as inputs. Note that the eigenvalues E_1 and E_2 (or α_1 and α_2) and the moduli $|a_i|$, $|b_i|$ and $|c_i|$ for the Type A fits are practically the same. We have kept the number of places of decimal as given by the mathematica program. This ensures that unitarity etc. constraints are obeyed exactly and also this shows where slight difference in the values of parameters occurs in the different fits. See text for the definition of the parameters and details.

	Type A	Type A , $\alpha_2 = 0$
ρ	0.200284	0.4911
η	0.325685	0.3719
α	99.43°	106.70°
β	22.16°	36.16°
γ	58.41°	37.13°
$\sin 2\beta$	0.6986	0.953
$\sin 2\alpha$	-0.3232	-0.550

Table II: Values of the angles (α , β , γ) and parameters ρ and η connected with the unitary triangle (Eq(15)) are given. For the Type A fits both Cases (i) and (ii) give practically the same values. Column 1 gives the values for the asymmetric V ($\alpha_2 \approx -6.1^\circ$) while column 2 gives the values for the moduli symmetric case ($\alpha_2 = 0^\circ$).