14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

CRAMER-RAO BOUNDS FOR SOURCE LOCALIZATION IN SHALLOW OCEAN WITH GENERALIZED GAUSSIAN NOISE

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ABSTRACT

Localization of underwater acoustic sources is a problem of great interest in the area of ocean acoustics. There exist several algorithms for source localization based on array signal processing. It is of interest to know the theoretical performance limits of these estimators. In this paper we develop expressions for the Cramer-Rao-Bound (CRB) on the variance of direction-of-arrival (DOA) and range-depth estimators of underwater acoustic sources in a shallow range-independent ocean for the case of generalized Gaussian noise. We then study the performance of some of the popular source localization techniques, through simulations, for DOA/range-depth estimation of underwater acoustic sources in shallow ocean by comparing the variance of the estimators with the corresponding CRBs.

1. INTRODUCTION

Underwater acoustic source localization has numerous commercial as well as military applications. Source localization in ocean involves estimation of three parameters, viz. range, depth and DOA. Most localization techniques are based on the concept of matched field processing (MFP) [1, 2] wherein the measured field at the sensor array is matched with the replicas of the expected field for all possible source locations. The efficiency of different techniques can be assessed by comparing the variance of the estimators with the corresponding Cramer-Rao bounds (CRBs). All available results on CRB for the source localization problem are based on the assumption that the ambient noise is white Gaussian [3]. But, statistical analysis of ambient noise data in shallow coastal ocean indicates that the noise has strong impulsive components (due to snapping shrimp and local shipping activities)[4], and such noise is well modeled by generalized Gaussian distributions with heavy tails [5]. In this paper, the CRB for the problems of DOA and range-depth estimation of underwater acoustic sources in a shallow range-independent ocean are derived for the case of additive complex circular white generalized Gaussian noise. This derivation follows the procedure used by Stoica and Nehorai [6] for deriving the CRB for the problem of plane wave DOA estimation in white Gaussian noise. A simple homogeous model of the ocean, called the Pekeris model[7] which is sufficient to capture most salient features of acoustic propagation in shallow ocean, is used in this work. Performance analysis of four widely used localization techniques, viz. linear beamformer (Bartlett processor), Capon's minimum variance beamformer, MUSIC, and min-norm technique[8] is carried out, and simulation results are presented to compare the variance of these estimators in generalized Gaussian noise with the corresponding CRBs. It is shown that the CRBs under heavy-tailed generalized Gaussian noise are significantly lower than the corresponding CRBs under Gaussian noise.

This work was supported by the Defense Research and Development Organisation, Ministry of Defence, Government of India.

2. SOURCE LOCALIZATION

We consider the following well-known algorithms [8] for source localization, based on the use of a uniform linear array(ULA) of sensors.

1. MUSIC algorithm, which is based on the eigendecomposition of the data covariance matrix, estimates the localization parameters by maximising the MUSIC ambiguity function $\left[\mathbf{p}^H(\theta,r,z)\mathbf{V}\mathbf{V}^H\mathbf{p}(\theta,r,z)\right]^{-1}$, where $\mathbf{V}$ is the noise subspace.
2. Min-norm algorithm, unlike MUSIC, uses a single vector $\mathbf{v}$ that lies in the noise subspace and which satisfies suitable constraints. Min-norm achieves a satisfactory accuracy, but slightly inferior to MUSIC, at a reduced computational cost.
3. Linear beamforming (Bartlett Processor) estimates the source locations by maximising the spatial power spectrum $\mathbf{p}^H(\theta,r,z)\mathbf{R}\mathbf{p}(\theta,r,z)$. This technique can be shown to be not suitable for localization of multiple sources.
4. Minimum variance distortionless beamformer (Capon processor), which estimates the source position coordinates by maximising the ambiguity function $\left[\mathbf{p}^H(\theta,r,z)\mathbf{R}^{-1}\mathbf{p}(\theta,r,z)\right]^{-1}$.

A vertical ULA is used for range-depth estimation. It is assumed that the acoustic properties of the ocean are range-independent, and the consequent cylindrical symmetry of the ocean is exploited by the vertical ULA to restrict the search for the source position to two dimensions (range and depth). A horizontal ULA is used for DOA estimation, and it is assumed that the source ranges and depths are either known or have been estimated using a vertical ULA. All the algorithms mentioned above require an estimate of the array data covariance matrix which is obtained from a finite number of snapshots of the array data vector. The algorithms also require an acoustic propagation model of the ocean, which is briefly described in the next section.

3. ACOUSTIC PROPAGATION MODEL FOR SHALLOW OCEAN

Modeling of acoustic propagation in a shallow ocean has to account for the effects of the ocean boundaries and medium inhomogeneity on the propagation of acoustic waves. A simple and widely used acoustic propagation model for shallow ocean is the Pekeris model: a homogenous water layer of depth Δ, density ρ and sound speed c is assumed to be resting over a homogenous fluid half-space of density ρ_b and sound speed $c_b > c$. For a point source of frequency $\frac{\omega}{2\pi}$, in the water at a depth $z_s < \Delta$, the far-field approximation to the acoustic pressure at a range r and depth $z < \Delta$, assuming normal mode propagation [7] , is given by

$$p(r,z) = \frac{2\sqrt{2\pi}\, e^{j\frac{\pi}{4}}}{\Delta} \sum_{i=1}^{K} \alpha_i^2 \, sin(\gamma_i z_s)\, sin(\gamma_i z) \frac{e^{jk_i r}}{\sqrt{k_i r}} , \quad (1)$$

where K is the number of propagating modes and k_i is the wavenumber of the i^{th} normal mode.
The quantity α_i is defined as

$$\alpha_i^2 = \left[1 + \frac{\rho(k^2 - k_b^2) sin^2(\gamma_i \Delta)}{\rho_b \gamma_i^2 \beta_i \Delta} \right]^{-1} , \quad (2)$$

with $k = \omega/c$, $k_b = \omega/c_b$, $\beta_i = \sqrt{k_i^2 - k_b^2}$, and $k_i = \sqrt{k^2 - \gamma_i^2}$. The parameters k_i and γ_i are respectively the horizontal and vertical wavenumbers of the i^{th} normal mode. The values of γ_i, $i = 1, 2, \ldots, K$, can be obtained as the real roots of the equation

$$\rho_b \gamma cot(\gamma \Delta) + \rho \sqrt{k^2 - k_b^2 - \gamma^2} = 0 .$$

3.1 Horizontal Linear Array

Let J mutually uncorrelated narrowband sources of center frequency $\frac{\omega}{2\pi}$ be located at depths z_j and ranges r_j, $j = 1, 2, \ldots, J$, with respect to the first sensor of a horizontal ULA. The ULA consists of M narrowband sensors located at depth z_h and having intersensor spacing $d = \frac{\pi}{k}$. Let the bearing angle of the j^{th} source with respect to the endfire direction of the array be denoted by θ_j.
The noisy data vector at the array at time instant t can be written as

$$\mathbf{y}(t) = \mathbf{P}\mathbf{s}(t) + \mathbf{n}(t) , \ t = 1,2,\ldots,N , \quad (3)$$

where $\mathbf{s}(t)$ is the $J \times 1$ source signal amplitude vector given by

$$\mathbf{s}(t) = [s_1(t)\ s_2(t)\ \ldots\ s_J(t)]^T , \quad (4)$$

$\mathbf{n}(t) = [n_1(t)\ n_2(t)\ \ldots\ n_M(t)]^T$ is the array noise vector that is spatially *white*, and $\mathbf{P}$ is the $M \times J$ *steering vector* matrix given by

$$\mathbf{P} = [\ \mathbf{p}(r_1,\theta_1,z_1)\ \mathbf{p}(r_2,\theta_2,z_2)\ \ldots\ \mathbf{p}(r_J,\theta_J,z_J)] . \quad (5)$$

The noise variance σ^2 is assumed to be known.
The j^{th} column of $\mathbf{P}$ can be written as

$$\mathbf{p}(r_j,\theta_j,z_j) = \mathbf{A}(\theta_j)\mathbf{x}_j(r_j,z_j) , \quad (6)$$

where $\mathbf{A}(\theta_j)$ is an $M \times K$ matrix consisting of the steering vectors for each mode as given below

$$\begin{bmatrix} 1 & 1 & \ldots & 1 \\ e^{jk_1 dcos(\theta_j)} & e^{jk_2 dcos(\theta_j)} & \ldots & e^{jk_K dcos(\theta_j)} \\ e^{j2k_1 dcos(\theta_j)} & e^{j2k_2 dcos(\theta_j)} & \ldots & e^{j2k_K dcos(\theta_j)} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(M-1)k_1 dcos(\theta_j)} & e^{j(M-1)k_2 dcos(\theta_j)} & \ldots & e^{j(M-1)k_K dcos(\theta_j)} \end{bmatrix} , \quad (7)$$

and $\mathbf{x}_j(r_j,z_j) = [x_{j1}\ x_{j2}\ \ldots\ x_{jK}]^T, j = 1,2,\ldots,J$. For the Pekeris channel, x_{ji} is given by [see Eq (1)]

$$x_{ji} = \frac{2\sqrt{2\pi}\, e^{j\frac{\pi}{4}}}{\Delta} \, \alpha_i^2 \, sin(\gamma_i z_j)\, sin(\gamma_i z_h) \frac{e^{jk_i r_j}}{\sqrt{k_i r_j}} . \quad (8)$$

The $(m,j)^{th}$ element of $\mathbf{P}$ is given by

$$p_{m,j} = p_m(\theta_j) = \sum_{i=1}^{K} x_{ji} \exp\{-j(m-1)k_i d\cos(\theta_j)\}. \quad (9)$$

It is assumed that the signal and noise are uncorrelated. The covariance matrix of the array data vector is

$$\begin{aligned} \mathbf{R} &= E[\mathbf{y}(t)\mathbf{y}^H(t)] \\ &= \mathbf{P}\, E[\mathbf{s}(t)\mathbf{s}^H(t)]\, \mathbf{P}^H + E[\mathbf{n}(t)\mathbf{n}^H(t)] \\ &= \mathbf{P}\mathbf{\Sigma}\mathbf{P}^H + \sigma^2\mathbf{I} , \end{aligned} \quad (10)$$

3.2 Vertical Linear Array

The data model is similar to that for the horizontal ULA, barring a few modifications due to the vertical geometry of the array. The ULA consists of M narrowband sensors vertically positioned at depths, z_0, $z_0 + d$, $z_0 + 2d$, $\ldots$, $z_0 + (M-1)d$, where d is the spacing between sensors. Using the same notation and assumptions as in the previous case, the covariance matrix of the array data vector can again be written as

$$\mathbf{R} = \mathbf{P}\mathbf{\Sigma}\mathbf{P}^H + \sigma^2\mathbf{I} , \quad (11)$$

where the *steering vector matrix* $\mathbf{P}$ for the present case is independent of the bearing angles θ_j because of the cylindrical symmetry. Hence the *steering vector* matrix may be written as

$$\mathbf{P} = [\ \mathbf{p}(r_1,z_1)\ \mathbf{p}(r_2,z_2)\ \ldots\ \mathbf{p}(r_J,z_J)] , \quad (12)$$

The $(m,j)^{th}$ element of $\mathbf{P}$ is given by

$$p_{mj} = p_m(r_j,z_j) = \frac{2\sqrt{2\pi}e^{j\frac{\pi}{4}}}{\Delta} \sum_{i=1}^{K} \alpha_i^2 sin(\gamma_i z_j) sin(\gamma_i z_m) \frac{e^{jk_i r_j}}{\sqrt{k_i r_j}}. \quad (13)$$

4. CRAMER-RAO BOUND

In this section, the mathematical formulations of the Cramer-Rao bound (CRB) for the problems of DOA/range-depth estimation of underwater acoustic sources in a shallow range-independent ocean are presented for the case of additive complex circular white generalized Gaussian noise. Our method of derivation is similar to that of Stoica and Nehorai [6] for the case of plane wave DOA estimation in white Gaussian noise.

We model the noise samples $n_m(t)$ as complex valued. The complex noise samples are considered to be statistically independent both spatially (with respect to index m) and temporally (with respect to time t). The marginal pdf of the in-phase and quadrature components are considered to belong to the generalized Gaussian class [5]. The Complex-valued noise components $n_m(t) = \bar{n}_m(t) + j\ \tilde{n}_m(t)$ are considered to have a circularly symmetric pdf of the form $f(\bar{n},\tilde{n}) = g(\sqrt{\bar{n}^2 + \tilde{n}^2})$. Therefore, $\bar{n}_m(t)$ and $\tilde{n}_m(t)$ have zero mean and are uncorrelated.

The joint pdf $f(\bar{n},\tilde{n})$ is given by [9]

$$f(\bar{n},\tilde{n}) = \frac{k}{2\pi\Gamma\left(\frac{2}{k}\right)B^2(k)} exp\left[-\left(\frac{\sqrt{\bar{n}^2+\tilde{n}^2}}{B(k)}\right)^k\right] \quad (14)$$

$$\text{where } B(k) = \left[\sigma^2\frac{2\Gamma(\frac{2}{k})}{\Gamma(\frac{4}{k})}\right]^{\frac{1}{2}},$$

$$\text{and } \Gamma(a) = \int_0^\infty x^{a-1}e^{-x}dx, \text{ is the gamma function.}$$

Equation (14) represents circular Gaussian pdf for $k = 2$, and circular heavy tailed generalized Gaussian pdf for $k < 2$.
The likelihood function of the sampled data $\mathbf{y}(t)$ in Eq. (3) can be written as

$$l(\mathbf{y}) = \prod_{t=1}^{N}\prod_{m=1}^{M}\frac{k}{2\pi\Gamma\left(\frac{2}{k}\right)B^2(k)}e^{\left[-\left(\frac{\left|y_m(t)-\sum_{j=1}^{J}p_m(\theta_j)s_j(t)\right|}{B(k)}\right)^k\right]}. \quad (15)$$

Hence, the log likelihood function is given by

$$L(\mathbf{y};\mathbf{s},\mathbf{\Theta}) = N\,M\,ln\left(\frac{k}{2\pi\,\Gamma\left(\frac{2}{k}\right)\,B^2(k)}\right) + $$
$$+\frac{(-1)}{B^k(k)}\sum_{t=1}^{N}\sum_{m=1}^{M}\left(\left|y_m(t) - \sum_{j=1}^{J}p_m(\theta_j)s_j(t)\right|^k\right), \quad (16)$$

where $\mathbf{y}$ and $\mathbf{s}$ are respectively the data vector and signal vector of samples at all sensor points and all time instants. $\mathbf{\Theta}$ is the unknown parameter vector for which we calculate the CRB.
The Fisher Information Matrix $\mathbf{\Omega}$ is defined as [10]

$$\mathbf{\Omega} = E\left\{\left(\frac{\partial L(\mathbf{y};\mathbf{\Theta},\mathbf{s})}{\partial\mathbf{\Psi}}\right)\left(\frac{\partial L(\mathbf{y};\mathbf{\Theta},\mathbf{s})}{\partial\mathbf{\Psi}}\right)^T\right\}, \quad (17)$$

where $\mathbf{\Psi} = [\Theta^T, s^T]^T$. The CRB of the i^{th} component of Θ can be found as the $(i,i)^{th}$ element of the inverse of the Fisher Information matrix.

4.1 CRB for DOA

For DOA estimation problem, consider $\mathbf{\Theta} = [\theta_1,\ldots,\theta_J]^T$. Considering only DOAs as the unknown parameters, it can be shown that the CRB for DOA vector $\mathbf{\Theta}$ is given by

$$CRB(\mathbf{\Theta}_i) = \left(\frac{1}{I}\right)\cdot\left([\mathbf{\Sigma}]^{-1}\right)_{ii}, \quad (18)$$

$$\text{where } \mathbf{\Sigma} = \sum_{t=1}^{N}\Re\left\{\mathbf{S}^H(t)\mathbf{E}^H\left[\mathbf{I}-\mathbf{P}\left(\mathbf{P}^H\mathbf{P}\right)^{-1}\mathbf{P}^H\right]\mathbf{E}\mathbf{S}(t)\right\},$$

$$\mathbf{S}(t) = \begin{bmatrix} s_1(t) & & 0 \\ & \ddots & \\ 0 & & s_J(t), \end{bmatrix}$$

$$\mathbf{E} = [\mathbf{e}(\theta_1)\ldots\mathbf{e}(\theta_J)],$$

$$e_m(\theta_j) = \frac{\partial p_m(\theta_j)}{\partial\theta_j} = \sum_{i=1}^{K}x_{ji}d_{mj},$$

$$d_{mj} = e^{\{-j(m-1)k_i d\cos(\theta_i)\}}\left[j(m-1)k_i d\sin(\theta_j)\right],$$

$$I = \frac{k^2\Gamma(\frac{4}{k})}{4\sigma^2\Gamma^2(\frac{2}{k})}.$$

4.2 CRB for Range and Depth

For range-depth estimation problem, let $\Theta = \left[\mathbf{z}^T\;\mathbf{r}^T\right]^T = [\,z_1,\ldots,z_J,\;r_1,\ldots,r_J\,]^T$ be the vector of the unknown depth and range of the sources in Eq. (16).
Now, the CRB for Θ is given by

$$(CRB(\Theta))_i = \left(\frac{1}{I}\right)\cdot\left(\begin{bmatrix}\mathbf{\Sigma}_1 & \mathbf{\Sigma}_2\\ \mathbf{\Sigma}_3 & \mathbf{\Sigma}_4\end{bmatrix}^{-1}\right)_{ii} \quad (19)$$

$$\text{where } \mathbf{\Sigma}_1 = \sum_{t=1}^{N}\Re\left\{\mathbf{S}^H(t)\mathbf{F}^H\left[\mathbf{I}-\mathbf{P}\left(\mathbf{P}^H\mathbf{P}\right)^{-1}\mathbf{P}^H\right]\mathbf{F}\mathbf{S}(t)\right\}$$

$$\mathbf{\Sigma}_2 = \sum_{t=1}^{N}\Re\left\{\mathbf{S}^H(t)\mathbf{F}^H\left[\mathbf{I}-\mathbf{P}\left(\mathbf{P}^H\mathbf{P}\right)^{-1}\mathbf{P}^H\right]\mathbf{G}\mathbf{S}(t)\right\}$$

$$\mathbf{\Sigma}_3 = \sum_{t=1}^{N}\Re\left\{\mathbf{S}^H(t)\mathbf{G}^H\left[\mathbf{I}-\mathbf{P}\left(\mathbf{P}^H\mathbf{P}\right)^{-1}\mathbf{P}^H\right]\mathbf{F}\mathbf{S}(t)\right\}$$

$$\mathbf{\Sigma}_4 = \sum_{t=1}^{N}\Re\left\{\mathbf{S}^H(t)\mathbf{G}^H\left[\mathbf{I}-\mathbf{P}\left(\mathbf{P}^H\mathbf{P}\right)^{-1}\mathbf{P}^H\right]\mathbf{G}\mathbf{S}(t)\right\}$$

$$\mathbf{F} = [\mathbf{f}(r_1,z_1)\ldots\mathbf{f}(r_J,z_J)]$$

$$f_{mj} = f_m(r_j,z_j) = \frac{\partial p_m(r_j,z_j)}{\partial z_j} = $$
$$= \frac{2\sqrt{2\pi}e^{j\frac{\pi}{4}}}{\Delta}\sum_{i=1}^{K}\alpha_i^2 cos(\gamma_i z_j)\gamma_i sin(\gamma_i z_m)\frac{e^{jk_ir_j}}{\sqrt{k_ir_j}}$$

$$\mathbf{G} = [\mathbf{g}(r_1,z_1)\ldots\mathbf{g}(r_J,z_J)]$$

$$g_{mj} = g_m(r_j,z_j) = \frac{\partial p_m(r_j,z_j)}{\partial r_j} = $$

$$= \frac{2\sqrt{2\pi}\,e^{j\frac{\pi}{4}}}{\Delta}\sum_{i=1}^{K}\alpha_i^2\,sin(\gamma_i z_j)\,sin(\gamma_i z_m)e^{jk_ir_j}\frac{\left(jk_i-\frac{1}{2r_j}\right)}{\sqrt{k_ir_j}}$$

5. PERFORMANCE ANALYSIS : SIMULATIONS

In this section, simulation results are presented to compare the estimation performance of the MUSIC, min-norm, Bartlett, and Capon algorithms with the CRB. These comparisons are similar to those presented by Stoica and Nehorai [11] for the case of plane wave DOA estimation in white Gaussian noise.

The acoustic signals used for simulations were generated following the normal mode model discussed in Section 2 for the Pekeris channel with the following parameters: $\rho = 1000\ kg/m^3$, $\rho_b = 1500\ kg/m^3$, $c = 1500\ m/s$, $c_b = 1700\ m/s$, channel depth $\Delta = 100\ m$. We considered a uniform linear array of $M = 25$ sensors, with intersensor spacing $d = \lambda/2$. For DOA estimation we considered a horizontal ULA at depth $z_h = 50\ m$ from the surface, and for range-depth estimation we considered a vertical ULA with the first sensor just below the ocean surface. Two narrowband sources with common centre frequency $\frac{\omega}{2\pi} = 100\ Hz$ were assumed to be present at ranges $r_1 = 3000\ m$, $r_2 = 3150\ m$, azimuths $\theta_1 = 40°$, $\theta_2 = 50°$ and depths $z_1 = 60\ m$, $z_2 = 75\ m$. At this frequency 6 normal modes propagate in the medium. $N = 200$ data samples at each sensor were used for the estimation of the covariance matrix R.

Following figures illustrate the performance of the estimators in two different conditions in the shallow ocean environment, firstly when the noise statistics is Gaussian in nature and secondly when the noise has heavy-tailed generalized Gaussian distribution. In this work, generalized Gaussian noise with value of $k = 0.5$ in Eq. (14)

has been used to represent impulsive noise, and $k = 2$ corresponds to Gaussian noise.

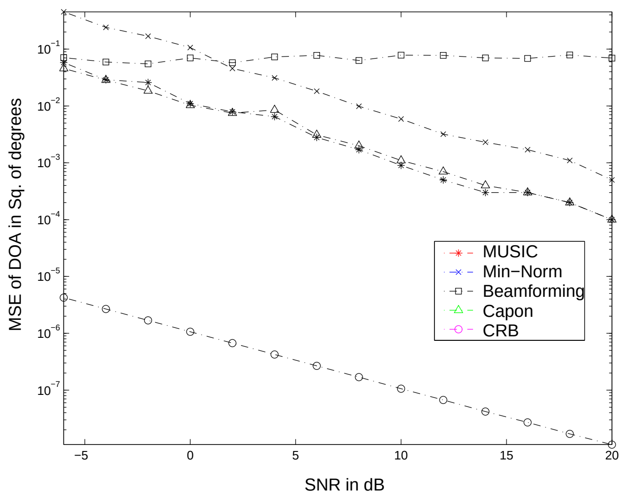

Figure 1: MSE and CRB for DOA estimates in Gaussian noise

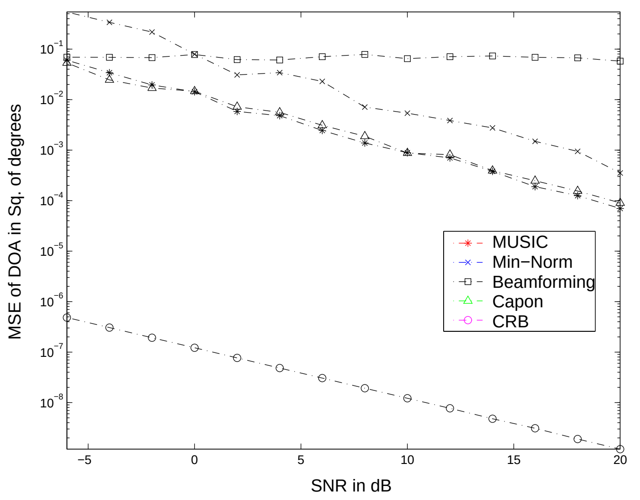

Figure 2: MSE and CRB for DOA estimates in GGN (k=0.5)

Figures 1 and 2 show the plots of MSE of DOA estimates (of the source at $\theta = 40^\circ$) versus input SNR in dB for Gaussian noise and generalized Gaussian noise (GGN) with $k = 0.5$ respectively. Performance of different estimators are compared with the corresponding Cramer-Rao bounds for the above mentioned source and channel parameters. The MSE for each algorithm at each SNR was estimated by averaging over 200 Monte Carlo simulations. The CRB shown in these figures is the average CRB over 200 Monte Carlo realizations of the random narrowband signal.

Figures 3 and 4 show the plots of MSE of depth estimates (of the source at $z = 60\ m$) versus input SNR in dB for Gaussian noise and GGN with $k = 0.5$ respectively. Figures 5 and 6 show the plots of MSE of range estimates (of the source at $r = 3000m$) versus input SNR in dB for Gaussian noise and GGN with $k = 0.5$ respectively.

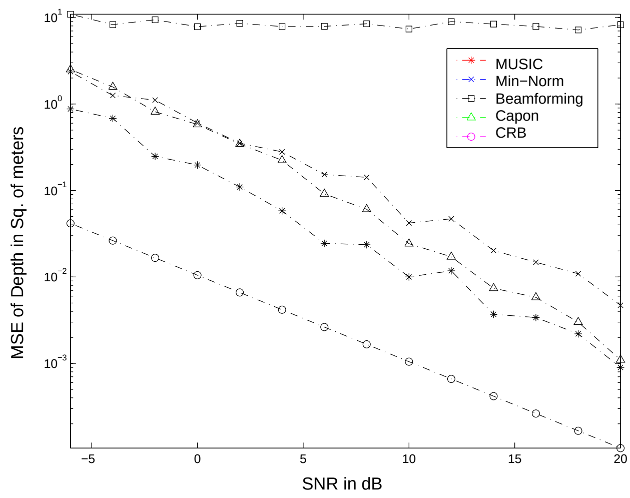

Figure 3: MSE and CRB for depth estimates in Gaussian noise

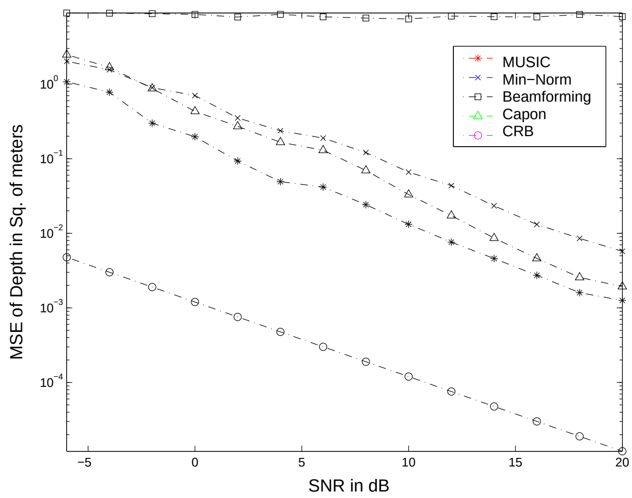

Figure 4: MSE and CRB for depth estimates in GGN (k=0.5)

From all these plots we observe that the CRB for non-Gaussian noise is lower than that for Gaussian noise. This result is in conformity with the known theoretical result that the optimal estimator has the worst performane for Gaussian noise. The MSEs of all the algorithms are independent of the noise pdf since these algorithms depent only on the covariance matrix $\mathbf{R}$ and not on the noise pdf. Also, the performance of MUSIC is the best among all the estimators we have considered, followed by Capon, Min-norm and the linear beamformer (Bartlett). The linear beamformer yields a poor performance as it is not suitable for localization of multiple sources.

6. CONCLUSIONS

In this paper we have developed an expression for CRB for the problem of DOA/Range-Depth estimation of underwater acoustic sources in a shallow ocean for the case of generalized Gaussian noise. The performance of MUSIC, Min-norm, Linear beamformer and Capon beamformer have been studied for estimation of all the above three parameters in shallow ocean, in terms of the variance

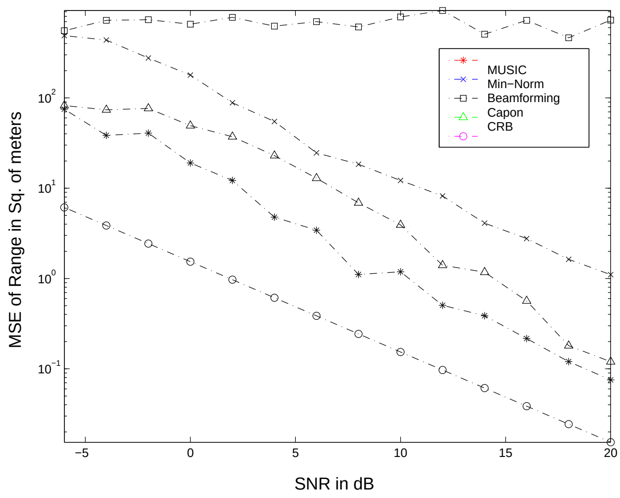

Figure 5: MSE and CRB for range estimates in Gaussian noise

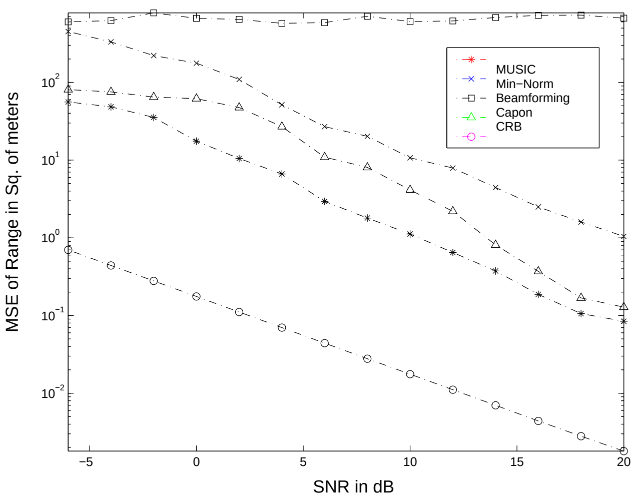

Figure 6: MSE and CRB for range estimates in GGN (k=0.5)

of the estimation error. This has been carried out in two different scenarios: ambient noise being Gaussian and ambient noise being impulsive. The CRB is higher for Gaussian noise, which is in accordance with the known result that the optimal estimator has the worst performance in Gaussian noise. None of the estimation algorithms considered here is dependent on noise pdf, and hence the MSEs of these algorithms are not affected by the change in noise distribution. The usefulness of the CRB formula derived in this work is not limited to the performance studies reported in this paper. It may also be used to establish the relative efficiency of other estimators in shallow ocean.

REFERENCES

[1] Tolstoy A., Matched Field Processing for Underwater Acoustics, World Scientific, Singapore (1993).

[2] Baggeroer A.B., Kuperman W.A. and Mikhalevsky P.N., ”An overview of matched field methods in ocean acoustics”, *I*EEE Journal of Oceanic Engineering, Vol. 18, Issue. 4, pp. 401-424, October 1993.

[3] Baggeroer A.B., Kuperman W.A. and Henrik Schmidt, ”Matched Field Processing: Source Localization in Correlated Noise as an Optimum Paramter Estimation Problem”, *J. Acoust. Soc. Am. 83(2), pp. 571-587, February 1988.*

[4] E. J. Wegman, S. C. Schwartz and J. B. Thomas,*Topics in Non-Gaussian Signal Processing*,Springer-Verlag New York Inc.,1989.

[5] S. A. Kassam, *”Signal Detection in non-Gaussian noise”*, Springer-Verlag, 1988.

[6] P. Stoica and A. Nehorai, ”MUSIC, Maximum Likelihood, and Cramer-Rao Bound”, *IEEE Trans. Acoust., Speech, Signal Processing*, Vol. ASSP -37, No. 5, pp. 720-741, May 1989.

[7] L. M. Brekhovskikh and Yu. Lysanov, *Fundamentals of Ocean Acoustics*, II ed., Springer-Verlag, Berlin (1991).

[8] H. Krim adn M. Viberg, ”Two Decades of Array Signal Processing Research”, *IEEE SP Mag., pp. 67-94, Jul. 1996.*

[9] N. H. Lu and B. A. Eisenstein, ”Detection of Weak Signals in Non-Gaussian Noise”, *IEEE Transactions on Information Theory*, Vol. IT-27, Issue. 6, pp. 755-771, November 1981.

[10] S. M. Kay, *Statistical Signal Processing Vol. 1: Estimation Theory*, Prentice-Hall Inc., Englewood Cliffs, New Jersey (1995).

[11] P. Stoica and A. Nehorai, ”MUSIC, Maximum Likelihood, and Cramer-Rao Bound : Further Results and Comparisons”, *IEEE Trans. Acoust., Speech, Signal Processing*, Vol. ASSP-38, No. 12, pp. 2140-2150, Dec 1990.