

A THÉOREM ON SUMS OF POWERS WITH APPLICATIONS TO THE ADDITIVE THEORY OF NUMBERS (III).

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1. LET $G(k)$ denote the least value of s such that every large number is a sum of at most s k th powers of positive integers. Vinogradow¹ has recently announced the results :

$$(1) \quad G(k) < 32 (k \log k)^2 \quad [k > k_0].$$

More recently Vinogradow² claims to have proved [for $k > k_0$]

$$(2) \quad G(k) \leq 2 \{k(k-2) \log 2 + 2k\}.$$

i.e.,

$$(3) \quad G(k) \leq 2k^2 \log 2 + o(k^2)$$

From (3) it would follow immediately that

$$(4) \quad v(k) \leq 2 (\log 2) k^2 + o(k^2)$$

In II it was proved that

$$(5) \quad v(k) \leq k^2 + 7k - 2$$

is true for infinitely many k , and this result cannot be deduced from (4), since $2 \log 2 > 1$.

2. From (5) it can be deduced that

THEOREM. *There are infinitely many odd k such that for any given ϵ , every positive integer is a sum of at most $\{(2+\epsilon)k^2\}$ k th powers of integers (positive or negative).*

This is also a consequence of Vinogradow's (2).

3. IMPORTANT CORRECTION. In my preceding paper the last line at the bottom of page 704 is incorrect. For "is true for $\theta \leq k+m$ but not true for $\theta = (k+m+1)$ " read "is true for $\theta \leq t(k+m)$ but not true for $\theta = t(k+m+1)$ ". The argument that follows remains unaltered.

NOTE.—The theorem (proved in I and II) that

$$N(k) \leq \frac{k^2 + k}{2} + 1$$

has also been proved independently by E. Maitland Wright.

¹ *C. R. Acad. Sci. U.R.S.S.* (1934). "A new solution of Waring's problem."

² *C. R. (Paris)*, 1934. "sur quelques résultats nouveaux . . ."

The numbers I and II in the sequel refer to the earlier papers of this title in *Proc. Ind. Acad. Sc. (A)*, 1935, 1, 698-700 and 701-706.