SOURCE LOCALISATION IN SHALLOW OCEAN USING A VERTICAL ARRAY OF ACOUSTIC VECTOR SENSORS

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ABSTRACT

This paper introduces a new approach to 3D localisation of a narrowband acoustic source in a shallow ocean using acoustic vector sensors (AVS). Assuming a horizontally stratified and range-independent model of the ocean, it is shown that the azimuth of the source can be determined from the estimates of the horizontal components of the acoustic intensity vector obtained from the measurements of an AVS. The range and depth of the source could then be estimated through a 2D search to match the computed complex acoustic intensity vector expressed as a function of these parameters with its estimate obtained from the AVS measurements. However the search in range is computationally intensive as the range parameter is unbounded. We propose an alternative approach employing a vertical array of AVS, based on eigen-decomposition of the spatial correlation matrix of the data vector, leading to a closed form solution for the range parameter. The source depth is then estimated through a 1D search of this bounded parameter.

1. INTRODUCTION

An acoustic vector sensor (AVS) simultaneously measures the acoustic pressure and the three Cartesian components of the particle velocity vector at a point in an acoustic field. A single AVS is capable of finding the direction of an acoustic source using the information provided by the particle velocity measurements unlike a conventional sensor which measures only the scalar acoustic pressure. Hence, an array of vector sensors can outperform a similar array of conventional scalar sensors. The AVS has received considerable attention from the signal processing community after the publication of two seminal papers by D'Spain *et al* [1, 2] in 1991. This is exemplified by a host of papers that appeared in the literature starting with Neharoi and Paldi [3] who presented the AVS measurement model for plane wave propagation and an acoustic intensity based method for localising acoustic sources using an array of such sensors. Later, Hawkes and Neharoi [4] considered the effect of the presence of a reflecting boundary near the sensor. However none of the data models considered so far is applicable to the shallow ocean scenario which is characterised by multimode acoustic propagation in a channel with reflecting boundaries at the top and the bottom.

In this paper, we present an AVS data model that is appropriate for measurements in a shallow ocean. The ocean is modelled as a range-independent channel consisting of a horizontally stratified water layer of constant depth overlying a horizontally stratified bottom. The pressure field due to a harmonic source is made up of two components, viz., the sum of normal modes *and* an integral of continuous spectrum [5]. Since a hydrophone does not measure particle velocity, the particle velocity field generated by the source is not considered in the conventional approach to source localisation. We derive an expression for the particle velocity field, combine it with the acoustic pressure field in a convenient vector form and add the measurement noise to obtain the data model of a single AVS. We then proceed to derive expressions for the three Cartesian components of the active and reactive acoustic intensity vectors at the AVS. The azimuth angle of the source is determined from the estimates of the two horizontal components of the active intensity vector. One can then obtain an estimate of the range and depth of the source through a 2D search to match the computed complex acoustic intensity vector with its estimate obtained from the AVS measurements. Towards this end we define an ambiguity function whose maximum would correspond to the actual source position, and demonstrate the application of this method through a numerical case study. But this method is highly computation intensive and also it may not provide unambiguous estimates of range and depth. Next we propose a method, employing a vertical AVS array, based on eigen-decomposition of the spatial correlation matrix to estimate the range of the source without the need for a numerical search. After estimating azimuth and range, the depth can be easily estimated through a one-dimensional numerical search within a bounded region.

2. MEASUREMENT MODEL

Consider an acoustic narrowband point source of centre frequency $\omega/2\pi$ in a shallow ocean. Fig. 1 shows the source-receiver geometry. Far-field spatial distribution of the resulting pressure signal at a range r and depth z, ignoring the negligible contribution due to continuous modal spectrum, is well approximated by a sum of normal modes given by [5]

$$p(r,z,t) = \left(\frac{2\sqrt{2\pi} e^{j\pi/4}}{h} \sum_{n=1}^{N} \psi_n(z_s)\psi_n(z) \frac{e^{jk_n r}}{\sqrt{k_n r}} \right) \tilde{p}(t) e^{-j\omega t}, \qquad (1)$$

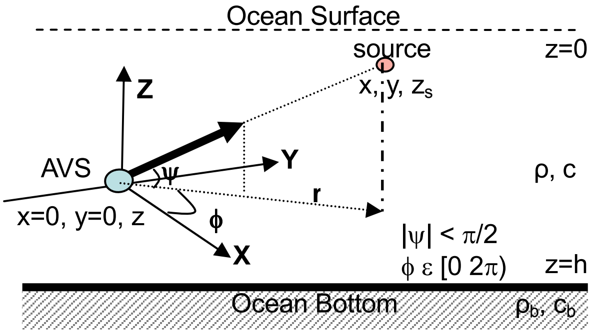

Figure 1 - Source-receiver geometry

where,

$r = \sqrt{x^2 + y^2}$ is the range of the source,

h is the depth of the water layer,

z_s is the source depth,

$\rho(z)$ is the density of water layer at depth z ($< h$),

$c(z)$ is the sound speed in water layer at depth z ($< h$),

$\rho_b(z)$ is the density of bottom at depth z ($\geq h$),

$c_b(z)$ is the sound speed in bottom at depth z ($\geq h$),

N is the maximum number of normal modes,

$\psi_n(z)$ is the eigenfunction and k_n is the eigenvalue obtained as the solution of the Sturm-Liouville type characteristic differential equation (with appropriate boundary conditions at $z = 0$ and $z = h$): $\frac{d^2\psi_n(z)}{dz^2} + \left[k^2(z) - k_n^2\right]\psi_n(z) = 0,$

$k(z) = \omega / c(z), \quad k_b(z) = \omega / c_b(z),$

and $\tilde{p}(t)$ is the slowly varying envelope of the narrowband signal.

The expression for particle velocity follows from the acoustic vector field equation [1], viz.,

$$\rho(\mathbf{r})\frac{\partial \mathbf{v}(\mathbf{r},t)}{\partial t} + \nabla p(\mathbf{r},t) = 0,$$

from which we get,

$$\mathbf{v}(\mathbf{r},t) = -\int_0^t \frac{1}{\rho(\mathbf{r})}\nabla p(\mathbf{r},\tau)d\tau, \tag{2}$$

where $\mathbf{r}$ = (x, y, z).

We shall denote the complex amplitudes of the pressure and the x, y and z-components of particle velocity by u, v_x, v_y and v_z respectively. Using Eqs. (1) and (2) it can be shown that

$$u = \sum_{n=1}^{N} u_n, \tag{3}$$

$$v_x = \frac{1}{\omega\rho}\left(\frac{x}{r}\right)\sum_{n=1}^{N}\left(k_n + j\frac{1}{2r}\right)u_n, \tag{4}$$

$$v_y = \frac{1}{\omega\rho}\left(\frac{y}{r}\right)\sum_{n=1}^{N}\left(k_n + j\frac{1}{2r}\right)u_n, \tag{5}$$

$$v_z = \frac{1}{\omega\rho}\sum_{n=1}^{N}\left(\frac{-j\psi_n'(z)}{\psi_n(z)}\right)u_n. \tag{6}$$

where

$$u_n = \frac{2\sqrt{2\pi}e^{j\pi/4}}{h}\psi_n(z_s)\psi_n(z)\frac{e^{jk_n r}}{\sqrt{k_n r}}. \tag{7}$$

is the complex amplitude of pressure due to n^{th} mode and $\psi_n'(z)$ denotes derivative of $\psi_n(z)$ with respect to z.

Thus the narrowband measurement model for a single AVS in shallow water due to a point source, along with additive noise, may be written as

$$\mathbf{y}(t) = [y_u(t)\ y_{vx}(t)\ y_{vy}(t)\ y_{vz}(t)]^T = [u\ v_x\ v_y\ v_z]^T\, \tilde{p}(t) + \mathbf{n}(t), \tag{8}$$

where $\mathbf{n}(t)$ is the noise vector, and $y_u(t)$, $y_{vx}(t)$, $y_{vy}(t)$ and $y_{vz}(t)$ denote the measured pressure and particle velocity components respectively. We shall consider the measurements to have been sampled at Nyquist rate so that the time index t will now on represent the discrete time index.

3. COMPLEX ACOUSTIC INTENSITY VECTOR

The real part of the complex acoustic intensity vector (i.e., active intensity vector) due to a narrowband source is given by [6]

$$\mathbf{I}(\mathbf{r}) = \frac{j}{4\omega\rho}\left[u(\mathbf{r})\nabla u^*(\mathbf{r}) - u^*(\mathbf{r})\nabla u(\mathbf{r})\right]. \tag{9}$$

From Eq. (9) we obtain the x, y and z-components of active intensity as

$$I_x(\mathbf{r}) = \left(\frac{x}{r}\right)\boldsymbol{\alpha}_{r,z_s}{}^T(\mathbf{C}_r\mathbf{K})\boldsymbol{\alpha}_{r,z_s}, \tag{10}$$

$$I_y(\mathbf{r}) = \left(\frac{y}{r}\right)\boldsymbol{\alpha}_{r,z_s}{}^T(\mathbf{C}_r\mathbf{K})\boldsymbol{\alpha}_{r,z_s}, \tag{11}$$

$$I_z(\mathbf{r}) = \boldsymbol{\alpha}_{r,z_s}{}^T(\mathbf{S}_r\mathbf{J})\boldsymbol{\alpha}_{r,z_s}, \tag{12}$$

where the vector $\boldsymbol{\alpha}_{r,zs}$ and matrices $\mathbf{C}_r$, $\mathbf{S}_r$, $\mathbf{J}$, $\mathbf{K}$ are defined as

$$[\boldsymbol{\alpha}_{r,z_s}]_m = \frac{1}{\sqrt{\omega\rho}}\frac{2\sqrt{\pi}}{h}\frac{\psi_m(z_s)\psi_m(z)}{\sqrt{k_m r}},$$

$$[\mathbf{C}_r]_{m,n} = \cos(k_m - k_n)r,$$

$$[\mathbf{S}_r]_{m,n} = -\sin(k_m - k_n)r,$$

$$[\mathbf{J}]_{m,n} = \delta_{m,n}\frac{\psi_n'(z)}{\psi_n(z)}$$

$$[\mathbf{K}]_{m,n} = \delta_{m,n}k_m,$$

and $\delta_{m,n}$ is the kronecker-delta.

Similarly the imaginary part of the complex acoustic intensity vector (i.e., the reactive intensity vector [6]) due to a narrowband source can be shown to be

$$\mathbf{Q}(\mathbf{r}) = -\frac{1}{4\omega\rho}\left[u(\mathbf{r})\nabla u^*(\mathbf{r}) + u^*(\mathbf{r})\nabla u(\mathbf{r})\right]. \tag{13}$$

From Eq. (13) we obtain the x, y and z-components of reactive intensity as

$$Q_x(\mathbf{r}) = \left(\frac{x}{r}\right)\boldsymbol{\alpha}_{r,z_s}{}^T\left(\frac{\mathbf{C}_r}{2r} + \mathbf{S}_r\mathbf{K}\right)\boldsymbol{\alpha}_{r,z_s}, \tag{14}$$

$$Q_y(\mathbf{r}) = \left(\frac{y}{r}\right)\boldsymbol{\alpha}_{r,z_s}{}^T\left(\frac{\mathbf{C}_r}{2r} + \mathbf{S}_r\mathbf{K}\right)\boldsymbol{\alpha}_{r,z_s}, \tag{15}$$

$$Q_z(\mathbf{r}) = \boldsymbol{\alpha}_{r,z_s}{}^T(\mathbf{C}_r\mathbf{J})\boldsymbol{\alpha}_{r,z_s}. \tag{16}$$

4. ESTIMATION OF SOURCE LOCATION

It follows from Eqs. (10) and (11) that

$$\phi = \tan^{-1}(y/x) = \tan^{-1}(I_y/ I_x).$$

Thus if we estimate I_x, and I_y we may unambiguously estimate the azimuth angle as,

$$\hat{\phi} = \tan^{-1}\left\{\frac{\hat{I}_y}{\hat{I}_x}\right\}, \tag{17}$$

where $\hat{I}_x$, and $\hat{I}_y$ are the estimates of I_x, and I_y respectively. The components of active intensity vector may be estimated consistently from the AVS measurements as [3]

$$\begin{bmatrix}\hat{I}_x\\ \hat{I}_y\\ \hat{I}_z\end{bmatrix} = < y_u(t) \begin{bmatrix} y_{vx}(t)\\ y_{vy}(t)\\ y_{vz}(t)\end{bmatrix}^* >_T,$$

while those of reactive intensity may be obtained as

$$\begin{bmatrix}\hat{Q}_x\\ \hat{Q}_y\\ \hat{Q}_z\end{bmatrix} = < y_u(t) \begin{bmatrix} y_{vx}(t-l)\\ y_{vy}(t-l)\\ y_{vz}(t-l)\end{bmatrix}^* >_T,$$

where $<.>_T$ denotes time averaging over T time samples and $l=\pi/2\omega$ is the time delay to effect a phase shift of 90^0 in velocity measurements. Since l need not be an integer, in general it is required to obtain the phase shifted sequence either through an appropriate interpolation scheme or using Hilbert transform technique [7].

In order to find range and depth one may numerically search for r and z_s trying to match the acoustic intensity vector computed as a function of these parameters against the estimate of the intensity vector obtained from the AVS measurements. Towards this end we define an ambiguity function as

$$A_F(r,z_s) = \left\{ \left(\mathbf{f}\begin{bmatrix} r\\ z_s\end{bmatrix} - \hat{\mathbf{f}}\begin{bmatrix} r_0\\ z_{s0}\end{bmatrix}\right)^T \left(\mathbf{f}\begin{bmatrix} r\\ z_s\end{bmatrix} - \hat{\mathbf{f}}\begin{bmatrix} r_0\\ z_{s0}\end{bmatrix}\right)\right\}^{-1}, \quad (18)$$

where,

$$\mathbf{f}\left(\begin{bmatrix} r\\ z_s\end{bmatrix}\right) = \begin{bmatrix} I_r(r,z_s)\\ I_z(r,z_s)\\ Q_r(r,z_s)\\ Q_z(r,z_s)\end{bmatrix} = \begin{bmatrix} \boldsymbol{\alpha}_{r,z_s}{}^T(\mathbf{C}_r\mathbf{K})\boldsymbol{\alpha}_{r,z_s}\\ \boldsymbol{\alpha}_{r,z_s}{}^T(\mathbf{S}_r\mathbf{J})\boldsymbol{\alpha}_{r,z_s}\\ \boldsymbol{\alpha}_{r,z_s}{}^T(\frac{\mathbf{C}_r}{2r}+\mathbf{S}_r\mathbf{K})\boldsymbol{\alpha}_{r,z_s}\\ \boldsymbol{\alpha}_{r,z_s}{}^T(\mathbf{S}_r\mathbf{J})\boldsymbol{\alpha}_{r,z_s}\end{bmatrix},$$

$$\hat{\mathbf{f}}\left(\begin{bmatrix} r_0\\ z_{s0}\end{bmatrix}\right) = \begin{bmatrix} \sqrt{\hat{I}_x^2+\hat{I}_y^2}\\ \hat{I}_z\\ \sqrt{\hat{Q}_x^2+\hat{Q}_y^2}\\ \hat{Q}_z\end{bmatrix},$$

and r_0, z_{s0} represent the true source range and depth respectively. Clearly, the peaks of this ambiguity function correspond to possible source locations. However, this 2-dimensional search is computationally intensive, especially since the range parameter r is unbounded. In the next section we propose a method to avoid the search in r.

5. RANGE ESTIMATION THROUGH EIGEN DECOMPOSITON OF SPATIAL CORRELATION MATRIX

We now propose a method to solve for r uniquely by employing a *vertical* array of M AVS. Let the sensors of the AVS array be located at $(0,0,z_1)$, $(0, 0, z_2)$…and $(0,0, z_M)$. Let the composite data vector be denoted by

$$\mathbf{d}(t) = [\mathbf{y}^T(t; z_1)\ \mathbf{y}^T(t; z_2)\ \ldots\ \mathbf{y}^T(t; z_M)]^T \in \mathbf{C}^{4M\times 1},$$

where

$$\mathbf{y}(t; z_m) = [u(z_m)\ v_x(z_m)\ v_y(z_m)\ v_z(z_m)]^T\ \tilde{p}(t) + \mathbf{n}(t; z_m) \in \mathbf{C}^{4\times 1},$$

is the data vector at the m^{th} AVS in accordance with Eq. (8). The composite data vector can be written in the form

$$\mathbf{d}(t) = \mathbf{A}\ \mathbf{s}(t) + \mathbf{w}(t), \quad (19)$$

where

$$\mathbf{A} = \mathbf{D}(\phi)\ \boldsymbol{\Gamma} \in \mathbf{C}^{4M\times N}, \quad (20)$$

$$\mathbf{D}(\phi) = \mathbf{I}_M \otimes \begin{bmatrix} 1 & & & \\ & \frac{\cos\phi}{\omega\rho} & & \\ & & \frac{\sin\phi}{\omega\rho} & \\ & & & \frac{1}{\omega\rho}\end{bmatrix} \in \mathbf{C}^{4M\times 4M},$$

$$\boldsymbol{\Gamma} = \begin{bmatrix} \psi_1(z_1) & \psi_2(z_1) & .. & \psi_N(z_1)\\ \psi_1(z_1)\left(k_1+\frac{j}{2r}\right) & \psi_2(z_1)\left(k_2+\frac{j}{2r}\right) & .. & \psi_N(z_1)\left(k_N+\frac{j}{2r}\right)\\ \psi_1(z_1)\left(k_1+\frac{j}{2r}\right) & \psi_2(z_1)\left(k_2+\frac{j}{2r}\right) & .. & \psi_N(z_1)\left(k_N+\frac{j}{2r}\right)\\ -j\psi_1'(z_1) & -j\psi_2'(z_1) & .. & -j\psi_N'(z_1)\\ \ldots & \ldots & \ldots & \ldots\\ \psi_1(z_M) & \psi_2(z_M) & .. & \psi_N(z_M)\\ \psi_1(z_M)\left(k_1+\frac{j}{2r}\right) & \psi_2(z_M)\left(k_2+\frac{j}{2r}\right) & .. & \psi_N(z_M)\left(k_N+\frac{j}{2r}\right)\\ \psi_1(z_M)\left(k_1+\frac{j}{2r}\right) & \psi_2(z_M)\left(k_2+\frac{j}{2r}\right) & .. & \psi_N(z_M)\left(k_N+\frac{j}{2r}\right)\\ -j\psi_1'(z_M) & -j\psi_2'(z_M) & .. & -j\psi_N'(z_M)\end{bmatrix},$$

$$\mathbf{s}(t) = [s_1\ s_2\ \ldots\ s_N]^T\ \tilde{p}(t) \in \mathbf{C}^{N\times 1},$$

$$s_n = u_n(r,z)/\psi_n(z) = \frac{2\sqrt{2\pi}e^{j\pi/4}}{h}\psi_n(z_s)\frac{e^{jk_n r}}{\sqrt{k_n r}},$$

$$\mathbf{w}(t) = [\mathbf{n}^T(t; z_1)\ \mathbf{n}^T(t; z_2)\ \ldots\ \mathbf{n}^T(t; z_M)]^T \in \mathbf{C}^{4M\times 1},$$

$\mathbf{I}_M$ denotes $M\times M$ identity matrix and $\otimes$ denotes the kronecker product.

The data correlation matrix is given by,

$$\mathbf{R}_d = E\{\mathbf{d}\mathbf{d}^H\} = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \mathbf{R}_w \in \mathbf{C}^{4M\times 4M},$$

where $\mathbf{R}_s=E\{\mathbf{s}\mathbf{s}^H\}$ and $\mathbf{R}_w=E\{\mathbf{w}\mathbf{w}^H\}$. The noise at different sensors can be assumed to be uncorrelated. Therefore the noise covariance matrix $\mathbf{R}_w$ has the block diagonal form

$$\mathbf{R}_w = \text{diag}\ \{\mathbf{R}_n(z_1)\ \mathbf{R}_n(z_2)\ \ldots\ \mathbf{R}_n(z_M)\} \in \mathbf{C}^{4M\times 4M},$$

where

$\mathbf{R}_n(z_m) = E\{\mathbf{n}(t; z_m)\ \mathbf{n}^H(t; z_m)\} \in \mathbf{C}^{4\times 4}$ is the noise covariance matrix of the m^{th} AVS located at depth z_m. Using the model of ambient noise proposed by Buckingham [8], it can be shown that [9]

$$\mathbf{R}_n(z) = \sigma^2\boldsymbol{\Upsilon}_n(z),$$

where

$$\boldsymbol{\Upsilon}_n(z) = \begin{bmatrix} \xi & & & \zeta(z)\\ & \xi_x & & \\ & & \xi_y & \\ \zeta(z) & & & \xi_z\end{bmatrix} \in \mathbf{C}^{4\times 4},$$

$$\xi = \sum_{n=1}^{N}\psi_n^2(z),\ \xi_x = \xi_y = \frac{1}{2}\left(\frac{k}{\omega\rho}\right)^2\sum_{n=1}^{N}\psi_n^2(z),\ \xi_z = \left(\frac{1}{\omega\rho}\right)^2\sum_{n=1}^{N}\psi_n'^2(z),$$

$$\zeta(z) = \frac{1}{\omega\rho}\sum_{m=1}^{N}\sum_{n=1}^{N} mn\,\psi_m(z)\psi_n'(z),$$

and σ^2 is a positive scale factor independent of z. Let us define $\boldsymbol{\Upsilon}_w = \text{diag}\ \{\boldsymbol{\Upsilon}_n(z_1)\ \boldsymbol{\Upsilon}_n(z_2)\ \ldots\ \boldsymbol{\Upsilon}_n(z_M)\} \in \mathbf{C}^{4M\times 4M}$. Therefore, through a linear transformation of the data vector $\mathbf{d}(t)$, we may obtain

$$\tilde{\mathbf{d}}(t) = \boldsymbol{\Upsilon}_w^{-1/2}\mathbf{d}(t) = \boldsymbol{\Upsilon}_w^{-1/2}\mathbf{A}\mathbf{s}(t) + \tilde{\mathbf{w}}(t),$$

where now $\tilde{\mathbf{w}}(t) = \boldsymbol{\Upsilon}_w^{-1/2}\mathbf{w}(t)$ has the covariance given by $E\{\tilde{\mathbf{w}}\tilde{\mathbf{w}}^H\} = \sigma^2\mathbf{I}$. The covariance matrix of the transformed vector $\tilde{\mathbf{d}}(t)$ is given by

$$\mathbf{R} = E\{\tilde{\mathbf{d}}\tilde{\mathbf{d}}^{\mathrm{H}}\} = \boldsymbol{\Upsilon}_w^{-1/2}\mathbf{A}\mathbf{R}_s\mathbf{A}^{\mathrm{H}}\boldsymbol{\Upsilon}_w^{-1/2\,\mathrm{H}} + \sigma^2\mathbf{I}.$$

Noting that the rank of $\mathbf{R}_s$ is one, we conclude that rank of $\boldsymbol{\Upsilon}_w^{-1/2}\mathbf{A}\mathbf{R}_s\mathbf{A}^{\mathrm{H}}\boldsymbol{\Upsilon}_w^{-1/2\,\mathrm{H}}$ is also one. The eigen-decomposition of $\mathbf{R}$ can be expressed as

$$\mathbf{R} = [\mathbf{e}_s \mid \mathbf{E}_n]\begin{bmatrix}\lambda_s + \sigma^2 & \\ & \sigma^2\mathbf{I}_{4M-1}\end{bmatrix}\begin{bmatrix}\mathbf{e}_s^H \\ \mathbf{E}_n^H\end{bmatrix},$$

where $\mathbf{e}_s$ is a $4M$x1 vector spanning the signal subspace and $\mathbf{E}_n$ is a $4M$x($4M$-1) matrix whose columns span the noise subspace. We have

$$\mathbf{e}_s \in \mathrm{range}\{\boldsymbol{\Upsilon}_w^{-1/2}\mathbf{A}\},$$

and hence we may conclude from Eq. (20) that

$$\tilde{\mathbf{e}}_s = \mathbf{D}^{\dagger}(\phi)\,\boldsymbol{\Upsilon}_w^{1/2}\,\mathbf{e}_s \in \mathrm{range}\{\boldsymbol{\Gamma}\}, \tag{21}$$

where, $\mathbf{D}^{\dagger}=(\mathbf{D}^{\mathrm{T}}\mathbf{D})^{-1}\mathbf{D}^{\mathrm{T}}$ is the Moore-Penrose pseudo-inverse of $\mathbf{D}$. It follows from Eq. (21) that there exists a vector $\boldsymbol{\eta}=[\eta_1,\ldots\eta_N]^{\mathrm{T}}\in\mathbf{C}^{N\times 1}$, such that

$$\boldsymbol{\Gamma}\boldsymbol{\eta} = \tilde{\mathbf{e}}_s. \tag{22}$$

Using the following M equations from Eq. (22),

$$\sum_{n=1}^{N}\eta_n\psi_n(z_m) = \tilde{e}_{s(4m-3)},\ \ m = 1:M, \tag{23}$$

the $2M$ equations which involve r in Eq. (22) can be written in the form

$$\mathbf{b}\left(\frac{j}{2r}\right) + \mathbf{B}\boldsymbol{\eta} = \mathbf{g}, \tag{24}$$

where

$\mathbf{b} = [\tilde{e}_{s1}\ \tilde{e}_{s1}\ \tilde{e}_{s5}\ \tilde{e}_{s5}\ \ldots\ \tilde{e}_{s(4M-3)}\ \tilde{e}_{s(4M-3)}]^{\mathrm{T}} \in \mathbf{C}^{2M\times 1}$,

$\mathbf{g} = [\tilde{e}_{s2}\ \tilde{e}_{s3}\ \tilde{e}_{s6}\ \tilde{e}_{s7}\ \ldots\ \tilde{e}_{s(4M-2)}\ \tilde{e}_{s(4M-1)}]^{\mathrm{T}} \in \mathbf{C}^{2M\times 1}$,

$$\mathbf{B} = \begin{bmatrix} k_1\psi_1(z_1) & k_2\psi_2(z_1) & \ldots & k_N\psi_N(z_1) \\ k_1\psi_1(z_1) & k_2\psi_2(z_1) & \ldots & k_N\psi_N(z_1) \\ \ldots & \ldots & \ldots & \ldots \\ k_1\psi_1(z_M) & k_2\psi_2(z_M) & \ldots & k_N\psi_N(z_M) \\ k_1\psi_1(z_M) & k_2\psi_2(z_M) & \ldots & k_N\psi_N(z_M) \end{bmatrix} \in \mathbf{C}^{2M\times N}.$$

Assuming that at least one element of the set $\{\tilde{e}_{s(4m-3)}$; $m=1,\ldots,M\}$ is different from zero, the least squares solution for $j/2r$ parameterised by $\boldsymbol{\eta}$, is obtained from Eq. (24) as

$$j/2r = \mathbf{b}^{\dagger}(\mathbf{g} - \mathbf{B}\boldsymbol{\eta}) = \alpha - \boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{\eta}, \tag{25}$$

where $\alpha=\mathbf{b}^{\dagger}\mathbf{g}$ and $\boldsymbol{\beta}=(\mathbf{b}^{\dagger}\mathbf{B})^{\mathrm{T}}\in\mathbf{C}^{N\times 1}$. Now substituting for $j/2r$ from Eq. (25) into Eq. (22) and using Eq. (23) we obtain the system of linear equations

$$\mathbf{H}\boldsymbol{\eta} = \boldsymbol{\chi}, \tag{26}$$

where

$\mathbf{H} = [\mathbf{H}_1^{\mathrm{T}}\ \mathbf{H}_2^{\mathrm{T}}\ \ldots\ \mathbf{H}_M^{\mathrm{T}}]^{\mathrm{T}} \in \mathbf{C}^{4M\times N}$,

$\boldsymbol{\chi} = [\boldsymbol{\chi}_1^{\mathrm{T}}\ \boldsymbol{\chi}_2^{\mathrm{T}}\ \ldots\ \boldsymbol{\chi}_M^{\mathrm{T}}]^{\mathrm{T}} \in \mathbf{C}^{4M\times 1}$,

$$\mathbf{H}_m = \begin{bmatrix} \psi_1(z_m) & \psi_2(z_m) & \ldots & \psi_N(z_m) \\ k_1\psi_1(z_m)-\tilde{e}_{s(4m-3)}\beta_1 & k_2\psi_2(z_m)-\tilde{e}_{s(4m-3)}\beta_2 & \ldots & k_N\psi_N(z_m)-\tilde{e}_{s(4m-3)}\beta_N \\ k_1\psi_1(z_m)-\tilde{e}_{s(4m-3)}\beta_1 & k_2\psi_2(z_m)-\tilde{e}_{s(4m-3)}\beta_2 & \ldots & k_N\psi_N(z_m)-\tilde{e}_{s(4m-3)}\beta_N \\ -j\psi_1'(z_m) & -j\psi_2'(z_m) & \ldots & -j\psi_N'(z_m) \end{bmatrix} \in \mathbf{C}^{4\times N},$$

$\boldsymbol{\chi}_m = [\tilde{e}_{s(4m-3)}\ \ \tilde{e}_{s(4m-2)} - \alpha\tilde{e}_{s(4m-3)}\ \ \tilde{e}_{s(4m-1)} - \alpha\tilde{e}_{s(4m-3)}\ \ \tilde{e}_{s(4m)}]^{\mathrm{T}} \in \mathbf{C}^{4\times 1}$,

and $\beta_1, \beta_2,\ldots \beta_M$ are the elements of the vector $\boldsymbol{\beta}$. We assume that the matrix $\mathbf{H}$ has full column rank. A *sufficient* condition for this to hold is $2M \geq N$, i.e., $M \geq \lceil N/2 \rceil$. The least squares solution to Eq. (25) is then given by

$$\boldsymbol{\eta} = (\mathbf{H}^{\mathrm{T}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}\boldsymbol{\chi}, \tag{27}$$

Substituting for $\boldsymbol{\eta}$ from Eq. (27) into Eq. (25), we obtain the estimate

$$\hat{r} = j\,(\alpha - \boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{\eta})^{-1}/2.$$

In practice, we estimate $\mathbf{R}$ from T snapshots of data vector as $\hat{\mathbf{R}}=\langle\tilde{\mathbf{d}}(t)\tilde{\mathbf{d}}^{\mathrm{H}}(t)\rangle_T$. The eigenvector $\tilde{\mathbf{e}}_s$ is estimated by $\hat{\tilde{\mathbf{e}}}_s = \mathbf{D}^{\dagger}(\hat{\phi})\boldsymbol{\Upsilon}_w^{1/2}\ \hat{\mathbf{e}}_s$, where $\hat{\mathbf{e}}_s$ is the eigenvector of $\hat{\mathbf{R}}$ corresponding to the largest eigenvalue and $\hat{\phi}$ is the estimate of ϕ obtained by averaging the estimates determined using Eq. (17) at each AVS. To estimate the source depth we need to now perform only a 1D search for the parameter $0<z_s<h$, which maximises the ambiguity function $A_F(\hat{r}, z_s)$ given by Eq. (18).

6. NUMERICAL STUDY

For the purpose of illustrating the approach described above, we consider the case of an idealized ocean model comprising a homogenous water layer of constant depth 75 m with a sound speed of c = 1500 m/s over a rigid bottom. A 25 Hz acoustic source is located at an azimuth of 45^0, range of 10 km and a depth of 37.5 m. The receiver is placed at z=18.75 m. For a channel with a hard bottom [5], the eigenfunctions and the eigenvalues are $\psi_n(z)=\sin(\gamma_n z)$ and $k_n^2=k^2-\gamma_n^2$ respectively, where γ_n for n=1,…,N are given by N real roots of the equation, $\gamma\cot(\gamma h)=0$. Further, the number of normal modes is given by $N=\lfloor 2h/\lambda \rfloor$, λ being the wavelength of the source. In the present numerical example, N works out to be 2.

Fig. 2 shows the ambiguity function plot in a search region of 1-15 km for range and 0-75 m for depth at an SNR of 20 dB. The AVS is at a depth of 18.75 m. Fig. 3 shows the results of range estimation through eigen-decomposition of spatial correlation matrix. Fig. 3 (a)-(c) shows root mean square error in the estimation of the azimuth angle, range and depth parameters, obtained through 300 Monte Carlo runs at various input SNR, employing $M=\lceil 2/2 \rceil=1$ sensor placed at 18.75 m. Fig. 3 (d)-(f) shows the RMS errors in the estimates corresponding to a channel of depth 105 m for which N works out to be 3. For this case, a vertical array of M=4 AVS was considered. CRB for azimuth and range estimation, computed for the present measurement model, is also shown for comparison. To compute CRB, the source is considered as a vector source consisting of N scalar sources corresponding to each normal mode in the oceanic waveguide [10, Theorem 3.1].

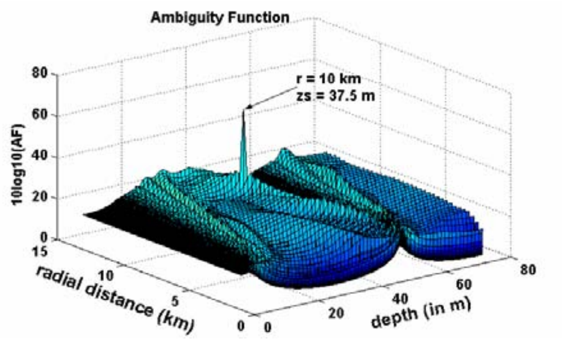

Figure 2 - Ambiguity Function. Range parameter is discretised into 2048 points from 1 km to 15 km and source depth parameter is discretised into 64 points from 0 to 75 m. Receiver depth is 18.75 m. N=2. SNR= 20 dB. r= 10 km, z_s= 37.5 m.

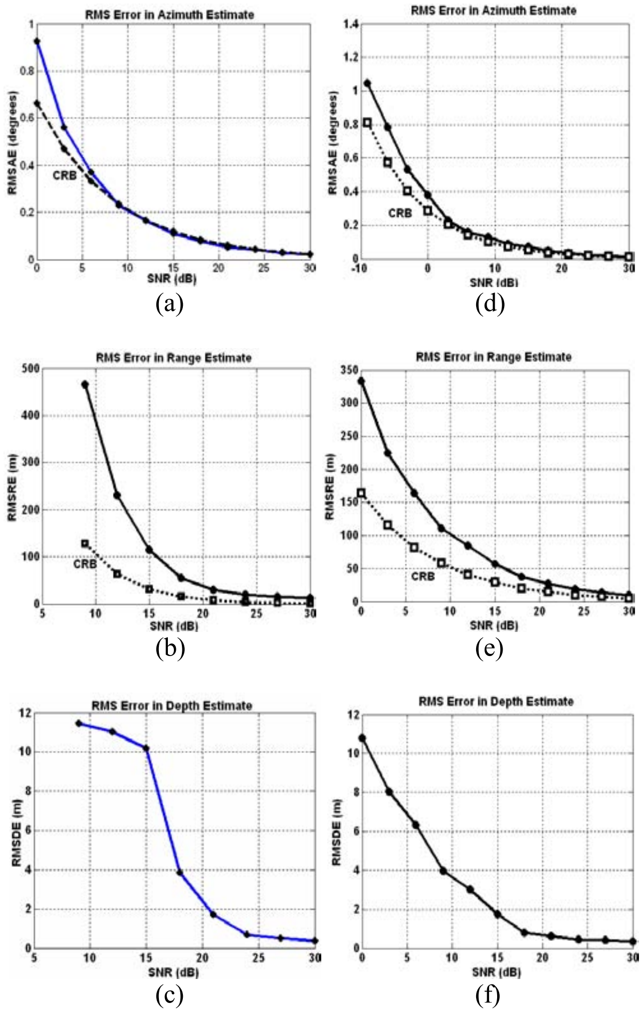

Figure 3 – RMS errors in azimuth angle, range and depth estimates at various SNR. Panels (a)-(c): *h*=75 m, *N*=2, and *M*=1. Panels (d)-(f): *h*=105 m, *N*=3, *M*=4.

7. CONCLUSION

A method was presented for 3D localisation of a narrowband acoustic source in shallow ocean using a vertical array of acoustic vector sensors. After developing an AVS measurement model using the normal mode theory of underwater sound propagation, it was shown that the horizontal component of the active intensity vector at an AVS points in the direction of the azimuth angle of the source. Range was then shown to be uniquely computable through an eigen-decomposition of the spatial autocorrelation matrix of the data vector from a vertical array of AVS avoiding the need to perform a computationally intensive numerical search. It was seen from simulation studies that azimuth estimation, with a single AVS, is quite accurate (RMS error < 1^0) above an SNR of 0 dB. For the numerical example here, the azimuth estimator achieved CRB for SNR>9 dB. However, range estimation error approaches CRB only for SNR above 20 dB. The performance can be improved by increasing the number of sensors in the array.

REFERENCES

[1] G.L. D'Spain, W.S. Hodgkiss, and G.L. Elmonds, "Energetics of the deep ocean's infrasonic sound field", *J. Acoust. Soc. Amer.*, vol. 89, no.3, pp. 1134-1158, March 1991.

[2] G.L. D'Spain, W.S. Hodgkiss, and G.L. Elmonds, "The Simultaneous Measurement of Infrasonic Acoustic Particle Velocity and Acoustic Pressure in the Ocean by Freely Drifting Swallow Floats", *IEEE Journal of Oceanic Engineering*, vol.16, no.2, pp. 195-207, April 1991.

[3] A. Nehorai and E. Paldi, "Acoustic Vector-Sensor Array Processing", *IEEE Transactions on Signal Processing*, vol. 42, no. 9, pp. 2481-2491, Sept. 1994.

[4] M. Hawkes and A. Nehorai, "Acoustic Vector-Sensor Processing in the Presence of a Reflecting Boundary", *IEEE Transactions on Signal Processing*, vol. 48, no.11, pp 2981-2992, Nov. 2000.

[5] B.G. Katsnelson and V.G. Petnikov, *Shallow-Water Acoustics*, Springer, London, 2002.

[6] J.A. Mann, III, J. Tichy, and A.J. Romano, "Instantaneous and time-averaged energy transfer in acoustic fields", *J. Acoust. Soc. Amer.*, vol.82, no.1, pp. 17-30, July 1987.

[7] S.S. Abeysekera, "A Non-linear Algorithm for Digital Beamforming of a Wideband Linear Active Sonar Array", *IEEE EURASIP Workshop on Nonlinear Signal and Image Processing*, vol. II, pp. 612-616, 1999.

[8] M.J. Buckingham, "A Theoretical Model of Ambient Noise in a Low-Loss Shallow Water Channel", *J. Acoust. Soc. Am.*, vol. 67, no. 4, pp. 1186-1192, April 1980.

[9] Arunkumar K.P., *3-Dimensional Localisation of Acoustic Sources using Vector Sensors*, M.E. Project Report, Indian Institute of Science, Bangalore (2007).

[10] A. Nehorai and E. Paldi, "Vector-Sensor Array Processing for Electromagnetic Source Localization", *IEEE Transactions on Signal Processing*, vol. 42, no. 2, pp. 376-398, Feb. 1994.