SOLAR CYCLE THEORY OF PULSARS

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Abstract

The plausibility of the Solar Cycle type of mechanism for pulsar phenomenon is examined. The relevant time scales are found to be within the range of the observed pulsar periods.

We wish to examine the proposition that the qualitative features involved in the pulsar mechanism are probably very similar to those responsible for the solar cycle. It is now generally accepted that most solar activity is related to changes occurring in the magnetism of the sun. The main dipole field of the sun which is essentially axisymmetric and has a mean intensity of order 1 gauss is usually limited to high latitudes and its polarity is found to reverse in successive 11-year cycles. The sun is by no means unique in exhibiting a magnetic cycle. Babcock[1] has reported magnetic variations over a period of 4–9 days for stars of spectral type Ap. The fact that the typical magnetic variables are characterised by the presence of fields which are some thousand times larger than that of the sun and that these fields exhibit reversal of polarity within relatively short periods suggests that such variations could conceivably occur for magnetic fields of order 10^{11} gauss which are usually associated with neutron stars.[2] It would seem worthwhile to examine if the same physical process is operating for the sun magnetic variables and neutron stars. We set out in Table I some typical characteristics of these three types of stars, where the entries for the Sun and Magnetic Variables are based on observations, while for Pulsars the magnetic field strength is deduced from the compression of the interstellar gas with its associated magnetic field to the typical density appropriate for neutron stars.

The table clearly indicates an inverse correlation between the period and the strength of the magnetic field. It is tempting to think of pulsars as magnetic neutron stars with the magnetic variations occurring at a much faster rate and the resulting emission of energy arising from a flare-type of activity analogous to the solar flares.

TABLE I

	The Sun	Magnetic variables	Pulsars
Magnetic field (gauss)	1	10^{3}–10^{4}	10^{10}–10^{12}
Observed cycle period (sec.)	3×10^{8}	10^{5}–10^{6}	3×10^{-2}–3

The solar cycle theory[3, 4] makes use of the observed differential rotation of the sun whose equatorial regions rotate faster than the higher latitudes. The initial poloidal magnetic field that connects the polar caps can be drawn out longitudinally by the differential rotation to form a much stronger toroidal field in the sub-surface layers. The conversion of the rotational energy into magnetic energy proceeds until a critical limit is reached when an instability sets in and the magnetic buoyancy is then supposed to lift the flux tubes to the surface to produce bipolar regions and spot groups which usually occur in high latitudes in the early part of the cycle and later near the equatorial region reaching it towards the end of the cycle. As the cycle progresses it is then assumed that the preceding parts of the bipolar regions drift towards the equator from both sides and get neutralised there by merging, while following spots migrate towards the poles where the neutralisation of their lines of force results in a new reversed poloidal field. In Parker's model the twisting by coriolis forces exerted on the material moving in convective cells is invoked to yield a new reversed poloidal field.

The obvious difference between solar and stellar fields is in magnitude and there is no reason to suppose that the solar cycle type of theory cannot be extended to other stars. Babcock[1] has, in fact, proposed that the stellar magnetic variables which have periods about one thousandth of that of the sun probably follow an enhanced solar cycle. The solar cycle theory is essentially based on the hydromagnetic interaction between the magnetic field, rotation and convection. It involves a mechanism which demands the occurrence of a certain type of motion which must consist of differential rotation which is an observational fact and also meridional circulation which could be inferred from the evidence of the migration of the sunspot zone.

The periods of variation of the magnetic field would depend on the angular velocity and on the sub-surface convection.[5] Admittedly the surface convection zone in magnetic stars is much thinner and weaker than in the sun, but then the angular velocity, with which the hydromagnetic process is rather intimately connected, is some thirty times that of the sun and it is conceivable that there is a much faster hydromagnetic cycle of a similar kind operating in magnetic variables. As for degenerate stars it has also become clear from the recent work of Böhm[6] and Hubbard[7] that there is a possibility of convection in the envelope of white dwarfs. It does not therefore seem altogether implausible to imagine that neutron stars have convective envelopes resulting from the super-adiabatic temperature gradient in the layers extending over a few cm. below the surface, but whether convection as we understand it for ordinary matter occurs for neutron stars has not yet been decisively settled. In what follows we shall assume that the usual hydromagnetic equations are applicable for neutron star matter.

In order to maintain a quasi-steady state for a rotating star with a poloidal magnetic field which is symmetrical about the axis of rotation, the angular velocity Ω has to be constant along a line of force in the absence of any circulation. But if there is a meridional circulation of matter, clearly there will be a transport of angular momentum causing a variation of the angular velocity along a line of force. The angular velocity Ω and the circulation velocity v then satisfy the equation[8] $\Omega - v\mathrm{H}_\phi/r\mathrm{H}_p = \text{constant}$ along a line of force. Here H_p is the strength of the poloidal component and H_ϕ that of the toroidal component of the magnetic field and r is the perpendicular distance from the axis of rotation. This gives us on an order-of-magnitude basis $v \leqslant \Omega r \mathrm{H}_p/\mathrm{H}_\phi$. Furthermore, balancing the angular momentum transported by circulation against the transport by magnetic stresses we get

$$\rho v \cdot \nabla (\Omega r^2) = r \left| \frac{\text{curl } H \wedge H}{4\pi} \right|_\phi$$

which, again, on an order-of-magnitude basis yields

$$v\Omega r = \frac{\mathrm{H}_p \mathrm{H}_\phi}{4\pi\rho}$$

and then for sub-Alfve'nic velocity of circulation the differential angular velocity Ω is given by[8]

$$\frac{d\Omega}{\Omega} = \frac{4\pi\rho v^2}{\mathrm{H}_p^2} \cdot$$

The above set of equations is adequate to give an estimate of H_ϕ, v and $d\Omega$ once we prescribe typical values for H_p, Ω and ρ, provided r is taken to be of the order of the radius of the star. With the insertion of typical numbers for the solar surface layers we recover $H_\phi \simeq 400$ gauss, the average field strength for the toroidal flux tube responsible for producing sunspots; the circulation velocity $v \lesssim 5 \times 10^2$ cm./sec. in reasonable agreement with the drift of the sunspot zone; and the differential rotation $d\Omega \simeq 0 \cdot 1\ \Omega$, consistent with the observed equatorial acceleration. The corresponding numbers for a neutron star turn out to be $H_\phi \simeq 10^{11}$ gauss, $v \lesssim 10^7$ cm./sec. and $d\Omega \simeq 0 \cdot 1\ \Omega$. Alternately we could have obtained an estimate for H_ϕ from the energy released in flare radiation during a cycle. For the case of the sun the flare energy of the order of 10^{35} ergs is released within a volume with linear dimension $\sim 10^9$ cm. and this leads to $H_\phi \simeq 500$ gauss. For the neutron star in the Crab nebula taking the energy release in a cycle to be of order 10^{37}–10^{38} ergs and the associated volume to have a linear dimension $\sim 10^5$ cm., we recover $H_\phi \simeq 10^{11}$ gauss.

The critical limit reached by the magnetic field which is being drawn out longitudinally by the differential rotation is presumably set by conditions of instability and buoyancy of the flux tubes, which come into operation when the toroidal field has become sufficiently strong. The time taken to reach this critical value essentially places a lower limit on the period of the cycle τ; it can be deduced from the hydromagnetic equation:

$$\frac{\partial H}{\partial t} = \text{curl}\,(V \wedge H)$$

which, on an order-of-magnitude basis may be written as $\tau \gtrsim H_\phi / 10\ H_p d\Omega$. We have assumed here that the lines of force of the toroidal field are submerged only to a relatively thin layer of the order of (1/10) R below the surface; this is not altogether unnatural if we suppose that the toroidal girdle lies at a depth comparable with the separation of sunspot pairs. For the case of the sun with $H_\phi \simeq 400$ gauss, $H_p \simeq 1$ gauss and $d\Omega \simeq 0 \cdot 1\ \Omega$, we get $\tau \gtrsim 1 \cdot 3 \times 10^8$ sec. (~ 4 yrs.), while for a neutron star, with $H_\phi \simeq 10^{11}$ gauss, $H_p \simeq 10^{11}$ gauss and $d\Omega \simeq 10 \cdot 1\ \Omega$, we obtain $\tau \simeq 1/\Omega$, which can be as low as 10^{-3} sec. leading to possible periods in the range of milliseconds.

Before we put forward the argument that the solar cycle type of phenomenon can occur in neutron stars, we should be able to establish that the flux tubes of the toroidal girdle do indeed float to the surface sufficiently fast so that the time of rise is shorter than the period itself. It is evident that in the flux tubes of locally enhanced toroidal field, the gas pressure must necessarily be lower than that outside at the same level, since we have for quasi-static equilibrium the pressure balance.

$$P_{\text{flux tube}} + \frac{H^2}{8\pi} = P_{\text{outside}}.$$

This leads to a reduction in the density in the tubes which accordingly acquire a magnetic buoyancy and rise to the surface. The r.m.s. velocity of rise of a flux tube may be approximately given by

$$\langle u^2 \rangle = gh \frac{\triangle\rho}{\rho}$$

where g is the acceleration due to gravity, $\triangle\rho$ is the density difference between the flux-tube and outside and h the scale-height. The use of the equation of pressure balance and the equation of state in the form $P = \mathcal{R}\rho T$ yields

$$\langle u^2 \rangle \simeq gh \frac{H^2/8\pi}{\mathcal{R}\rho T}$$

which, for neutron stars, produces the time of rise of the order of a few milliseconds.

The energy for pulsar emission is supposed to arise from the conversion of magnetic energy into electromagnetic radiation. The time-scale for the dissipation of magnetic energy in a neutron star is estimated to be of the order of 10^8 years.[9] However, it is known from the case of the sun that the dissipation time for the local activity (flares) is very much shorter than the time-scale for the overall ohmic dissipation. It is therefore conceivable that there exists a sufficiently fast local hydromagnetic dissipation mechanism operating in an active region on a neutron star on a much shorter time-scale.

We have so far demonstrated the plausibility of the pulsar phenomenon being similar to the mechanism responsible for the solar type of cycle. Many details of course remain to be explored; for example, the division of the available energy released in a flare-type of activity at the surface into the electromagnetic radiation of different wavelengths; there will also be electromagnetic radiation emitted by the magnetic field fluctuating because of the reversal of its polarity during each cycle similar to the radiation in the oblique rotator model for pulsars.[10, 11] There then remains the question about the double pulses and other structures seen in the pulsar emission, It may be noted that such a double structure is to be observed even in the case of solar activity.[12]

It may be argued that the solar cycle is rather imprecise when compared with the great precision observed in the pulsar cycle. But it is just as well to remember that the high precision of the pulsar period is deduced, not by comparing the cycle times of neighbouring pulses, but by averaging over a larger number ($\sim 10^4$–10^5) of cycles. By contrast only about 25 cycles have been observed for the sun and averaging the period over twelve successive cycles already improves the accuracy from about 25% to about 1%. In any case the method of determining the period of the solar cycle using sunspots cannot be expected to lead to an accurate determination. It is probable that the very method of detection of pulsars selectively chooses precise pulsars and the analogy of magnetic variables could be extended to include a class of 'precise' pulsars and a class of 'imprecise' pulsars. It may then be not too unlikely that the recent observations of pulsars with change in arrival times would be a reflection of some sort of impreciseness.

The emission from pulsars is found to be largely polarised which is only to be expected of the emitted radiation due to the high magnetic field. If the solar cycle model has a physical relevance, the alternate cycles should show some correlation between polarization characteristics. Such is the case with some magnetic variables, where the change of the polarisation vector is correlated with change in the polarity of the magnetic field.[13] It should also be noted that on our model the slowing down of pulsar periods could be attributed to the depletion of the energy of differential rotation which is being converted into magnetic energy in every cycle. The pulse width for pulsars is demonstrably much smaller than the period when compared with the solar 'pulse'. This is almost certainly to be associated with the relative magnitudes of the time-scales for build-up and discharge for the two types of stars.

It is not too unreasonable to suppose that the solar cycle is not unique and that stars in other parts of the HR-diagram display a similar pattern. Recently, Wilson[14] has reported variations in the H and K line fluxes of dwarf stars on the main sequence; it remains to be seen if these are indicative of the solar type of activity. The cyclical behaviour of the magnetic field and the release of magnetic energy in the form of emission is perhaps characteristic exhibited by a large number of stars. The periodic emission may sometimes be overshadowed by a steady emission. In the case of white dwarfs the periods are expected to be in the range of 10–100 sec. However, the periodic energy emission may be far too weak to be detectable in the radio and optical regions.

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