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Matrix Models and Nonperturbative String Propagation in Two-dimensional Black Hole Backgrounds

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ABSTRACT

We identify a quantity in the $c = 1$ matrix model which describes the wavefunction for physical scattering of a tachyon from a black hole of the two dimensional critical string theory. At the semiclassical level this quantity corresponds to the usual picture of a wave coming in from infinity, part of which enters the black hole becoming singular at the singularity, while the rest is scattered back to infinity, with nothing emerging from the whitehole. We find, however, that the exact nonperturbative wavefunction is nonsingular at the singularity and appears to end up in the asymptotic region "behind" the singularity.

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In a previous paper[1] it was suggested that the one dimensional matrix model[2] may be used to study the nonperturbative behavior of massless "tachyons" coupled to the black hole background of the two dimensional critical string[3] [4]. This follows from the fact that at the semiclassical level a certain integral transform of the fluctuation of the collective field of the matrix model[5] [6] around the classical ground state value satisfies the same *linearized* equation of motion as that of the massless tachyon in a blackhole background. Indeed this connection between the matrix model and the black hole is essentially the same as that between the liouville theory and the $SL(2, R)/U(1)$ coset model discovered earlier by Martinec and Shatashvilli[7]. It was soon found in[8] that a similar transformation exists in the bosonized form of the fermionic field theory of the matrix model developed in[9] and leads to the same black hole interpretation. A large class of transformations between collective field theory and black hole wavefunctions have been discussed in[10]. In fact, there are several pieces of evidence from continuum treatments that the liouville and black hole backgrounds are closely related[11].

The presence of this connection does not imply that the physics of the black hole background is identical to that of the liouville background. Rather, at the semiclassical level, the integral transform allows one to obtain the behaviour of a tachyon in a black hole background from matrix model quantities just as singular gauge transformations in gauge theories allow one to obtain wavefunctions in presence of vortices or monopoles from those in free space. However, if one examines the structure of the integral transform, one finds that near the singularity of the black hole this receives contributions from the collective field in regions where the coupling is strong. Far from the black hole the semiclassical approximation is good so that one can meaningfully talk about incoming and outgoing tachyon states.

Beyond the semiclassical expansion the matrix model quantities transformed as above would represent correlations in *some* quantum field theory of scalar particles whose semiclassical behaviour is identical to that of tachyons moving in a static black hole. We could thus use matrix model results to study non-perturbative behaviour of tachyon propagation. Indeed one feature of such non-perturbative effects have been observed in [8] where it was shown that, with the particular definition of this transform used in that paper, the classical value of the *background* is singular at the location of the black hole singularity, but the exact value is completely regular. A similar behavior has been found for the transform of a class of wavepackets which represent "spikes" on the fermi sea [12]. However these spikes transform into tachyon wavefunctions which do not represent scattering processes as described above.

What is lacking in all the papers quoted above is a clear identification of the quantities which have to be computed in the *matrix model* which represent a *physical* scattering process in the black hole background. For example, at the semiclassical level, the transforms defined in either [1] or [8] represent a process where there is a non-zero flux emerging from the past null infinity and hence do not represent a process in which the scalar particle comes in from infinity, part of the wave being scattered and part of the wave absorbed by the black hole. One of the transforms considered in [10] does represent a scattering type wavefunction in the exterior region, but not in the interior region - where the wave has a component which does not vanish on the extension of the past horizon.

In this paper we make this identification by noting that the large class of transforms which take matrix model quantities into single particle wavefunctions in a black hole background correspond to different boundary conditions [7] [10]. We find the correct linear combination of these transforms which corresponds to a wave vanishing on the past horizon and hence represents a physical scattering process from a black hole. We show that, as expected, single particle wave functions which represent such a scattering process diverge at the black hole singularity at the semiclassical level. We then consider quantum corrections to this single particle wave function using the exact results of the matrix model. We show that while to all orders of string perturbation theory the divergence of the wavefunction at the location of the singularity persists, the exact non-perturbative answer is completely nonsingular. We show that in the aymptotic region "beyond" the singularity this wave corresponds to a purely outgoing wave. Thus at the exact quantum level the particle thrown in from the standard asymptotic region in the exterior of the black hole ends up in a different asymptotic region beyond the black hole.

The semiclassical limit

Consider the two types of loop operators in the matrix model of a matrix $M_{ij}(t)$ The macroscopic loop operator with real loop lengths is defined by the quantity

$$W(p,t) \equiv \mathrm{Tr}\ e^{-pM} = \int_{2\sqrt{\mu}}^{\infty} dx\ e^{-px}\ \phi(x,t) \tag{1}$$

We will also define macroscopic loop operators with *imaginary* loop lengths which are defined by

$$V(z,t) \equiv \mathrm{Tr}\ e^{izM} = \int_{-\infty}^{\infty} dx\ e^{izx}\ \phi(x,t) \tag{2}$$

The field $\phi(x,t)$ is the density of eigenvalues of M_{ij}, the collective field. In terms of the fermionic fields $\psi(x,t)$ of the fermionic field theory of[13] one has $\phi(x,t) = \psi^\dagger(x,t)\psi(x,t)$. In the fermionic field theory the correlations of $W(p,t)$ may be obtained by first computing the correlations of $V(z,t)$ and then performing a suitable analytic continuation, as explained in[14]. However, in the considerations which follow it is important to keep in mind the difference between the two quantities.

In the lowest order of the semiclassical expansion one can obtain linearized equations for the matrix elements of the loop operators between one particle states and the ground state, which we will denote by $\tilde{V}$ and $\tilde{W}$. Semiclassically these may be obtained by expanding the collective field $\phi(x,t)$ around its classical ground state value $\phi_0(x,t) = \frac{\sqrt{2}}{\pi}(x^2 - 4\mu)^{\frac{1}{2}}$

$$\phi(x,t) = \phi_0(x,t) + \partial_x \eta(x,t) \tag{3}$$

and replace $\phi(x,t)$ in (1) and (2) by $\partial_x \eta(x,t)$. Using the equations of collective field theory it may be seen that at the classical level the linearized equation of motion for $\eta(x,t)$ leads to the following linearized equation for $\tilde{W}(p,t)$ and $\tilde{V}(z,t)$

$$
\begin{aligned}
&[(p\partial_p)^2 - \partial_t^2 - 4\mu p^2]\tilde{W}(p,t) = 0 \\
&[(z\partial_z)^2 - \partial_t^2 + 4\mu z^2]\tilde{V}(z,t) = 0
\end{aligned}
\tag{4}
$$

The first equation in (4) is identical to the Wheeler-de Witt equation obtained in the liouville theory. Indeed using the semiclassical expression for energy eigenstates $\eta(x,t) = e^{i\omega t}\ sin(\omega\ cosh^{-1}(\frac{x}{2\sqrt{\mu}}))$ of the matrix model satisfying the correct Dirichlet boundary condition the fluctuation of $W(p,t)$ is given by the modified Bessel function $e^{i\nu t}\ K_{i\nu}(2\sqrt{\mu}\ p)$ which represents (in the coordinate $log\ p$) a wave coming in from infinity and getting reflected perfectly from the "liouville barrier". This is of course the basic picture of scattering in the liouville theory. Formally $\tilde{V}(z,t)$ may be regarded as the liouville wave function with *negative* cosmological constant. We shall not, however, assign any physical meaning to $\tilde{V}(z,t)$ and we will not need any.

Consider now the following classes of transforms of $\tilde{V}$ and $\tilde{W}$

$$
\begin{aligned}
\tilde{T}^{\pm}_{\pm}(u,v) &\equiv \int_0^\infty dz \int_{-\infty}^\infty dt\ e^{\pm iz(e^t v - e^{-t}u)}\ \tilde{V}(\pm z,t) \\
\tilde{S}_{\pm}(u,v) &\equiv \int_0^\infty dp \int_{-\infty}^\infty dt\ e^{\pm iz(e^t v + e^{-t}u)}\ \tilde{W}(p,t)
\end{aligned}
\tag{5}
$$

Using equations (4) it may be easily verified that each of these quantities $\tilde{T}^{\pm}_{\pm}, \tilde{S}_{\pm}$ satisfy the following linearized equation

$$
[4(uv+\mu)\partial_u\partial_v + 2(u\partial_u + v\partial_v) + 1]\tilde{X}_i(u,v) = 0 \tag{6}
$$

where $\tilde{X}_i$ stands for any of the $\tilde{T}^{\pm}_{\pm}, \tilde{S}_{\pm}$ This is precisely the linearized equation of motion for the massless tachyon of the two dimensional critical string moving in a black hole background. The coordinates (u, v) are the Kruskal-like coordinates in terms of which the background metric and dilaton fields are*

$$\begin{aligned} G_{uv} = G_{vu} &= \frac{1}{2(\frac{2}{\alpha'}\ uv + a)} \quad G_{uu} = G_{vv} = 0 \\ D(u,v) &= -\frac{1}{2}\log\left(\frac{2}{\alpha'}\ uv + a\right) \end{aligned} \tag{7}$$

The black hole mass is given by $a = \frac{2}{\alpha'}\mu$. In terms of the coordinates (u, v) the various regions of the black hole geometry are

$$\begin{aligned} &u = r\ e^{\theta} \quad v = r\ e^{-\theta} \quad \text{for} \quad u, v \geq 0 \quad \text{Region I} \\ &u = -r\ e^{\theta} \quad v = r\ e^{-\theta} \quad \text{for} \quad u < 0, v > 0 \quad \text{Region II} \\ &u = -r\ e^{\theta} \quad v = -r\ e^{-\theta} \quad \text{for} \quad u, v < 0 \quad \text{Region III} \\ &u = r\ e^{\theta} \quad v = -r\ e^{-\theta} \quad \text{for} \quad u > 0, v < 0 \quad \text{Region IV} \end{aligned} \tag{8}$$

Thus the Region II contains the future singularity while the Region IV contains the past singularity.

The transform defined in [1] is given by $\tilde{S}_+$, while the transform defined in [8] is a specific linear combination of all of these independent transforms†. Given a space-time history of $\phi(x, t)$ these various transforms provide *different* space time histories of a tachyon in a black hole background. Semi classically these histories may be easily evaluated using the expression for the fluctuations of the collective field given above and are given by hypergeometric functions, as done for $\tilde{S}_+$ in [1]. However, the different transforms lead to different hypergeometric functions, i.e. waves with different boundary conditions [15]. For example $\tilde{S}_+$ with $\tilde{W}$ chosen to

$\star$ Our conventions are those of [3]

$\dagger$ In [8] the transform involves a collective field which is the density of fermions in the momentum space conjugate to the eigenvalue coordinate. However, in the double scaling limit the interchange of momenta and coordinates simply changes the sign of the cosmological constant. This may be then used to relate the part of the transform of [8] involving momentum space collective fields in terms of the quantities we have defined above.

be of the form $\tilde{W} \sim e^{i\nu t}$ is given in Region I by

$$\tilde{S}_{+}(r,\theta) = \frac{e^{-i\nu\theta}}{r}[(\frac{\mu}{r^2})^{\frac{i\nu}{2}} A(\nu) F(\frac{1}{2} - i\nu, \frac{1}{2}; 1 - i\nu; -\frac{r^2}{\mu}) + (-1)^{i\nu}(\frac{\mu}{r^2})^{\frac{-i\nu}{2}} A(-\nu) F(\frac{1}{2} + i\nu, \frac{1}{2}; 1 + i\nu; -\frac{r^2}{\mu})] \tag{9}$$

where $A(\nu) \equiv \frac{\Gamma(i\nu)}{\Gamma(\frac{1}{2}+i\nu)\Gamma(\frac{1}{2})}$ and we have omitted an overall ν-dependent constant. It is clear from (9) that near the horizon at $r = 0$ this represents a linear combination of a wave coming out of the past event horizon as well as one going into the future event horizon. This is not what we want to study for the scattering of a particle from a black hole. For the latter physical problem we must ensure that there is nothing emerging from the past horizon. We specifically want to study what happens to the wave at the singularity. The transform of[8] also consists of a wave which is nonvanishing at the past horizon.

It is indeed possible to write down a transform which represents a physical scattering of a tachyon from the black hole, i.e. a wave coming in from infinity - part of which gets inside the horizon while a part is scattered back with nothing emerging from the past horizon. This is obtained by first using the time translation invariance of the theory to work in terms of fourier transforms of the operators $V(z,t)$ defined by

$$V(z,t) = \int d\nu \; e^{i\nu t} \; V(z,\nu) \tag{10}$$

The correct transform is then given, in terms of the fourier transformed quantities $\tilde{V}(z,\nu)$ by

$$T^{(\nu)}_{scat}(u,v) = \int_0^\infty dz \int_{-\infty}^\infty dt \; [e^{\frac{\pi\nu}{2}} \; e^{iz(e^t v - e^{-t}u)} + e^{-\frac{\pi\nu}{2}} \; e^{-iz(e^t v - e^{-t}u)}] \; e^{i\nu t} \; [V(z,t) + V(-z,t)] \tag{11}$$

The above definition may be regarded as an operator definition. To extract single particle wave functions and hence obtain $\tilde{T}_{scat}$ one has to take suitable matrix elements which amounts to the replacement of $V(z,t)$ by $\tilde{V}(z,t)$. The integration over t in (11) may be explicitly performed separately in the different regions, with the result

$$\tilde{T}^{(\nu)}_{scat} = e^{i\nu\theta} \int_0^\infty dz\ [K_{i\nu}(2zr)][\tilde{V}(z,\nu) + \tilde{V}(-z,\nu)] \qquad \text{Region I}$$
$$\tilde{T}^{(\nu)}_{scat} = e^{i\nu\theta} \int_0^\infty dz\ [H^{(1)}_{i\nu}(2zr) - H^{(2)}_{i\nu}(2zr)][\tilde{V}(z,\nu) + \tilde{V}(-z,\nu)] \qquad \text{Region II} \tag{12}$$

The expressions in the other regions may be trivially obtained from these by changing the sign of one of the coordinates appropriately.

Let us first evaluate this transform semiclassically using collective field theory as discussed above. This leads to

$$\tilde{V}^{sc}(z,\nu) = A(\nu)\ H^{(1)}_{i\nu}(2\sqrt{\mu}z) \qquad \tilde{V}^{sc}(-z,\nu) = -A(\nu)\ H^{(2)}_{i\nu}(2\sqrt{\mu}z) \tag{13}$$

where $H^{(1,2)}_{i\nu}$ denote the standard Hankel functions, and $A(\nu)$ is a constant which is irrelevant to our discussion. The superscript sc stands for semiclassical. Substituting these expressions in (12) and performing the integral over z [16] one obtains

$$T^{sc}_{scat}(u,v) = B(\nu)\ \mu^{\frac{i\nu}{2}}\ v^{-i\nu}\ F(\frac{1}{2} - i\nu, \frac{1}{2}; 1 - i\nu; -\frac{uv}{\mu}) \tag{14}$$

where $B(\nu) = -\frac{i\sqrt{\pi}\Gamma(\frac{1}{2}-i\nu)\ sinh(\pi\nu)}{\sqrt{\mu}\Gamma(1-i\nu)}$, and F stands for the hypergeometric function. This expression is valid throughout Region I while in Region II it is valid for $-uv < \mu$ (note in Region II uv is negative), i.e. in the region between the horizon and the singularity. In the exterior region this represents a wave which vanishes on the past horizon and may be written (by using standard relations between hypergeometric functions) in terms of a specific linear combination of right and left moving waves at an infinite distance away from the horizon. Thus we have an incoming wave and an outgoing wave, but nothing emerging from the past horizon

- precisely what we wanted. At the black hole singularity $uv = -\mu$ the wave is singular. The singularity comes from the upper limit of integration in (12). For large z the asymptotic forms of the Hankel functions are

$$H^{(1)}_{i\nu} \sim \frac{1}{\sqrt{z}}\, e^{i(z-\frac{\pi}{2}i\nu)} \quad H^{(2)}_{i\nu} \sim \frac{1}{\sqrt{z}}\, e^{-i(z-\frac{\pi}{2}i\nu)} \tag{15}$$

Using this it may be seen that in (12) the integral of products $H^{(1)}_{i\nu}(2rz)\, H^{(1)}_{i\nu}(2\sqrt{\mu}z)$ and $H^{(2)}_{i\nu}(2zr)H^{(2)}_{i\nu}(2\sqrt{\mu}z)$ are finite while the cross terms lead to a logarithmic singularity at $r = \sqrt{\mu}$. Therefore in the semiclassical limit a part of the wave that came in from infinity and went inside the horizon "crashes" into the singularity*.

The Exact wavefunctions

For non-perturbative computations of $\tilde{V}(z,t)$ we use the fermionic field theory of the matrix model[13] [14]. As explained in[14], this is done as follows. Consider the two point function $< V(z_1,\nu)V(z_2,-\nu) >$. The single particle tachyon wavefunction in the standard interpretation of the matrix model is obtaining by computing the quantity

$$\tilde{V}(z_2,\nu) \equiv \oint \frac{dz_1}{2\pi i}\, z_1^{-i\nu-1} \;\; < V(z_1,\nu)V(z_2,-\nu) > \tag{16}$$

and then performing a suitable analytic continuation to real loop lengths. The reasoning behind this identification is that the various powers of z_1 or z_2 denote contributions from various operators and the above contour intgeral simply isolates the part which is the contribution of the tachyon operator as dictated by the correct scaling dimension.(We will restrict our attention to nonintegral ν. The interpretation for integer ν is more involved, see e.g.[17][18][19] and related to the symmetry structure of the theory[20] [21] [18] [22]) It may be also simply verified that

$\star$ The transform considered in[1] does not diverge at the singularity. However, as noted above this does not describe a physical scattering process with the correct boundary conditions.

the quantity $\tilde{V}(z,\nu)$ satisfies the Wheeler de Witt equation with *negative* μ at the semiclassical level. Thus after analytic continuation $z \to il$ (in the sense explained in Ref[14]) this does represent the physical tachyon single particle wave function. We are, however, not interested in extracting liouville wave functions; we will, therefore, make no such analytic continuation†

The two point function in (16), or rather its μ-derivative may be computed exactly in the fermionic field theory, either directly as in[14] or using Ward identities to relate it to the one point function as shown in[19]. The result is (after a change of integration variables in the formulae of these papers)

$$\frac{\partial}{\partial\mu} < V(z_1,\nu)V(z_2,-\nu) >= \mathrm{Im} \int\limits_0^\infty \frac{d\xi}{\mu sinh(\xi/2\mu)} \; e^{i\xi+\frac{i}{2}\;(z_1^2+z_2^2)\; coth\;(\xi/2\mu)} \times$$
$$\{2\pi\; e^{\frac{\pi\nu}{2}} \; \frac{sinh(\nu\xi/2\mu)}{sin(\pi\nu)} \; J_{i\nu}[\frac{z_1 z_2}{sinh(\xi/2\mu)}] + \sum_{r=1}^{\infty} \frac{4i^r\; r\; sinh(r\xi/2)}{r^2+\nu^2} \; J_r[\frac{z_1 z_2}{sinh(\xi/2\mu)}]\} \tag{17}$$

The semiclassical expansion is obtained by expanding in powers of the string coupling $\frac{1}{\mu}$ keeping $l = \sqrt{\mu}z$ fixed. From (17) it is clear that this means replacing the hyperbolic functions by their power series expansions. To the leading order it is easy to see that the answers depend on the combinations $\sqrt{\mu}z_1$ and $\sqrt{\mu}z_2$, as one would expect on the basis of the Wheeler de Witt equations.

The single particle wave functions $\tilde{V}(z,\nu)$ comes from the first term of the series expansion of the Bessel function $J_{i\nu}(z_1 z_2/sinh(\xi/2))$, which goes as $(z_1 z_2)^{i\nu}$. The result is

$$\partial_\mu \tilde{V}(z,\nu) = \mathrm{Im}\; A(\nu)\; (z/2)^{i\nu} \int\limits_0^\infty \frac{d\xi}{[sinh\;(\xi/2)]^{i\nu+1}} \; e^{i[\mu\xi+\frac{1}{2}z^2\; coth\;\frac{\xi}{2}]} \; sinh(\nu\xi/2) \tag{18}$$

where $A(\nu) = \frac{2\pi e^{\frac{\pi\nu}{2}}}{sinh(\pi\nu)\;\Gamma(1+i\nu)}$. The integration over ξ may be performed in terms

† We are using a definition of the nonperturbative theory which has a potential behaving as $-x^2$ both for positive and negative x[23]. Since we do not make any analytic continuation, the quantities we define are unambiguous as opposed to non-perturbative S-matrices which have to be defined by analytic continuation to real loop lengths.

of Whittaker functions[14]. The result is

$$\partial_\mu \tilde{V}(z,\nu) = \text{Im } B(\nu)\frac{1}{z}[\Gamma(\frac{1}{2}-i\mu)\, W_{i(\mu+\frac{\nu}{2}),\frac{i\nu}{2}}(-iz^2)-\Gamma(\frac{1}{2}-i\mu+i\frac{\nu}{2})\, W_{i(\mu-\frac{\nu}{2}),\frac{i\nu}{2}}(-iz^2)] \tag{19}$$

where $W_{\lambda,\kappa}$ stands for the Whittaker function. For zero energy $\nu = 0$ this is the μ- derivative of the vacuum expectation value of the operator $V(z,t)$ as expected. This takes the simple form

$$\tilde{V}(z,0) = \text{Im } [e^{\frac{-3\pi i}{4}}\ \Gamma(\frac{1}{2}-i\mu)\ \frac{1}{z}\ W_{i\mu,0}(-iz^2)] \tag{20}$$

Several properties of these exact expressions for $\tilde{V}(z,\nu)$ are worth noting and will be useful in a moment. First, to take the semiclassical limit one has to remember that $l = \sqrt{\mu}z$ has to be kept fixed while $\sqrt{\mu} \to \infty$. Using standard asymptotic forms of Whittaker functions[24] it may be checked that the above expression reduce to the semiclassical expressions involving Hankel functions in this limit. Secondly, *for small values of z these wavefunctions reduce to the semiclassical wavefunctions in the corresponding limit.* This may be seen from (18) from where it is clear that for small z only large values of *coth* $\frac{\xi}{2}$, or only small values of ξ contribute. In that case one can replace the hyperbolic functions by their power series expansions. But this is precisely what yields the semiclassical limit. Alternatively one can directly take the result (19) and use the small-z behaviour of the Whittacker functions[24]and verify that one indeed recovers the same answer as one obtains from the small z behaviour of the Hankel functions involved in the semiclassical wave function. This is simply a reflection of the fact that the coupling constant in the string field theory of the matrix model goes rapidly to zero away from the wall. In our case this region corresponds to the region of small z.

The quantity $\partial_\mu T_{scat}(u,v)$ can be now calculated by inserting (19) in (12). The first point to note about the reaulting expressions for the exact $T_{scat}(u,v)$ is that for large values of $|uv|$, i.e. for large values of r the exact expressions agree with the semiclassical expressions in this region. This follows from the large r behavior of the Bessel functions in (12). However, we have just argued that for small z the exact wavefunctions $\tilde{V}(z,\nu)$ approach their semiclassical estimates. Thus for large r the black hole wavefunctions also agree with their semiclassical answers. This fact shows that in the asymptotic region of the black hole the coupling of the theory is indeed weak. By the same token we expect that near the black hole there will be significant deviations from the semiclassical behavior.

We saw in that in the semiclassical limit the logarithmic singualrity of the scattering wavefunction T_{scat} came from the upper limit of integration, i.e. from large z. This came about from a competition between the exponentials coming from the matrix model wavefunctions and the Bessel functions in (12). However it is precisely in the large z region that the matrix model wavefunctions differ most from the semiclassical answers. In fact, the asymptotic behavior of the Whittaker function is given by

$$W_{\lambda,\kappa}(-iz^2) \sim z^{2\lambda}\ e^{\frac{iz^2}{2}} \tag{21}$$

and there is no competition between this and the e^{2izr} coming from the Hankel functions. The result is in fact **finite at the singularity**.

It is important to note that the scattering wavefunction remains infinite at the location of the black hole singularity to *all orders in the semiclassical expansion.* This is similar to the result of[8]about the value of the one point function. The smearing of the behavior of the wavefunction near the singularity is a genuinely nonperturbative effect.

Discussion

What does this result mean ? Our matrix model black hole connection shows that one can obtain wavefunctions (and presumably the S-matrix) of tachyons moving in a black hole background at the semiclassical level by evaluating suitable quantities in the matrix model. Given this connection we evaluate the *same* quantities non-perturbatively in the matrix model. Our interpretation of these non-perturbative answers is that we are evaluating correlations or wavefunctions in *some* (possibly nonlocal) theory of a scalar field $T(u,v)$ whose semiclassical behavior is identical to that of tachyons in a black hole background.

We found that while the exact wavefunctions approach the semiclassical wavefunctions in the asymptotic region, they depart significantly inside the black hole, which is therefore a region of strong coupling. In this region it is not very meaningful to expand around the standard classical ground state solution and interpret the resulting fluctuations as "particles". Nevertheless $\tilde{T}_{scat}(u,v)$ can be correctly interpreted as single particle tachyon wavefunctions in the aymptotic regions and hence it is meaningful to talk of a scattering process.

As emphasized in[15] the region "beyond the singularity", i.e. the portion of Region II with $-uv > \mu$ has to be taken seriously for the string theory black hole. Since the exact answer for $\tilde{T}_{scat}(u,v)$ is nonsingular it is now meaningful to continue it across the singularity and ask what happens "beyond the singularity". For large r in Region II we can either look at the small z behavior of the Whittaker functions[24] or simply look at the semiclassical answer. This can be easily computed (the answer (14) is not valid here since $-uv > \mu$)

$$T^{(\nu)}_{scat}(u,v) \sim \frac{1}{r}\ (\frac{r}{\sqrt{\mu}})^{i\nu}\ e^{i\nu\theta} \tag{22}$$

which shows that the wavefunction has support on *one* of the null infinities (corresponding to large values of v). The wave simply lands up in the asymptotic region beyond the singularity.

To really see whether we are describing the behavior of tachyons of *two dimensional string theory* in a black hole background, one has to compute S-matrices and compare them with continuum S-matrices, e.g. those obtained from the coset model. After all, the identification of quantities in the matrix model with tachyon fields in a liouville background can be considered as correct because the tree level

S-matrix computed by matrix model methods[25] agreed with those obtained from the continuum[26]. Without a similar continuum calculation in a black hole background the relationship between the theory of tachyon fields which result from the transform discussed above and string theory is unclear. Nevertheless our results show that in this exactly solvable theory of tachyons nonperturbative effects may drastically alter the behavior near the singularity.

In the standard interpretation of the matrix model it is troublesome to assign a clear spacetime interpretation in the strongly coupled region[9] [14]. For example there is no relativistic invariance in this region. In our case this implies that the region inside the black hole does not have a clear relativistic space time interpretation. This is, in fact, expected in a string theory. As noted above, the fact that the theory is weakly coupled far from the black hole allows us to meaningfully desribe scattering processes involving tachyons. However this makes the absence of singularities of the exact wavefunction difficult to understand. Assuming that our complicated field theory of massless "tachyons" is indeed a *string* theory, the only statement we can make is that the string does not "see" any singularity even though it perceives a singularity at the semiclassical level. Since the graviton and dilaton fields are not explicitly present in the formalism it is confusing to ask whether the singularity itself has vanished.

Finally, it would be very interesting if one can understand time dependent backgrounds in string theory, like formation of black holes, in terms of the matrix model. Since there is no explicit field for the graviton or the dilaton it would not be possible to see the black hole "evaporate away", but its effects on the scattering of tachyons can still be studied.

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