J. Astrophys. Astr. (1980) **1,** 165–175

Convective Instability in the Solar Envelope

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Received 1980 September 9; accepted 1980 November 5

Abstract. The characteristics of the most unstable fundamental mode and the first harmonic excited in the convection zone of a variety of solar envelope models are shown to be in reasonable agreement with the observed features of granulation and supergranulation.

Key words: sun—granulation—supergranulation—convective instability

1. Introduction

The existence of preferred length-scales in the solar atmosphere is a well established observational phenomenon which has a satisfactory explanation based on convective motions in the sub-photospheric layers of the Sun (Beckers and Canfield 1976). A considerable amount of effort has been invested in the investigation of polytropic atmospheres (Skumanich 1955; Böhm and Richter 1959). The fluid mechanical equations for an ideal gas with constant coefficients of viscosity and heat conductivity were set up by Spiegel (1965) for studying the convective instability. He found that for a layer of sufficiently small vertical extent the problem of compressible convection was essentially similar to the Boussinesq approximation. In order to understand the length scales and lifetimes observed on the solar surface Böhm (1963) calculated the linear growth rates of convective modes by perturbing the equilibrium solar convection zone model of Böhm-Vitense (1958). In this study the growth rates were found to increase monotonically with the wave number well past the observed cut-off and the size-distribution of the observed cells on the solar surface could not be satisfactorily explained by Böhm's calculations. Later Böhm (1976) attempted to include the effects of turbulent conductivity and viscosity on the convective modes. This investigation which was restricted to the problem of the onset of instability indicated that the fundamental mode with a wavelength of ~ 1500 km could be identified with the granulation by choosing the parameter α occurring in the expression for turbulent

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viscosity to be of order unity; for a larger value of α the first harmonic with a wavelength of ~ 30,000 km was identified with supergranulation.

Any reasonable theoretical model must account for the distinct peaks exhibited by the cellular structures observed on the solar surface. The work of Antia, Chitre and Pandey (1980; hereinafter referred to as Paper I) was an attempt to explain the observed motions on the solar surface in terms of linear convective modes excited in a realistic solar envelope model (Spruit 1977) by incorporating the mechanical and thermal effects of turbulence through the eddy transport coefficients (Unno 1967).

In the absence of a satisfactory theory of time-dependent compressible convection the effects of turbulence on the mean flow were parametrized through turbulent transport coefficients calculated in the framework of the mixing-length formalism of Böhm-Vitense (1958). Since in the convection zone the turbulent heat conductivity is orders of magnitude larger than the radiative conductivity, the turbulence is expected to have a significant influence on the growth rates both through the modulation of the heat flux and through the Reynolds stresses. This was indeed borne out by the detailed computations of Paper I and it was demonstrated that the most rapidly growing fundamental mode and the first harmonic are in reasonable accord with the scales of motion corresponding to granulation and supergranulation.

The stability analysis in the earlier calculation of Antia, Chitre and Pandey (1980) was performed under certain approximations. In order to make the problem tractable certain simplifying assumptions were introduced, like the neglect of the perturbation of the urbulent thermal conductivity in the expression for the convective flux and of the perturbation in the adiabatic term occurring in the superadiabatic temperature gradient. Moreover, the effect of variation of the degree of ionization in the convective elements was not taken into consideration. This situation is remedied in the present work to find that all these effects in combination lead to a damping of the convective growth rates. The turbulent Prandtl number which is a measure of the relative importance of turbulent viscosity over the turbulent heat conductivity is treated as a free parameter in the investigation. Thus, in order to bring the scales of the most unstable modes in accord with granulation and supergranulation it is necessary to lower the value of the Prandtl number compared to the value found appropriate in the earlier calculation reported in Paper I.

2. Governing equations

We adopt the usual hydrodynamical equations for the conservation of mass, momentum and energy as being applicable to a thermally conducting viscous fluid layer. In the notation of Paper I these equations take the following form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = - \nabla P + \rho \mathbf{g} - \tfrac{2}{3} \mu \nabla (\nabla \cdot \mathbf{v}) - \tfrac{2}{3} (\nabla \cdot \mathbf{v}) \nabla \mu + \nabla \cdot [\mu (\nabla \mathbf{v} + \mathbf{v} \nabla)],$$

$$\rho C_P \left[\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla) T - \nabla_{ad} \frac{T}{P} \left(\frac{\partial P}{\partial t} + (\mathbf{v} \cdot \nabla) P \right) \right] = - \nabla \cdot \mathbf{F} + \Phi,$$

where Φ is the rate of viscous dissipation given by

$$\Phi = \tfrac{1}{2}\,\mu\,(\nabla \mathbf{v} + \mathbf{v}\nabla)\cdot(\nabla \mathbf{v} + \mathbf{v}\nabla) - \tfrac{2}{3}\,\mu\,(\nabla\cdot\mathbf{v})^2.$$

We treat the medium as a perfect gas undergoing ionization and we include the contribution to the pressure due to radiation. In the foregoing equations μ is the coefficient of dynamic viscosity, C_P the specific heat at constant pressure, ∇_{ad} is the logarithmic adiabatic gradient $(\partial \ln T/\partial \ln P)_{ad}$ and F is the total heat flux which is the sum of the radiative flux, F^R and the convective flux, F^C.For the computation of the radiative flux we use the Eddington approximation (Ando and Osaki 1975) and write

$$\mathbf{F}^R = -\frac{4}{3\kappa\rho}\nabla J,$$

where

$$J = \sigma T^4 + \frac{C_P}{4\kappa}\left\{\left[\frac{\partial T}{\partial t} + (\mathbf{v}\cdot\nabla)\,T\right] - \nabla_{ad}\frac{T}{P}\left[\frac{\partial P}{\partial t} + (\mathbf{v}\cdot\nabla)\,P\right]\right\}$$

is the intensity of radiation and κ the mean Rosseland opacity. The convective flux is computed in the usual mixing length formalism by writing

$$\mathbf{F}^C = -K_t\left[\nabla T - \nabla_{ad}\frac{T}{P}\nabla P\right],$$

where the coefficient of turbulent heat conductivity is taken to be of the form

$$K_t = \alpha\,\rho\,C_P\,WL.$$

Here α is the efficiency factor which is of order unity, L is the mixing length and the mean convective velocity W is given by

$$W = \left[\beta\,\frac{g}{H_P}\,Q\,L^2\,(\nabla - \nabla_{ad})\right]^{1/2}.$$

H_P is the pressure scale-height, β represents the effect of viscous braking on the convective elements and the factor

$$Q = -\frac{T}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_P$$

takes into account the variation of the degree of ionization in the moving element. The turbulent dynamic viscosity is chosen to have the expression,

$$\mu_t = P_t\,\alpha\,\rho\,WL,$$

where the turbulent Prandtl number P_t is treated as a free parameter in the present investigation.

We have adopted a number of equilibrium solar envelope models in order to study the instability of convective modes. The requirement for the models is that the physical run of variables should match with the interior solutions and they should also be consistent with the solar evolution generating the present radius and luminosity of the sun. In the solar envelope model due to Spruit (1977) the mixing length parameters take the form:

$$\alpha = \tfrac{1}{4},\ \beta = \tfrac{1}{8},\ Q = 1 \text{ and } L = z + 459 \text{ km},$$

z measured downwards from the top of the convection zone. We have also generated a model with the above set of parameters for $Q \neq 1$. Moreover, we have computed the solar convection zone models with the following sets of mixing-length parameters:

$$\alpha = \tfrac{1}{2},\ \beta = \tfrac{1}{4},\ Q \neq 1,\ L = 1{\cdot}5\ H_P,$$

$$\alpha = \tfrac{1}{2},\ \beta = \tfrac{1}{4},\ Q \neq 1,\ L = H_\rho\ (H_\rho, \text{ density scale-height}),$$

$$\alpha = \tfrac{1}{2}, \beta = \tfrac{1}{8},\ Q \neq 1, L = 1{\cdot}25\ H_\rho.$$

For the atmosphere we have adopted the empirical temperature-optical depth (T—τ) relationship given by Vernazza, Avrett and Loceser (1976), with the upper boundary chosen a little below the temperature minimum where $\tau = 7\times10^{-4}$. The lower boundary for the layer is fixed at a depth of $3{\cdot}3 \times 10^5$ km. There is a penetration of convective elements into these overlying stable layers. Clearly the velocity does not drop abruptly to zero at the boundary of the convection zone ($\tau \simeq 1$), and there is an overshoot of convective motion into the bounding regions. The height variation of the convective velocity field given by Canfield (1976) suggests that the amplitude of granular velocities is an exponentially decreasing function with a scale height ~150 km. In order to estimate the coefficient of dynamic viscosity in the atmosphere we assume a Kolmogoroff spectrum with turbulent velocities proportional to one-third power of the scale-length. We ensure the continuity of the viscosity coefficient across the interface between the convection zone and the atomsphere. After taking into account the almost exponential decrease of density with height, we find the viscosity coefficient drops exponentially with a scale height of approximately 25 km.

We adopt the spherical geometry and assume that any physical quantity can be expressed as

$$f(r, \theta, \phi, t) = f_0(r) + f_1(r)\ Y_l^m(\theta, \phi) \exp(\omega t),$$

where the subscripts 0 and 1 respectively refer to the equilibrium and perturbed quantities, (r, q, ϕ) are the spherical polar coordinates with the radial coordinate r measured from the centre of the Sun, $Y_l^m(\theta, \phi)$ are the spherical harmonics and ω is the growth rate. We linearize the governing equations in the usual manner to get the following system of equations:

$$\omega \rho_1 + \nabla \cdot (\rho_0 \mathbf{v}) = 0,$$

$$\omega \rho_0 \mathbf{v} = \rho_1 \mathbf{g} - \nabla P_1 - \tfrac{2}{3} \mu_t \nabla (\nabla \cdot \mathbf{v}) - \tfrac{2}{3} (\nabla \cdot \mathbf{v}) \nabla \mu_t + \nabla \cdot [\mu_t (\nabla \mathbf{v} + \mathbf{v} \nabla)],$$

$$\rho_0 C_P \left[\omega T_1 + (\mathbf{v} \cdot \nabla) T_0 - \nabla_{ad} \frac{T_0}{P_0} (\omega P_1 + (\mathbf{v} \cdot \nabla) P_0)\right] = - \nabla \cdot \mathbf{F}_1,$$

$$\mathbf{F}_1 = - \frac{4}{3\kappa_0 \rho_0} \nabla J_1 - \mathbf{F}_0^R \frac{\kappa_1}{\kappa_0} - \mathbf{F}_0^R \frac{\rho_1}{\rho_0}$$

$$- K_{t0} \left(\nabla T_1 - \nabla_{ad} \frac{T_0}{P_0} \nabla P_1 - \left(\nabla_{ad} \frac{T}{P}\right)_1 \nabla P_0 \right) - K_{t1} \left(\nabla T_c - \nabla_{ad} \frac{T_c}{P_0} \nabla P_0 \right),$$

where

$$J_1 = 4\sigma T_0^3 T_1 + \frac{C_P}{4\kappa_0} \left[\omega T_1 + (\mathbf{v} \cdot \nabla) T_0 - \nabla_{ad} \frac{T_0}{P_0} (\omega P_1 + (\mathbf{v} \cdot \nabla) P_0)\right],$$

$$\kappa_1 = \left(\frac{\partial \kappa}{\partial P}\right)_T P_1 + \left(\frac{\partial \kappa}{\partial T}\right)_P T_1,$$

$$\rho_1 = \left(\frac{\partial \rho}{\partial P}\right)_T P_1 + \left(\frac{\partial \rho}{\partial T}\right)_P T_1$$

Here

$$\mathbf{F}_0^R = - \frac{16\sigma T_0^3}{3\kappa_0 \rho_0} \nabla T_0$$

is the radiative flux in the steady state and the velocity v is assumed to have the form,

$$\mathbf{v} = \left(v_r (r), v_h (r) \frac{\partial}{\partial \theta}, v_h (r) \frac{1}{\sin \theta} \frac{\partial}{\partial \phi} \right) Y^m (\theta, \phi) \exp (\omega t).$$

In deriving these equations we have incorporated the perturbation in the turbulent heat conductivity K_t including the variation in the specific heat at constant pressure, C_P, the perturbation in the adiabatic term (∇_{ad} T/P) occurring in the superadiabatic temperature gradient and also included the factor Q in the expression for the convective velocity arising from the variation of the degree of ionization. We have, however, neglected the effect due to viscous dissipation in the energy equation which is liable to influence to certain extent the length scales of most unstable modes, especially the higher harmonics.

The total system of equations governing the perturbations is of the sixth order and we therefore require three boundary conditions at each interface. We have emphasized in Paper I that the exact conditions are not very important for convective growth rates since the boundary conditions are applied a little beyond the convection zone where in any case the amplitude of the modes falls off very rapidly. Consequently the convective growth rates turn out to be insensitive to the particular choice of the boundary conditions. For the purpose of the present analysis we have selected free boundary conditions, that is, the Lagrangian perturbation in the pressure and the tangential components of the viscous stress tensor vanish at the surface to give

$$\omega P_1 - g \rho_0 v_r = 0,$$

$$v_r + r \frac{dv_h}{dr} - v_h = 0.$$

Furthermore, we impose the thermal boundary condition which demands that the radiation does not come in from infinity, and this gives

$$\omega F_r - \frac{2 F_0^R}{r} v_r - \frac{J_1 F_0^R \omega}{\sigma T_0^4} - \frac{4 F_0^R v_r}{T_0} \frac{dT_0}{dr} = 0.$$

The boundary conditions at the lower interface are found to have no effect whatsoever on the convective growth rates since the eigenfunctions decay exponentially with depth in these regions. We therefore adopt the rigid conditions with no momentum flux and with the temperature maintained constant at the interface; that is, we take

$v_r = 0$, $v_h = 0$ and $T_1 = 0$.

The numerical scheme for solving this generalised eigenvalue problem is the same as that adopted in Paper I.

3. Numerical results and discussion

We attempt to account for the observed motions on the solar surface in terms of linear convective modes excited in the subphotospheric convection zone. For this purpose we shall first compute a variety of solar envelope models by integrating the standard equations of stellar structure. The mixing-length theory first developed, in the context of stellar structure, by Böhm-Vitense (1958) still remains the only viable method for treating convective transport in the model calculations. In so far as the choice of the mixing-length itself is concerned there is no compelling reason why it should be a constant multiple of the pressure scale height or the density scale height, or should be proportional to the distance from the boundary of the convection zone. We have therefore selected different sets of mixing-length parameters and the characteristic physical values for five models, I—V are summarised in Table 1. It is clear that at the base of the convection zone all the models essentially converge

Table 1. Physical parameters for various models, ρ_b, T_b denote the density and temperature at the base of the convection zone measured from the level $\tau = 1$ and $V_{\max}$ the maximum convective velocity km s^{-1}. L is the mixing-length, H_p, the pressure scale height and H_ρ the density scale height. The parameter $Q \neq 1$ except in Spruit's model (IV).

Model	Mixing-length parameters α	β	L	ρ_b (g cm^{-3})	T_b (K)	Z_0 (km)	$V_{\max}$ (km s^{-1})
I	1/2	1/4	$1{\cdot}5\ H_p$	0·234	$2{\cdot}178\times10^6$	$1{\cdot}90\times10^5$	3·19
II	1/2	1/4	H_ρ	0·226	$2{\cdot}174\times10^6$	$1{\cdot}86\times10^5$	3·61
III	1/2	1/8	$1{\cdot}25\ H_\rho$	0·243	$2{\cdot}180\times10^6$	$1{\cdot}90\times10^5$	3·06
IV	1/4	1/8	$z+459$ km $Q=1$ (Spruit 1977)	0·228	$2{\cdot}179\times10^6$	$1{\cdot}92\times10^5$	3·85
V	1/4	1/8	$z+459$ km	0·260	$2{\cdot}180\times10^6$	$1{\cdot}96\times10^5$	3·85

Table 2. Approximate *e*-folding times and preferred horizontal wavelengths corresponding to the most unstable fundamental mode ($C1$) and the first harmonic ($C2$) for a variety of models with different mixing-length parameters over a range of turbulent Prandtl numbers P_t.

					Fundamental mode ($C1$)		First harmonic ($C2$)	
Model	Mixing-length parameters α	β	L	P_t	*e*-folding time (min)	Preferred horizontal wave-length (km)	*e*-folding time (h)	Preferred horizontal wavelength (km)
				1/10	5·5	1900	0·4	2900
				1/4	6·3	2150	0·6	3400
I	1/2	1/4	$1{\cdot}5\ H_p$	1/3	6·7	2300	0·8	4400
				1	9·3	2900	4·1	8000
				$1\frac{1}{3}$	11·0	3100	39·0	11500
				1/10	5·3	1900	0·3	2500
				1/4	6·0	2200	0·4	3150
II	1/2	1/4	H_ρ	1/3	6·2	2300	0·5	3550
				1	9·0	2900	2·1	6600
				$1\frac{1}{2}$	10·7	3300	40·1	9100
				1/10	5·9	2100	0·3	2800
				1/4	6·4	2250	0·5	3250
III	1/2	1/8	$1{\cdot}25\ H_\rho$	1/3	6·8	2450	0·6	4150
				1	9·7	3000	3·1	7400
				$1\frac{1}{3}$	11·0	3200	21·0	8800
				1/10	6·0	1900	0·6	3100
				1/4	7·0	2200	1·5	4800
IV	1/4	1/8	$z+459$ km	1/3	7·5	2350	2·8	5500
	$Q=1$	(Spruit 1977)		1/2	8·6	2450	15·0	9000
				1	12·0	3100	—	—
				1/10	7·0	1800	1·2	3700
V	1/4	1/8	$z+459$ km	1/4	8·7	2200	7·7	8400
				1/3	9·6	2400	32·0	11000
				1	19·0	3400	—	—

to the same radiative interior solution. Having obtained the run of the equilibrium physical quantities with depth, the system of linearized equations is solved with the boundary conditions described earlier. The real growth rate ω is then computed as a function of the angular node number l which is related to the wavelength

$$\lambda = \frac{2\pi}{k_H} = \frac{2\pi R_{\odot}}{[l(l+1)]^{1/2}},$$

k_H being the horizontal wave number and R the solar radius. For a given value of l there exists a series of eigenvalues ω, which are classified according to the number of velocity nodes in the radial direction. Thus the fundamental mode ($C1$) which has the largest eigenvalue has no node in the radial velocity component, while the successive harmonics, $C2$, $C3$,.. have one extra node in the radial direction. It is of interest to enquire whether the resulting unstable convective modes for these models exhibit preferred length scales and time scales which are in some reasonable accord with the observed features on the solar surface.

In Table 2 we have summarised for various envelope models the time scales and scales of motion corresponding to the most unstable fundamental mode ($C1$) and the first harmonic ($C2$) for a number of turbulent Prandtl numbers, 0.1 P_t, 1.5. The *e*-folding times and the horizontal wavelengths for the maximally growing fundamental mode lie in the range 5–19 min and 1,900–3,400 km respectively over the range of the turbulent Prandtl numbers considered. The fundamental mode is evidently not very sensitive to the variation in different parameters, but the first harmonic is critically affected both by the choice of the mixing-length parameters and the value of the turbulent Prandtl number. The effect of turbulent viscosity on the convective growth rates of $C1$ and $C2$ modes is displayed in Fig. 1. Here we have shown for a typical solar envelope model with the mixing-length parameters, $\alpha = \frac{1}{2}$, $\beta = \frac{1}{4}$, L=l·5 H_P, the growth rate ω (in s^{-1}) as a function of the horizontal wave number for four values of the turbulent Prandtl number $P_t = 0, \frac{1}{3}, 1, 1\frac{1}{3}$. It is readily seen that for each value of the Prandtl number there is a maximal growth rate, and the damping influence of turbulent viscosity is clearly seen from the trend of the preferred length scales as well as the associated *e*-folding times to increase with P_t. For non-zero values of P_t, the maximum in the growth rates is found to shift to lower *l's for* successive harmonics, but for the inviscid case ($P_t = 0$) all the modes peak at about the same value of $l (\lesssim 3000)$. Thus, without the inclusion of viscosity in the problem it would not be possible to produce different scales of motion observed on the solar surface. An important effect brought about by viscosity is that the modes are highly damped for higher values of the harmonic number and for a given value of l only the first few harmonics turn out to be unstable.

An inspection of Table 2 immediately shows that for each of the models which we have investigated there exists a value of the turbulent Prandtl number for which the time scales and the associated wavelengths of the most unstable fundamental mode and the first harmonic can be made to agree reasonably well with the observed lifetimes and cell-sizes of granulation and supergranulation. Thus, for models I—III the choice of $P_t \sim 1\frac{1}{3}$ yields, for the most unstable $C1$ mode, time scale ~10 min and length scale ~ 3000 km which are fairly close to the characteristic scales corresponding to granulation. The time scale for the most rapidly growing $C2$ mode, ~30 hrs, is in accordance with the typical observed lifetime of supergranules, but the related wavelength tends to be on the lower side of the usually quoted diameters of super-

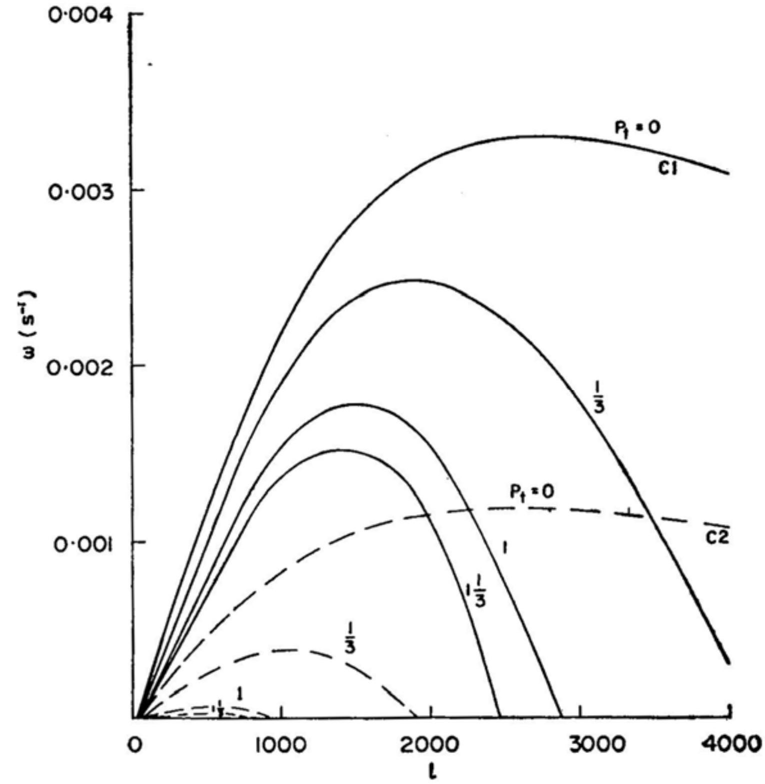

Figure 1. The growth rate ω (s^{-1}) of the fundamental mode ($C1$), shown by the full curves and the first harmonics ($C2$) shown by the broken curves, is displayed against the horizontal harmonic number l for a range of turbulent Prandtl numbers $P_t = 0,1/3,1, 4/3$.

granules ranging upwards of 10,000 km with a peak around 30,000 km. In models IV and V the value of $P_t \gtrsim \frac{1}{3}$ produces satisfactory lifetimes for granulation and supergranulation. But, while the preferred wavelength for the fundamental mode comes close to the granular cell-size, the corresponding wavelength for the first harmonic is somewhat on the shorter side.

In the earlier work of Antia, Chitre and Pandey (1980), with the choice of the turbulent Prandtl number $P_t \sim 1{\cdot}5$ the e-folding time and the associated wavelength for the most unstable fundamental mode and the first harmonic turned out to be not too far from the observed features of granulation and supergranulation respectively. The computation was based on Spruit's solar envelope model and there were some approximations introduced to make the calculation tractable. This situation has been remedied and the present investigation incorporates the additional terms arising from the perturbation of the turbulent heat conductivity, K_t and the adiabatic gradient, ($\nabla_{ad}\ T/P$). We have also taken account of the Q-factor in the mean convective velocity which was taken to be unity in the previous calculation. With the change in the degree of ionization in the moving convective elements, the value of Q varies between 1 and 2 and this leads to a damping of the convective modes because of the effective increase in the magnitude of turbulent heat conductivity. There is another difference in the way the viscosity is treated in the overlying atmosphere. In the present study we have taken a Kolmogoroff spectrum for the penetrative motion with turbulent velocities being proportional to one-third power of the scale length and also assumed a velocity profile from the observations of the motion at various heights. With the near-exponential decrease of the density in the atmospheric layers, the effective viscosity scale height in the regions above the convection zone turns out to be ~25 km, while in the earlier work we had assumed the viscosity scale height to be 10 km. The larger viscosity scale height has a more pronounced stabilizing effect on the convective modes, as in this case the effect of viscous damping in the overlying layers persists to a larger extent. All these effects combine to lower the resulting convective growth rates.

The influence of the new terms incorporated in the present investigation is shown ($\alpha = \frac{1}{2}$, $\beta = \frac{1}{4}$, $L = 2\ H_P$, $P_t = \frac{1}{4}$.) Case (a) corresponds to the earlier calculation of Antia, Chitre and Pandey (1980) where the perturbation in the turbulent conductivity, K_t and the perturbation of ($\nabla_{ad}\ T/P$) in the superadiabatic temperature gradient term are neglected. The dimensionless convective growth rates for $C1$ and $C2$ modes are displayed for the horizontal harmonic number l = 100, 500, 1000, 1500, 2000 and 3000 with the Q-factor taken to be unity. In case (b) the perturbations in K_t and ($\nabla_{ad}\ T/P$) are fully included, but the Q-factor is again set to unity, to find that the growth rates are lowered over those in case (a). In case (c) the perturbations in K_t and ($\nabla_{ad}\ T/P$) as well as the variation of the Q-factor are taken into account. The resulting growth rates are seen to be drastically reduced and in order to bring the time scales of the most unstable modes close to the observed solar motions it becomes necessary to lower the value of the turbulent Prandtl number over the corresponding value (P_t~1·5) found suitable in Paper 1.

We cannot show an overwhelming preference for any particular solar envelope model from the computed eigenvalue spectrum of the convective modes. In fact, it turns out that for a given set of parameters α and β, it is always possible to construct an envelope model which, for a selected value of the mixing-length L and the turbulent Prandtl number gives convective modes in reasonable accord with the characteristic features associated with the observed solar velocity fields. However,

Table 3. Convective growth rates in units of $(3263\ \mathrm{s})^{-1}$ corresponding to the fundamental mode and the first harmonic for the solar envelope model with the mixing-length parameters: α =1/2. β = 1/4, L/H_p=2·0 are listed for various values of the horizontal harmonic number.

(a)	$Q=1$;	$(K_t)_1$,	$(\nabla_{ad}\ T/P)_1$ are suppressed			
	l = 100	500	1000	1500	2000	3000
$C1$	0·34	3·23	5·79	6·76	6·50	3·52
$C2$	0·08	0·81	1·16	0·77	—	—
(b)	$Q=1$;	$(K_t)_1$,	$(\nabla_{ad}\ T/P)_1$ are included			
	l = 100	500	1000	1500	2000	3000
$C1$	0·25	2·59	4·81	5·75	5·61	3·04
$C2$	0·03	0·50	0·76	0·39	—	—
(c)	$Q\neq1$;	$(K_t)_1$,	$(\nabla_{ad}\ T/P)_1$ are included			
	l=100	500	1000	1500	2000	3000
$C1$	0·16	1·83	3·52	4·13	3·79	1·05
$C2$	0·01	0·20	0·12	—	—	—

any consistent solar envelope model should yield, for the same choice of the turbulent Prandtl number, not only the spectrum of most unstable convective modes corresponding to granulation and supergranulation, but should also reproduce the acoustic modes with the characteristics of five minute oscillations. We hope to discuss this problem in a separate communication.

It should be stressed that we have examined the instability of convective modes in the framework of the linearized theory. The linear stability analysis yields the growth rate indicating the manner in which a perturbation begins to grow from an equilibrium state, while the non-linear effects can alone limit the growing amplitudes of these instabilities. The *e*-folding time given by the inverse of the growth rate can only suggest the approximate time scale over which the instability grows until the effects neglected in the linear formulation become important. The lifetime of a granule must necessarily be determined by incorporating the nonlinear effects.

In conclusion, it may be stated that considering the uncertainties in observations as well as in the mixing-length formalism, our numerical results provide a reasonable explanation for granulation and supergranulation in terms of the unstable fundamental mode and the first harmonic excited in the solar convection zone. It should, however, be stressed that we have ignored potentially significant effects in our calculation like the contribution of the turbulent pressure and the presence of the turbulent energy and its dissipation into heat in the equation for conservation of energy. The neglect of these effects is liable to influence the growth rates at larger values of l especially for the higher harmonics. This, along with the extension into the non-linear regime will, hopefully, improve the agreement of the unstable convective modes with observed velocity fields on the solar surface.

Acknowledgement

It is a pleasure to thank Dr H. M. Antia for valuable correspondence.

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