ON THE CLASS NUMBER OF REAL QUADRATIC FIELDS

BY S. CHOWLA

UNIVERSITY OF NOTRE DAME

Communicated by Deane Montgomery, May 5, 1961

THEOREM.—*Let p be a prime* $\equiv 1 \pmod 4$, *h the class number of the real quadratic field* $R(\sqrt{p})$, $\epsilon = [(t + u\sqrt{p})/2] > 1$ *its fundamental unit. Then,*

$$\left(\frac{p-1}{2}\right)! \equiv (-1)^{(h+1)/2} \frac{t}{2} \pmod p. \tag{1}$$

Proof: From Dirichlet's classical formula we derive

$$\sqrt{p}\, \epsilon^h = \prod_n (1 - \theta^n), \tag{2}$$

where $\theta = e^{2\pi i/p}$ and n runs over the numbers with $0 < n < p$ and $(n/p) = -1$, where (x/p) is Legendre's symbol of quadratic residuacity. Working with integers of $R(\theta)$, we note

$$\sqrt{p} = \left(\frac{p-1}{2}\right)! \, (1-\theta)^{(p-1)/2} \pmod{(1-\theta)^{(p+1)/2}}, \tag{3}$$

$$\prod_n (1 - \theta^n) = (1-\theta)^{(p-1)/2} \pmod{(1-\theta)^{(p+1)/2}}. \tag{4}$$

(1) follows from (2), (3), and (4). Our result supplements formulae of Ankeny-Artin-Chowla in *Ann. Math.*, 1952, p. 479.

I am grateful to Professor A. Selberg for noticing an error in my original argument. Paromita Chowla has checked the formula in many special cases. See L. J. Mordell, *Amer. Math. Monthly* (Feb. 1961) for a similar result for primes $p \equiv 3 \pmod 4$.