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Tachyon emission from white-holes

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Abstract. Investigations are made about the motion of a radially outward propagating tachyon which is created in the singularity with the white-hole. The problem of confinement or escape of such a tachyon from a white-hole is discussed. It is shown that the confinement or escape of the tachyon depends on the maximum radius of the white-hole and also on a parameter k (defined in the text) associated with the momentum of the tachyon. Also it is shown that when a tachyon escapes it always escapes before the white-hole has expanded to half its Schwarzschild radius.

Keywords. General relativity; tachyons; white-holes.

1. Introduction

Recently considerable amount of research has been directed towards the involvement of tachyons in astrophysical and cosmological phenomena (Narlikar and Sudarshan 1976, Narlikar and Dhurandhar 1976, Davies 1975, Raychaudhari 1974, Honig *et al* 1975). Experimentally the attempts to produce or detect, tachyons have till now yielded null results. But as far as production is concerned one may look to high-energy astrophysics where phenomena are found to take place on a much grander scale, than can ever be achieved in terrestrial settings. One such large-scale phenomenon is the big-bang. Narlikar and Sudharshan (1976) have already discussed the behaviour of a primordial tachyon in the big-bang universe, whose sole interaction with the surrounding matter was gravitation. Such tachyons are shown to encounter a time-barrier, and the epoch of the time-barrier depends on the initial energy of the tachyon and also on the Friedmann model considered.

In this paper the propagation of a tachyon inside a white-hole is discussed. The geometry inside a homogeneous dust type of white-hole is the same as that of a big-bang Friedmann model. So to some extent we expect the situation to be similar to the above-mentioned problem discussed by Narlikar and Sudarshan (1976). There is, however, one essential difference. Here, we are also concerned with the problem of confinement or the escape of a tachyon from a white-hole. Such a problem did not arise, when a tachyon in the expanding universe was considered.

The role of white-holes in high energy astrophysics has been discussed by Narlikar and Apparao (1975). Exploding galactic nuclei, transient x-rays, gamma-ray bursts are some of the examples of likely white-hole phenomena. There have also been arguments (Eardley 1974, Zeldovich *et al* 1975) to show that white-holes cannot exist for a long enough time to be physically relevant. These objections have been successfully countered by Lake and Roeder (1976). We will not enter here into the discussion of these arguments or of the other implications of white-holes for astrophysics. There is already sufficient observational evidence for exploding objects in astrophysics (Hoyle 1975), apart from the varying degrees of faith among different astronomers, for the big-bang origin of the universe. The model of the white-hole considered here may be regarded as a simplified version of such exploding phenomena.

2. Geometry in the interior of a white-hole

We shall consider the homogeneous dust model of a white-hole, that is, a spherical object with uniform density and zero pressure. The object emerges from a singular state and subsequently obeys Einstein's field equations

$$R_{ik} - \tfrac{1}{2} R g_{ik} = -8\pi T_{ik}. \tag{1}$$

We have chosen units in which $C = 1$, $G = 1$. We shall consider the white-hole in the co-moving frame of reference of outward moving particles. In this frame the interior of the white-hole has the line-element,

$$ds^2 = dt^2 - S^2(t)\left[\frac{dr^2}{1 - ar^2} + r^2(d\theta^2 + \sin^2\theta \, d\phi^2)\right], \qquad r \leqslant r_b \tag{2}$$

where (r, θ, ϕ) are the constant co-ordinates of a co-moving particle and t the proper time of a co-moving observer. $r = r_b$ is the coordinate of a particle on the boundary of the white-hole. $S(t)$ is the expansion factor and it satisfies the differential equation

$$\left(\frac{dS}{dt}\right)^2 = \frac{a(1 - S)}{S} \tag{3}$$

where,

$$a = \frac{2m}{r_b^3} = \frac{8\pi\rho_0}{3}$$

$$m = \frac{4\pi}{3}\rho_0 r_b^3$$

ρ_0 is the lowest density attained by the white-hole when the expansion factor $S = 1$. m is the mass of the white-hole. It may be remarked that the line element (2) resembles the big-bang Friedmann line-elements for the closed universe.

Outside the white-hole the metric is Schwarzschild. The radial Schwarzschild co-ordinate is $rS(t)$, inside the white-hole.

3. Tachyon propagation

We assume the tachyon to be created when the white-hole is in the singular state given by $S = 0$ and its motion being directed radially outward. The tachyon interacts with the surrounding matter of the white-hole through the relativistic law of gravitation, but is otherwise free from any other interaction.

Under the above assumptions the tachyon motion is along a space-like geodesic starting at $r = 0$ and $t = 0$. Initially the tachyon momentum is radial, directed along $\theta = \theta_0$, $\phi = \phi_0$, say. The integration of the θ, ϕ geodesic equations lead to the result, that the tachyon motion continues to be radial and given by $\theta = \theta_0$, $\phi = \phi_0$.

For a tachyon trajectory we have $ds^2 < 0$, so that ds is imaginary. We define a real affine parameter σ by the relation,

$$d\sigma^2 = - ds^2. \tag{4}$$

For a radially moving tachyon we have,

$$d\sigma^2 = \frac{S^2(t)\, dr^2}{1 - ar^2} - dt^2 . \tag{5}$$

The geodesic equations corresponding to the radial motion given by (5), when integrated, result in the relation,

$$\frac{S^2}{\sqrt{(1 - ar^2)}} \frac{dr}{d\sigma} = k \text{ (a constant)} \tag{6}$$

where k is a real constant. Using (5) and (6) we get,

$$\left(\frac{dt}{d\sigma}\right)^2 = \frac{k^2 - S^2}{S^2} . \tag{7}$$

The interpretation of the constant k may be sought by considering the 3-velocity of the tachyon in the rest-frame of an outward moving particle of the white-hole, which coincides instantaneously with the tachyon at (r, t). This velocity $v(t)$ is given by

$$v(t) = \frac{S(t)}{\sqrt{(1 - ar^2)}} \frac{dr}{dt} . \tag{8}$$

Using (6), (7) and (8) we get,

$$k = \frac{S(t)\, v(t)}{\sqrt{[v^2(t) - 1]}} . \tag{9}$$

The momentum per unit meta-mass of the tachyon may be defined by the relation

$$P(t) = \frac{v(t)}{\sqrt{[v^2(t) - 1]}} . \tag{10}$$

From (9) and (10),

$$k = S(t)\, P(t) \tag{11}$$

(11) clearly gives the physical interpretation of the constant k.

4. Tachyon trajectories

It is convenient to investigate the radial trajectory of a tachyon in terms of the expansion factor S instead of t. Defining a parameter η by $S = \sin^2 \eta$ and then integrating (3) we get,

$$t = \frac{1}{\sqrt{a}} (\eta - \cos \eta \sin \eta) . \tag{12}$$

In the expansion phase of the white-hole, as S increases from 0 to 1, η ranges from 0 to $\pi/2$. In this range of η, it is seen that S is a monotonic function of t, so that our choice of S instead of t as the independent variable is not unjustified. (3), (6) and (7) give the differential equation of the trajectory in terms of r and S as,

$$\left(\frac{dS}{dr}\right)^2 = \frac{aS(k^2 - S^2)(1-S)}{k^2(1-ar^2)} \tag{13}$$

$$\frac{\pm\sqrt{a}\,dr}{\sqrt{(1-ar^2)}} = \frac{kdS}{\sqrt{[S(1-S)(k^2-S^2)]}}. \tag{14}$$

It is convenient to define a new variable R by the relation,

$$R = \sin^{-1}\sqrt{a}\,r. \tag{15}$$

The values over which R ranges are determined by the values over which r ranges. Since $0 \leq r \leq r_b$, we have $0 \leq R \leq R_b$ where,

$$R_b = \sin^{-1}\sqrt{a}\,r_b. \tag{16}$$

In view of $a = 2m/r_b^3$, (16) becomes

$$R_b = \sin^{-1}\sqrt{\frac{2m}{r_b}}. \tag{17}$$

We shall consider only those white-holes for which $r_b > 2m$, that is, those white-holes whose boundary crosses their Schwarzschild radius. (17) immediately implies that $R_b < \pi/2$. Henceforth we shall consider $R_b = \pi/2$ as the upper limit of the permitted range of R_b.

It can be remarked here that in (13), r is the co-moving radial coordinate of the tachyon, and hence the equation is valid physically only when the tachyon is inside the white-hole. In the region exterior to the white-hole, the geometry is different and consequently the tachyon would obey a different equation. It is meaningful to consider (13) as describing the trajectory of the tachyon only for $r \leq r_b$. The condition $r \leq r_b$, in our newly defined coordinate is equivalent to $R \leq R_b$. Equation (14) with the help of (15) becomes,

$$\pm\left(\frac{dR}{dS}\right) = \frac{k}{\sqrt{[S(1-S)(k^2-S^2)]}}. \tag{18}$$

Solving (18) we shall have R as a function of S. Initially, when the white-hole is in its singular state $S = 0$, we shall have $R = 0$. With the aid of (18) and the above initial condition, we can plot the trajectory of the tachyon in the R-S plane. However, since the maximum value attained by R_b is $\pi/2$ and $R \leq R_b$, we shall consider only the part of the plane described by $0 \leq R \leq \pi/2$.

5. Equations of trajectories

For a physically acceptable solution one would require dR/dS in (18) to be non-negative in the neighbourhood of $R = 0$, $S = 0$. Since we have the choice of sign in (18) we can choose the positive sign and hence restrict k to non-negative values. We note that $dS/dR = 0$ at $S = 0$ and at $S = k$ (the other roots of dS/dR are irrelevant) and

$$\left.\frac{d^2S}{dR^2}\right|_{S=0} = \tfrac{1}{2}, \qquad \left.\frac{d^2 S}{dR^2}\right|_{S=k} = -(1+k).$$

The trajectory reaches a maximum at $S = k$, when $R = R_m$ say. In the integration of (18) two cases arise according as

1. $k < 1$
2. $k \geq 1$

For $R \leqslant R_m$

$$R(S, k) = \begin{cases} f_1(S, k), & k < 1 \\ f_2(S, k), & k \geq 1 \end{cases} \tag{19}$$

where,

$$f_1(S, k) = \int_0^S \frac{k\,dS}{\sqrt{[S(1-S)(k^2 - S^2)]}} \qquad 0 \leq S \leq k$$

$$f_2(S, k) = \int_0^S \frac{k\,dS}{\sqrt{[S(1-S)(k^2 - S^2)]}} \qquad 0 \leq S \leq 1$$

$$R_m = \begin{cases} f_1(k, k), & k < 1 \\ f_2(1, k), & k \geq 1 \end{cases} \tag{20}$$

For $R \geqslant R_m$,

$$R(S, k) = \begin{cases} 2R_m - f_1(S, k), & k < 1 \\ 2R_m - f_2(S, k), & k \geq 1 \end{cases} \tag{21}$$

(19) and (21) comprise the equations of the trajectory. A typical trajectory is shown in figure 1, corresponding to $k = 0{\cdot}086$.

(19) and (21) show that $dS/dR > 0$ for $R < R_m$ and $dS/dR < 0$ for $R > R_m$. Since S is a monotonic function of time, the trajectory of $R < R_m$ represents a tachyon moving forward in time, while for $R > R_m$, the tachyon moves backward in time, which may be interpreted as an antitachyon moving forward in time (*see* Sudarshan 1970). Both the tachyon and the antitachyon annihilate each other at $R = R_m$.

One may remark at this stage that the trajectory is mathematically defined for $0 \leq R \leq 2R_m$. But if $R_b < 2R_m$, then the physically relevant portion of the trajectory would be that for which its R-coordinate is less than or equal to R_b.

6. Main problem

We ask the following question: Under what circumstances does the tachyon escape from the white-hole? The question is equivalent to the choice between the two conditions:

(*a*) $2R_m < R_b$ (*b*) $2R_m > R_b$

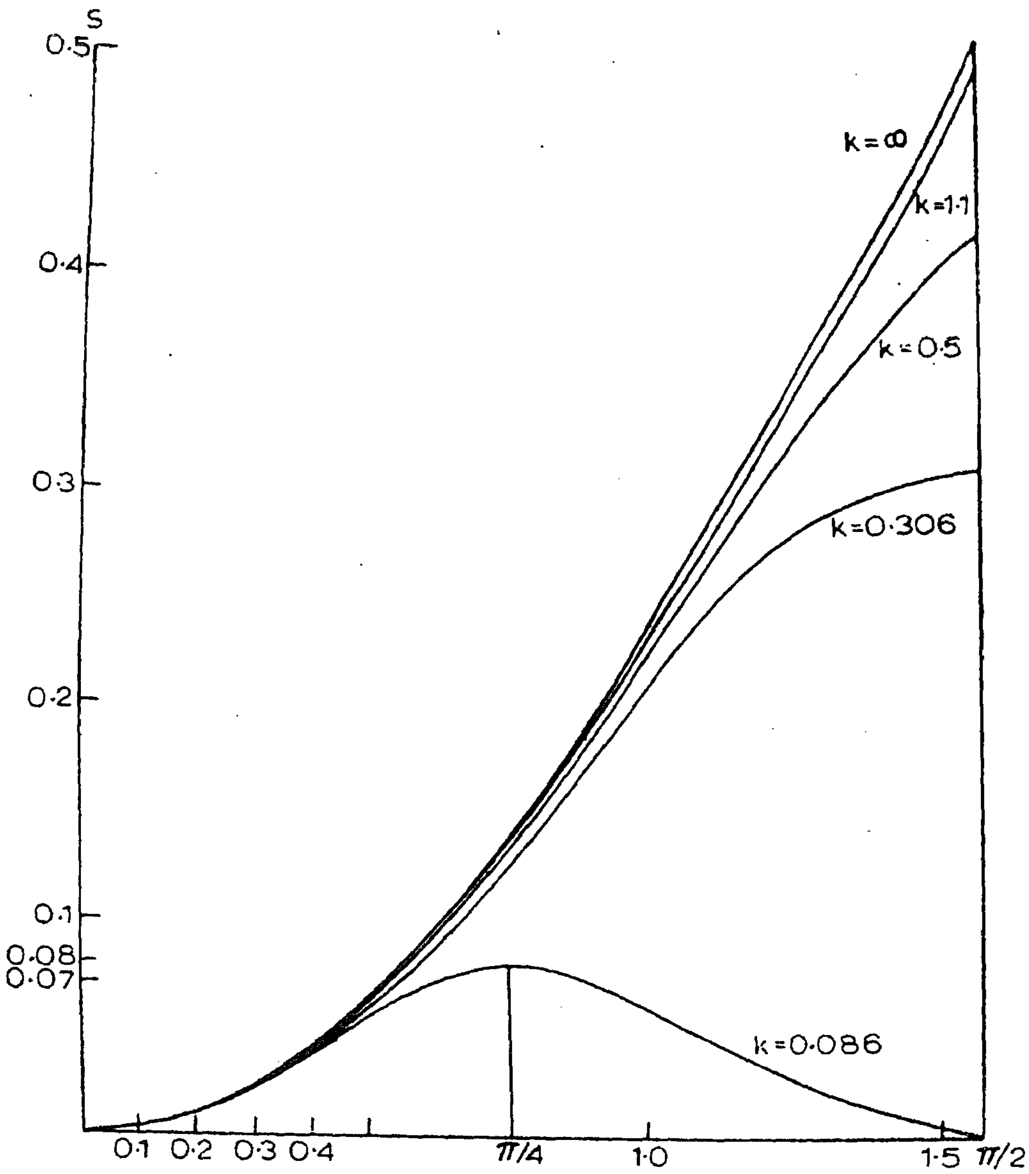

Figure 1. Tachyon trajectories with various values of k are shown. In particular the trajectories with $k = k_1 = 0{\cdot}306$ and $k = k_2 = 0{\cdot}086$ are also shown.

In case (a) the trajectory of the tachyon does not cross the boundary coordinate $R = R_b$ which implies that the tachyon is confined to the white-hole. Case (b) represents the trajectory of a tachyon, which crosses the boundary coordinate $R = R_b$, which means that the tachyon escapes. The two situations are shown in figure 2.

In case (b) when the tachyon escapes two cases arise according as,

(i) $R_m < R_b < 2R_m$

(ii) $R_b < R_m$.

The two cases are shown in figure 3.

In case (i) the trajectory bends back in time near $R = R_b$, so that one can interpret this situation as the annihilation of the tachyon and the antitachyon taking place inside the white-hole, with the antitachyon originating outside the white-hole. Or, one can say that the tachyon escapes from the white-hole, while moving

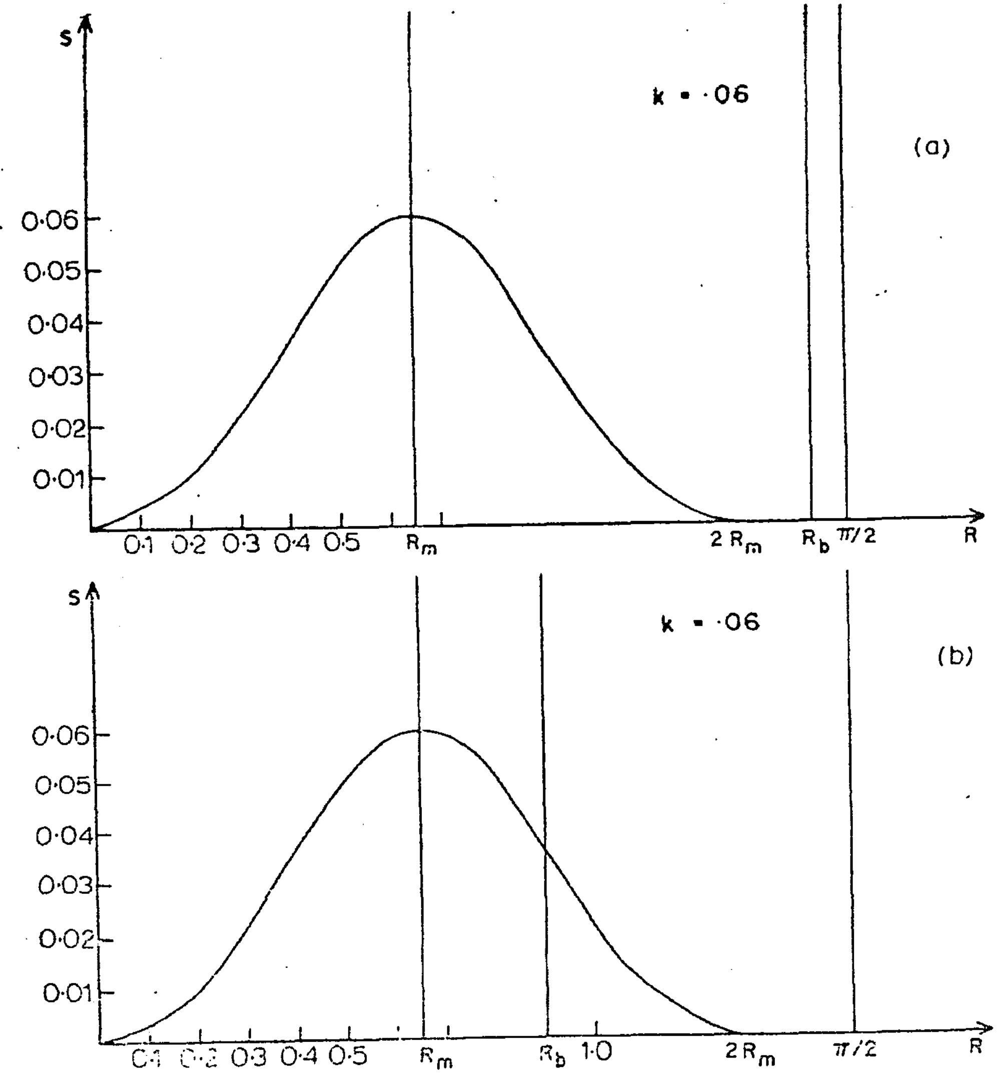

Figure 2. The two cases (*a*) $2R_m < R_b$ and (*b*) $2R_m > R_b$ are shown separately. In the first case the tachyon is confined, while in the second case the tachyon escapes.

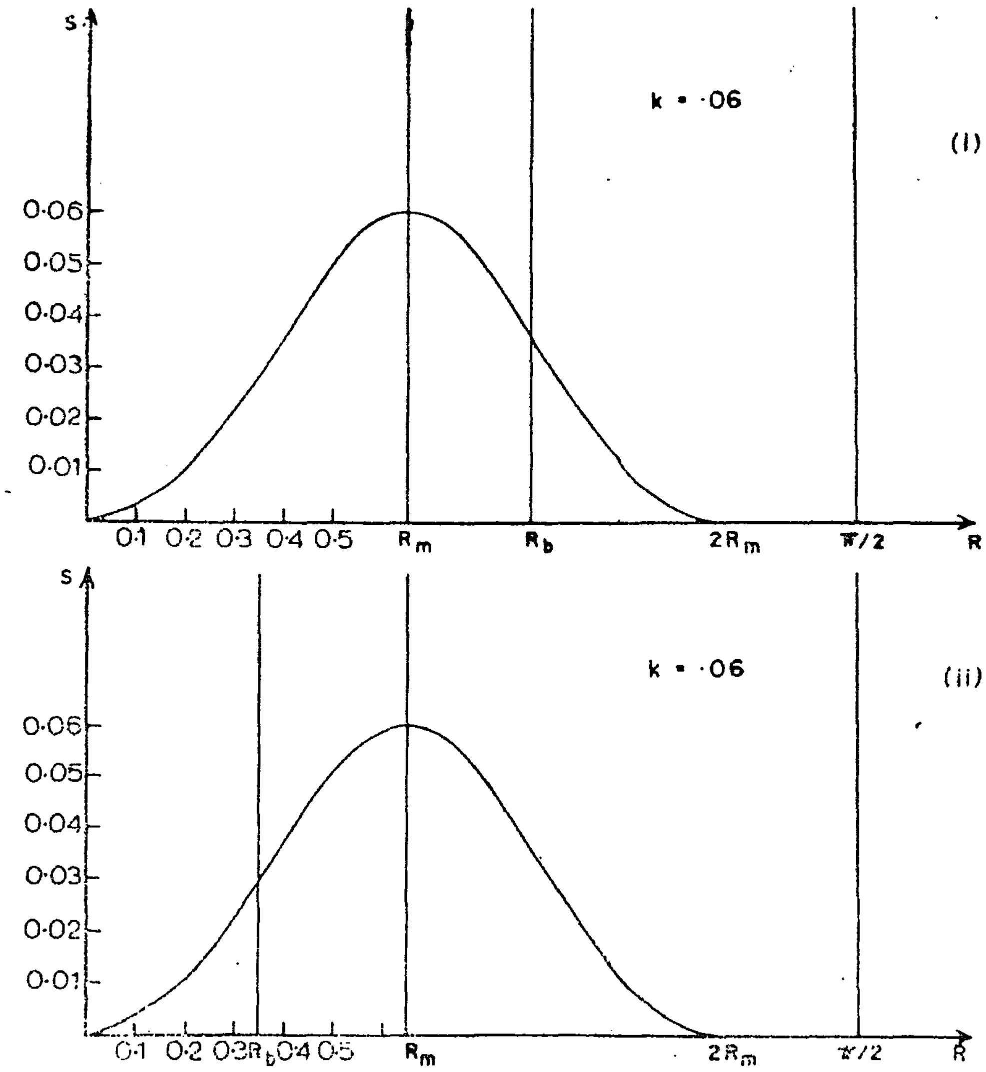

Figure 3. The two cases (i) $R_m < R_b < 2R_m$ and (ii) $R_b < R_m$ are shown. In case (i) the tachyon escapes moving backward in time, while case (ii) represents a tachyon which escapes moving forward in time.

backward in time. In case (ii) the tachyon always moves forward in time for $R \leq R_b$, reaches $R = R_b$ and then escapes. There is no annihilation in this case.

From the foregoing one can make the following statements:

(i) If $R_m > \pi/4$, then the tachyon surely escapes either going back in time or going forward in time.

(ii) If $R_m > \pi/2$, the tachyon surely escapes moving forward in time.

(iii) If $R_b > R_m > \pi/4$, the tachyon surely escapes moving forward in time.

In the light of the above statements it is necessary to investigate the behaviour of R_m as a function of k. (20) gives the required relations in terms of elliptic functions. For $k < 1$,

$$R_m(k) = \int_0^k \frac{k\,dS}{\sqrt{[S(1-S)(k^2 - S^2)]}} = \sqrt{2k}\, F\left(\sqrt{\frac{1+k}{2}}, \frac{\pi}{2}\right) \tag{22}$$

For $k \geq 1$,

$$R_m(k) = \int_0^1 \frac{k\,dS}{\sqrt{[S(1-S)(k^2 - S^2)]}} = 2\sqrt{\frac{k}{1+k}}\, F\left(\sqrt{\frac{2}{1+k}}, \frac{\pi}{2}\right) \tag{23}$$

The plot of $R_m(k)$ vs. k is shown in figure 4. As k increases from 0 to 1, $R_m(k)$ increases monotonically from 0 to ∞; further increase in k makes R_m decrease monotonically from ∞ until it asymptotically tends to π.

In view of statement (ii) the case for $k \geq 1$ becomes exceedingly simple, because then $R_m > \pi > \pi/2$ and the tachyon surely escapes moving forward in time, whatever be the value of R_b for the white-hole. The case of interest arises when $k < 1$.

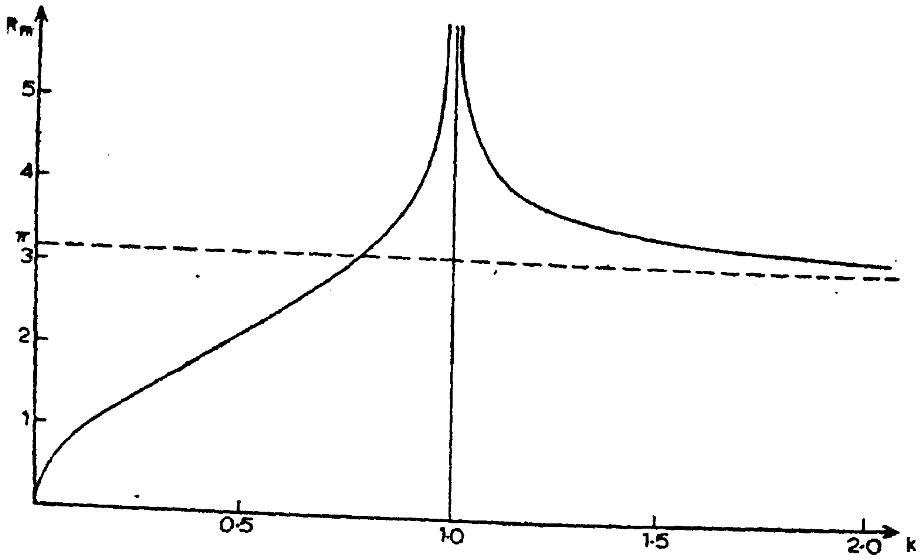

Figure 4. Curve showing the dependence of R_m on k. For large values of k, $R_m(k)$ asymptotically tends to π.

When $0 \leq k \leq 1$, $R_m(k)$ is monotonic increasing; we can use this fact to translate the three statements pertaining to R_m, into equivalent statements regarding k. Define k_1 and k_2 by,

$$\left.\begin{aligned} R_m(k_1) &= \pi/4 \\ R_m(k_2) &= \pi/2 \end{aligned}\right\} \tag{24}$$

and k_b by,

$$R_m(k_b) = R_b. \tag{25}$$

Now we can replace $\pi/4$, $\pi/2$, R_b, R_m by k_1, k_2, k_b, k respectively in statements (i) (ii) and (iii) and obtain equivalent statements.

From (22) it is possible to evaluate k_1 and k_2,

$$k_1 = 0{\cdot}086$$

$$k_2 = 0{\cdot}306.$$

We can get an upper-bound on k_b using the fact that $R_b < \pi/2$

$$k_b < k_2$$

that is,

$$k_b < 0{\cdot}306.$$

If a tachyon has $k > k_b$, the tachyon escapes moving forward in time. We shall list in table 1 a few values of k_b corresponding to various values of R_b.

7. Epoch of escape of a tachyon

The next problem of concern to us is summarised by the following question: Assuming that a tachyon does escape, when does it escape? In what state of expansion of the white-hole does the tachyon escape? The problem is to find the point of intersection of the tachyon trajectory $R = R(S, k)$ and the line $R = R_b$ and to observe how this intersection point behaves for different k's or R_b's. The S-coordinate of this point of intersection will determine the phase of expansion of the white-hole, when the tachyon escapes. To this end, we first propose to

Table 1.

Sl. No.	$\left(\frac{180}{\pi} R_b\right)$	$\frac{r_b}{2m}$	cosec^2 R_b
1.	15°	14·93	0·01
2.	30°	4·00	0·04
3.	45°	2·00	0·086
4.	60°	1·33	0·148
5.	75°	1·072	0·222
6.	90°	1·00	0·306

find an upper-limit on this S-coordinate of the point of intersection. Only the portion of the trajectories for which $R \leq R_m$ will be needed. We have,

$$R(S, k) = \int_0^S \frac{k dx}{\sqrt{[x(1-x)(k^2 - x^2)]}}, \quad 0 \leq S \leq \min(1, k)$$

where min $(1, k)$ denotes the minimum of the numbers 1 and k.

Now $k/\sqrt{k^2 - x^2}$ is a decreasing function of k for each x, hence $R(S, k)$ is a decreasing function of k for a fixed S. In figure 1 the trajectories are plotted for different k's. As k increases the trajectories move 'higher up' in the R-S plane. Finally the trajectory for which $k \to \infty$ will have the least R for a given S or equivalently a greatest S for a given R, when compared with other trajectories with finite k. So if we consider the extreme case, that is, a white-hole with $R_b = \pi/2$ and a tachyon with $k \to \infty$, the tachyon will escape in the greatest expansion phase of the white-hole, as compared with other white-holes or with tachyons with finite k. For $k \to \infty$, we have,

$$\lim_{k \to \infty} R(S, k) = R_\infty(S) = 2 \sin^{-1} \sqrt{S}. \tag{26}$$

From (26) we have,

$$R_\infty(S = 0{\cdot}5) = \pi/2.$$

Hence in the extreme case the tachyon escapes when $S = 0{\cdot}5$.

On this basis, one can now make a general statement, that when a tachyon does escape, it always escapes before the white-hole reaches half its ultimate linear size.

Incidently it may be remarked that the tachyon trajectory with $k \to \infty$ corresponds to the trajectory of a photon.

It is seen that a tachyon having $R_m = R_b$ escapes at an epoch t_0 given by $S(t_0) = k$. From (11), it is noted that such a tachyon has $P(t_0) = 1$, that is, in the frame of reference of the surface of the white-hole the tachyon has momentum unity or zero energy. In general the momentum of escape of the tachyon at an epoch t_0 is given by (11) to be $P(t_0) = k/S(t_0)$. Since $S(t_0) \leqslant 0{\cdot}5$, $P(t_0) \to \infty$ as $k \to \infty$. In the particular case of $R_b = \pi/2$, we have $S(t_0) = 0{\cdot}306$, and in the limit $k \gg 1$, $P(t_0) \sim 2k$.

Another aspect one can investigate is the radial distance at which the tachyon escapes from the white-hole. We intend to get an upper limit on this radial distance of escape of a tachyon.

For a given r_b, the maximum radial Schwarzschild coordinate of escape $r_b S(t_0)$ will be obtained by considering the trajectory $k \to \infty$, as is easily seen from figure 1 To this end we solve the equation $R_\infty(S) = R_b$ for r_b, that is

$$2 \sin^{-1} \sqrt{S_0} = \sin^{-1} \sqrt{\frac{2m}{r_b}} \tag{27}$$

where

$$S_0 = S(t_0).$$

after simplification of (27) one gets,

$$r_b S_0 = \frac{m}{2(1-S_0)} \quad . \tag{28}$$

Since S_0 is the epoch of escape, $S_0 \leq 0 \cdot 5$, (27) immediately gives the relation,

$$r_b S_0 \leq m. \tag{29}$$

Hence if a tachyon escapes, it always escapes before the white-hole has expanded to half its Schwarzschild radius.

In the limit $r_b \to \infty$, and k sufficiently large, we shall have from (28), $S_0 \to 0$ in which case, $R(S, k) \sim 2 \sin^{-1} \sqrt{S}$. One can apply eq. (28) in this case, which in the limit $S_0 \to 0$ yields,

$$r_b S_0 = m/2. \tag{30}$$

Hence it is seen that the tachyon always escapes, when the white-hole is entirely inside the Schwarzschild radius. After the escape of a tachyon its motion is governed by Schwarzschild geometry. Such a motion has already been discussed by Narlikar and Dhurandhar (1976).

8. Conclusion

From the foregoing one can conclude that not all tachyons escape from white-holes; some are confined, but generally the energetic ones escape. Hence one may look for tachyons which have velocities near the speed of light.

Acknowledgement

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