Substituting the value of 7 from (18) in (17), we get

\[(4^8 + 8^8)(x^8 + 16y^8)^8 + (16^8 + 15^8 + 2^8)\{(4x^5y^3)^8 + (8x^3y^5)^8\}\]

\[= (4^8 + 8^8)\{(x^8 - 16y^8)^8 + (2x^7y)^8 + (16xy^{-1})^8\} + 17^8\{(4x^5y^3)^8 + (8x^3y^5)^8\}.

Division by 4^8 now gives

\[(1^8 + 2^8)(x^8 + 16y^8)^8 + (16^8 + 15^8 + 2^8)\{(x^5y^3)^8 + (2x^3y^5)^8\}\]

\[= (1^8 + 2^8)\{(x^8 - 16y^8)^8 + (2x^7y)^8 + (16xy^7)^8\} + 17^8\{(x^5y^3)^8 + (2x^3y^5)^8\},

and the proof of (IX) is complete.

**LEUDESDORF'S GENERALIZATION OF WOLSTENHOLME'S THEOREM**

S. CHOWLA*

**Theorem†.** If \((n, 6) = 1\), then

\[\sum_{(m, n)=1}^{m \leq n} m^{-1} \equiv 0(n^2).\]

**Proof§.**

\[\sum_{(m, n)=1}^{m \leq n} \frac{1}{m} \equiv 0(n^2) \Rightarrow \sum_{(m, n)=1}^{m \leq n} \frac{1}{m(n-m)} \equiv 0(n),\]

\[\sum_{(m, n)=1}^{m \leq n} \frac{1}{m(n-m)} + \sum_{(m, n)=1}^{m \leq n} \frac{1}{m^2} \equiv 0(n).\]

It remains to prove that|| \(\sum m^{-2} \equiv 0(n)\), and this follows from the congruence

\[\sum m^{-2} \equiv \sum'(am)^{-2} \pmod{n},\]

if \(a\) is such that \(a^2 \not\equiv 1(p)\) for all prime factors \(p\) of \(n\); it is obviously possible to choose \(a\) in this way since \(n\)'s prime to 6.

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* Received 27 March, 1934; read 26 April, 1934.
† This is the simplest and most striking case of Leudesdorf's theorem. See Hardy and Wright, *Journal London Math. Soc.*, 9 (1934), 38-41. The theorem was rediscovered by S. S. Pillai.
‡ \(a \equiv b(c)\) means \(a \equiv b \pmod{c}\).
§ We write \(A \rightarrow B\) when proposition B follows from proposition A and conversely.
|| A dash attached to a \(\equiv\) indicates that \(m\) takes any incongruent set of values prime to \(n\).