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BLACK HOLE FORMATION AND SPACE-TIME FLUCTUATIONS IN TWO DIMENSIONAL DILATON GRAVITY AND COMPLEMENTARITY

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Abstract

We study black hole formation in a model of two dimensional dilaton gravity and 24 massless scalar fields with a boundary. We find the most general boundary condition consistent with perfect reflection of matter and the constraints. We show that in the semiclassical approximation and for the generic value of a parameter which characterizes the boundary conditions, the boundary starts receeding to infinity at the speed of light whenever the *total* energy of the incoming matter flux exceeds a certain critical value. This is also the critical energy which marks the onset of black hole formation. We then compute the quantum fluctuations of the boundary and of the rescaled scalar curvature and show that as soon as the incoming energy exceeds this critical value, an asymptotic observer using normal time resolutions will always mea-

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sure large quantum fluctuations of space-time near the *horizon*, even though the freely falling observer does not. This is an aspect of black hole complementarity relating directly to quantum gravity effects.
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I. INTRODUCTION

Recently, the question of information loss during the process of black hole evaporation [1] have been intensively studied [2]. The reason behind this renewed interest is the discovery of black hole solutions in two dimensonal dilaton gravity models [3] and exact conformal field theories describing two dimensional black holes [4] and the demonstration that black hole formation and subsequent evaporation can be *in principle* studied in two dimensional models [5].

Much of the work in the past two years have dealt with the problem in the semiclassical approximation [5], [6–18] in which the dilaton and graviton fields are treated as classical quantitites wheras the matter fields are treated quantum mechanically - an approximation which is valid if the number of matter fields N is large with the product Ne^{ϕ} kept finite (ϕ denotes the dilaton field). A more definitive analysis can be carried out for Reissner-Nordstrom black holes [19]. In most models the semiclassical approximation is valid in a region of space time bounded by a critical line where $\phi(x)$ becomes equal to a critical value $\phi_c(x)$. Nevertheless, the evolution of the system away from this boundary can be studied analytically for some models or numerically for others. The results of the semiclassical picture seems to converge on a picture where information is lost from the observable part of the universe.

It has been argued, however - most persuasively by 't Hooft - that one cannot really ignore quantum fluctuations of gravity even if the black hole is of large mass and N is large since high energy physics near the horizon appear as normal energy processes to the aymptotic observer due to the very large redshift.

A possible approach to an exact quantum treatment of black holes comes from the connection of matrix models with static black hole backgrounds, viz. a certain integral transform of the collective field of the matrix model behaves as a "tachyon" field in the black hole background of the two dimensional critical string. [20]. One remarkable result is that the singularity of the background disappears in the full nonperturbative answer [21] .

In fact the same holds true for the exact one particle wave function of the problem as well as for more general incoming waves [22] . A different matrix model black hole connection has been proposed in [23] . We expect the above conclusions to hold in this model as well [1]. However, this connection is as yet known only for *static* black holes and one does not have a description of black hole formation. Furthermore, in matrix models gravity and dilaton fields do not appear in the action explicitly : in a sense they are integrated out and their quantum effects are completely contained in the collective field theory or the fermionic field theory. It is useful to have models where one can study gravity fluctuations explicitly and exactly and infer how they affect the quantum evolution of black holes.

In a recent paper one such model with 24 scalar fields in the presence of two dimensional dilaton gravity with a boundary has been studied by Verlinde and Verlinde and by Schoutens,Verlinde and Verlinde [25] . In a fiducial coordinate system with light-cone coordinates (u, v) the boundary is taken to be $u = v$. However, in terms of physical coordinates which become minkowskian far away, the boundary is dynamically determined by the infalling matter and undergoes quantum fluctuations. In two dimensions there is no physical local degree of freedom of the graviton-dilaton system, but the degree of freedom corresponding to a dynamical boundary survives. Thus the effect of boundary fluctuations in this model serves as a useful toy model for studying effects of quantum gravity fluctuations. In fact this model has some features of the moving mirror problem [26] .

In this paper we study this model further with a view to understand quantum gravity effects. We first find the most general boundary conditions which leads to a perfect reflection of the matter energy momentum tensor and consistent with the constraints. These boundary conditions are parametrized by a single parameter β. The boundary conditions of [25] correspond to $\beta = 0$. In a recent paper [27] similar boundary conditions have been used to study the dynamics of the boundary as a dynamical moving mirror. Our discussion is

[1] For yet another proposal for description of black holes in matrix models see [24]

complementary to this paper and concentrates on the space-time features of the model.

The physics of the model is very different for $\beta = 0$ and $\beta \neq 0$. For $\beta = 0$ there is no meaningful classical solution. There is a semiclassical solution in which the boundary "jumps forward" and becomes spacelike as soon as the incident matter *energy density* exceeds a critical value. At the same time a black hole singularity forms which meets the apparent horizon at a finite time, like the other semiclassical models studied in the literature.

On the contrary the $\beta \neq 0$ model makes sense classically. The boundary now receedes away as the matter impinges on it. When the *total* energy of the incoming matter exceeds a critical value, the boundary starts receeding with the speed of light and with unbounded acceleration at a time much *later* than the time of impact. In the classical theory this is also the critical value beyond which a black hole singularity forms. At the semiclassical level, the critical value of the energy has a different dependence on β. The solution is, in fact, very similar to that in the RST model [7] with a similar picture of black hole evaporation.

Finally we quantize the $\beta \neq 0$ model fully by expanding around the classical solution representing black hole formation. We calculate some quantum fluctuations of quantities related to space-time in this theory, like the dispersion of the line element on the boundary and the dispersion of the rescaled scalar curvature. As in any field theory these dispersions depend on the ultraviolet cutoff which simply express the fact that one generically sees more and more quantum fluctuation as one goes to shorter and shorter distances. In other words, the dispersion depends on on the resolution time of the measuring apparatus. If all observers use a resolution time of order one (in natural units) we show that an infalling observer finds a finite dispersion. An asymptotic observer, however, has a rather different conclusion. So long as the incoming energy is below the critical value, the aymptotic observer also finds a finite dispersion with a finite resolution time. However as soon as the energy exceeds a critical value and a horizon is formed the dispersion grows as one approaches the horizon and becomes infinite at the horizon - even with a finite resolution time. Thus the aymptotic observer always finds that the quantum fluctuations of the dilaton and metric diverge at the horizon, which may be quite far from the singularity. The result is quite consistent with the ideas put forward by 't Hooft [28]and with ideas of black hole complementarity [29] . The phenomenon is quite similar to the infinite spreading of strings moving in black hole or rindler spacetimes considered by Susskind [30]. It is unclear whether the presence of these large fluctuations of space time as observed by an asymptotic observer signifies the invalidity of the semiclassical approximation even for large black holes.

II. THE MODEL AND BOUNDARY CONDITIONS

The model consists of a space-time metric in two dimensions $g_{ab}(u,v)$ and a dilaton field $\phi(u,v)$ and 24 massless scalar fields $f^i(u,v)$ with the action

$$S = \frac{1}{2\pi}\int dudv\ \sqrt{g}[e^{-2\phi}(R - 4(\nabla\phi)^2 -4\lambda^2) + \frac{1}{2}(\nabla f^i)^2] \tag{1}$$

Here R is the scalar curvature and λ^2 is a cosmological constant which can be set to one by a choice of scale. The space-time has a boundary, which is the fixed line $u = v$ in a fiducial set of null coordinates (u, v). These are defined in terms of the space coordinate x and the time coordinate t by $u = t + x$ and $v = t - x$.

We will work in the conformal gauge $g_{ab} = e^{2\rho}\ \eta_{ab}$. Let us introduce two free chiral fields $X^+(u)$ and $X^-(v)$. These are related to the metric and dilaton fields by

$$e^{2(\rho-\phi)} = \partial_u X^+(u)\partial_v X^-(v) \tag{2}$$

The remaining equation of motion is then

$$\partial_u\partial_v e^{-2\phi} = -\partial_u X^+(u)\partial_v X^-(v) \tag{3}$$

and constraints simply state the total energy momentum tensor vanishes

$$T^g_{uu} + T_{uu} = 0 \qquad T^g_{vv} + T_{vv} = 0 \tag{4}$$

Here T^g_{uu}, T^g_{vv} stand for the gravity-dilaton part of the eenergy momentum tensor while T_{uu}, T_{vv} the matter part. We will often omit the subscripts since they are chiral. Since the matter is conformally coupled we may write

$$f^i(u,v) = f^{i+}(u) - f^{i-}(v) \tag{5}$$

The general solution of (3) may be written as

$$e^{-2\phi} = -X^+(u)X^-(v) + g^+(u) + g^-(v) + K \tag{6}$$

where the functions $g^\pm$ have to be determined by solving the constraints (4), and K is an integration constant. The expressions for T^g become

$$\begin{aligned} T^g_{uu} &= -\partial_u g^+(u)\tfrac{\partial_u^2 X^+}{\partial_u X^+} + \partial_u^2 g^+(u) \\ T^g_{vv} &= -\partial_v g^-(v)\tfrac{\partial_v^2 X^-}{\partial_v X^-} + \partial_v^2 g^-(v) \end{aligned} \tag{7}$$

Now define two new fields $Y^+(v)$ and $Y^-(u)$ by

$$\begin{aligned} \partial_u g^+(u) &= Y^-(u)\partial_u X^+(u) \\ \partial_v g^-(v) &= Y^+(v)\partial_v X^-(v) \end{aligned} \tag{8}$$

One then has

$$T^g_{uu} = \partial_u Y^-(u)\partial_u X^+(u) \qquad T^g_{vv} = \partial_v Y^+(v)\partial_v X^-(v) \tag{9}$$

Finally we write down the expression for the scalar curvature in terms of the fields introduced above. This is given by

$$\begin{aligned} R &= 8e^{-2\rho}\partial_u\partial_v\rho \\ &= 4[1 + e^{2\phi}(X^-(v) - Y^-(u))(X^+(u) - Y^+(v))] \end{aligned} \tag{10}$$

We also introduce the "rescaled" scalar curvature

$$\tilde{R} \equiv e^{-2\phi}\ (R - 4) \tag{11}$$

which will be useful in later calculations. In fact this is the quantity that appears in the action. The expression (10) shows that the zeroes of $e^{-2\phi}$ would be generically curvature singularities, unless the expression which multiplies it vanishes. Furthermore any region where any of the fields $X^\pm, Y^\pm$ diverge is also potentially a region of curvature singularity.

A. Boundary Conditions

The boundary conditions we want to impose are (i) The dilaton field must be constant along the boundary, i.e. $(\partial_u + \partial_v)e^{-2\phi} = 0$ along $u = v$ and (ii) the matter must be perfectly reflected at the boundary. This sets $f^{i+}(u) = f^{i-}(u)$ and ensures that $T_{uu}(u) = T_{vv}(v)$ at $u = v$. Since the total energy momentum tensor must vanish (or a constant in the quantum theory) we must, for consistency, also require that (iii) the energy momentum tensor of gravity is also reflected off the boundary perfectly, i.e. $T^g_{uu}(u) = T^g_{vv}(v)$ at $u = v$.

The conditions on the dilaton and metric fields are nontrivial. The condition (i) is

$$[\partial_u X^+(Y^-(u) - X^-(v)) + \partial_v X^-(Y^+(v) - X^+(u))] = 0 \tag{12}$$

at $u = v$. In [25] each term in (12) was separately set to zero, which also automatically ensured (iii). This sets both $\partial_u e^{-2\phi}$ and $\partial_v e^{-2\phi}$ to zero at the boundary and corresponds to the boundary conditions used in [7]. Clearly this is not the general condition.

If there was no boundary we could have used the remaining reparametrization invariances to choose coordinates such that $X^+(u)$ and $X^-(v)$ are both of a desired form. However the boundary conditions relate left moving and right moving modes, which means that the remaining reparametrizations of u and v are not independent. We can, however, fix one of the fields $X^\pm$. In this paper we will often use a gauge such that $X^+(u) = u$. We will see later that this choice corresponds to a choice of Kruskal coordinates. The form of $X^-(v)$ has to be now determined by solving the constraints as we will show.

Let us introduce a new field $h_-(v)$ defined as

$$\partial_v X^-(v) = h_-^2(v) \tag{13}$$

The most general form for Y^+, Y^- which satisfies the condition (12) may be easily seen to be, in the gauge $X^+(u) = u$

$$\begin{aligned} Y^-(u) &= X^-(u) + F[u, h_-(u)] \\ Y^+(v) &= v - F[v, h_-(v)]\frac{1}{h_-^2} \end{aligned} \tag{14}$$

where $F[x, h_-(x)]$ is a general function of x and a functional of $h_-(x)$. Now substitute (14) into the expressions (9) and get

$$T^g_{uu} = h_-^2 + \frac{dF}{du}$$
$$T^g_{vv} = h_-^2 - \frac{dF}{dv} + \frac{2F\partial_v h_-}{h_-} \tag{15}$$

Requiring these two expression to match at $u = v$ we get the condition

$$\frac{\partial F}{\partial u} + \frac{\delta F}{\delta h_-}\partial_u h_- = \frac{F\partial_u h_-}{h_-} \tag{16}$$

whose unique solution is

$$F[u, h_-(u)] = \beta h_-(u) \tag{17}$$

where β is an integration constant. The boundary conditions in [25] corresponds to $\beta = 0$.

The boundary conditions (14) may be considered as operator conditions in the quantum theory. In the semiclassical theory, however, the expression for the energy momentum tensor has additional terms coming from the conformal anomaly and the form of the solution for $F[u, h_-(u)]$ are correspondingly different. This will be discussed in a later section.

III. THE CLASSICAL SOLUTION

Given some energy momentum tensor of incoming matter the classical solution is obtained by solving the equations $T^g(u) + T(u) = 0$ and $T^g(v) + T(v) = 0$ where T^g is obtained from (15) with F given by (17). These two equations are in fact identical (since the reflection condition on matter fields set $T(u) = T(v)$) and become

$$h_-^2 + \beta\partial_u h_- + T(u) = 0 \tag{18}$$

Given a solution of $h_-(u)$ it is straightforward to solve for $X^-(u)$. This is given by

$$X^- = \int^u h_-^2 + C \tag{19}$$

where C is an integration constant. This becomes, after using (18),

$$X^-(u) = -\int^u T(u) - \beta h_- + C \tag{20}$$

The gravity and dilaton fields may be now readily obtained. From the definition of $g^\pm$ and Y^+, Y^- in (6) and (8), the relations (14) and (17) and the equation (18) we get

$$\begin{aligned} g^+(u) &= uX^-(u) + \beta u h_-(u) - \int^u u'T(u') \\ g^-(v) &= -\beta v h_-(v) - \int^v dv'v'T(v') \end{aligned} \tag{21}$$

Putting these together we get

$$\begin{aligned} e^{-2\phi} = u[X^-(u) - X^-(v)] + \\ [\int^u du'u'T(u') - \int^v dv'v'T(v')] \\ +\beta[uh_-(u) - vh_-(v)] + K \end{aligned} \tag{22}$$

where K is an integration constant, which is the value of $e^{-2\phi}$ on the boundary.

Note that in this classical problem, β can be scaled out of the problem by rescaling $h_-(u) \to \beta h_-(u)$ and $T(u) \to \beta^2 T(u)$ in (18). This leads to a scaling of $X^- \to \beta^2 X^-$. This immediately shows that $\beta = 0$ is a rather singular limit. In the following we will work mostly with $\beta \neq 0$ unless otherwise stated.

A. Solution with Shock wave matter

We now find the classical solution in the presence incoming matter in the form of a shock wave whose matter energy momentum tensor is given by $T(u) = \frac{1}{u^2}T\delta(u-1)$ (we have chosen the location of the shock wave at $u = 1$ by a suitable shift of coordinates). We will solve (18) with the condition that the spacetime is flat linear dilaton vacuum before the shock wave arrives (i.e. in the region $u < 1$ for all v). With this condition the solution to (18) can be found in all of the $u - v$ space :

$$h_-(v) = \frac{\beta}{v}[1 - (\frac{a}{a + v(1-a)})\theta(v-1)] \tag{23}$$

where $a = \frac{T}{\beta^2}$. This is an explicitly real solution and since $\partial_v X^-(v) = h_-^2(v)$ it may appear that $X^-(v)$ is a monotonically increasing function, so that the boundary is everywhere timelike. However, (23) clearly shows that $h_-(v)$ blows up for some positive value of v whenever $a > 1$. This becomes clear when one looks at the solution for $X^-(v)$ obtained by plugging in (23) into (20)

$$X^-(v) = \beta^2[(-\frac{1}{v} + a)\theta(1-v) - (\frac{1-a}{a+(1-a)v})\theta(v-1)] \tag{24}$$

which shows that $X^-(v)$ blows up at $v = \frac{a}{a-1}$ when $a > 1$. The solution for the dilaton field is given by

$$\begin{aligned} e^{-2\phi} &= \beta^2 \frac{u}{v} & u, v < 1 \\ &= \beta^2(u(\frac{1}{v} - a) + a) & u > 1, v < 1 \\ &= \beta^2 \frac{(a+(1-a)u)}{(a+(1-a)v)} & u, v > 1 \end{aligned} \tag{25}$$

The meaning of the parameter β is now clear. This is simply the value of $e^{-\phi}$ at the boundary. The metric is best expressed in terms of the coordinates $z^\pm$ defined as $z^+ = u$ and $z^- = -\frac{1}{v}$. This becomes

$$\begin{aligned} ds^2 &= \frac{dz^+dz^-}{z^+z^-} & u, v < 1 \\ &= \frac{dz^+dz^-}{(a - z^+(z^- + a))} & u > 1, v < 1 \\ &= (\frac{az^- + (1-a)}{az^- + (1-a)z^-z^+})\ dz^+dz^- & u, v > 1 \end{aligned} \tag{26}$$

From (26) and (24) it may be verified that the asymptotically minkowskian coordinates are $\log(X^+(u))$ and $-\log(-X^-(v))$. As in theories of gravity field variables become physical coordinates. In the quantum theory these "coordinate fields" become operators and have important consequences [25].

The profile of the boundary in the X^+, X^- coordinates is simply the curve of $X^-(v)$ as a function of v. From the above expressions it is clear that as long as $a < 1$, the boundary is always timelike. At the point where the pulse hits the boundary there is a discontinuity, but it never has an unbounded acceleration. Furthermore $X^-(v)$ always stays negative, and no horizons are formed. For $a > 1$, $X^-(v)$ turns positive *before* the pulse hits the boundary. At the impact point, there is a usual discontinuity, but nothing dramatic happens at ths point. However at a later time the acceleration increases without bound, the velocity approaches the speed of light and X^- becomes infinite at a point $v = \frac{a}{a-1}$. As a increases this point approaches the point $u = v = 1$ where the shock have hits the boundary. It is not meaningful to continue the solution beyond this point. In fact the asymptotic observer will see the boundary running away at the point where X^- hits a zero, since this is the end of the asymptotic coordinate system. This instability has been observed in [27] and is different from the instability discussed above. The latter occurs for a Kruskal observer, using the $X^\pm$ coordinates.

Thus there is a critical value of the *total* energy of the incoming pulse beyond which the boundary runs away. Note that we are dealing with a delta function shock wave for which the *energy density* is always unbounded.

The interesting point in this model is that it is precisely at this value of the total energy that a black hole starts forming. This is clear from the solution to the dilaton and the metric fields in (25) and (26). Note that the conformal factor in the $z^\pm$ coordinates is exactly equal to $e^{2\phi}$. From (26) we see that there are no singularities in the region before the incident pulse. In the region $u > 1, v < 1$ the metric is exactly like the standard black hole metric. We thus have a potential curvature singularity along the spacelike line $z^+(z^- + a) = a$. Since this region in the u, v space corresponds to $z^+ > 1, z^- < -1$, there is no singularity when $a < 1$. For $a > 1$ there is a singularity which asymptotes to $z^- = -a$. The null line $z^- = -a$ which correspond to $v = \frac{1}{a}$ is the event horizon which in this case also coincides with the apparent horizon (defined in the usual way by $\partial_u \phi = 0$). In the region $u, v > 1$ the singularity is along the null line $u = \frac{a}{a-1}$ which begins at the point where the singularity in the region II intersects with the reflected wave.

The expression (10) may be used to compute the curvature scalar. The result is that R diverges on the part of the singularity in the $u > 1, v < 1$ region, but is finite along the null singularity in the $u, v > 1$ region. However, in this model the kinetic energy terms contain a factor of $e^{-2\phi}$ and zeroes of $e^{2\phi}$ are genuine singularities even when the curvature is finite.

The kruskal diagrams for the solution (in the coordinates $X^{\pm}$) are shown in Fig. 1 for $a < 1$ and in Fig. 2 for $a > 1$. Note the horizon is the line $X^- = 0$ while the reflected pulse is along $X^- = \beta^2(a-1)$.

IV. THE SEMICLASSICAL SOLUTION

We now outline what happens in the model when we take into account quantum fluctuations of the matter fields, but still treat the dilaton gravity sector classically. As is well known, this has several effects. First, there is a constant negative vacuum energy due to normal ordering effects. In the (u, v) coordinates this vacuum energy (for N scalar fields) is simply $-\frac{N}{48u^2}$ Secondly the conformal anomaly induces new terms in the action which modifies the equations of motion as well as the form of the energy momentum tensor for the dilaton gravity system. However, as demonstrated in [7] one can choose counterterms such that the dynamical equations of motion of the semiclassical theory are similar to those of the classical theory with the replacement

$$e^{-2\phi} \to \Omega \equiv \frac{1}{\kappa}[e^{-2\phi} + \kappa\phi] \tag{27}$$

Here $\kappa = \frac{N}{24}$. Thus the solution to Ω is still given by the general form in equation (6). With the same definition of the fields Y^-, Y^+ as in (8) the expressions for the gravity part of the energy momentum tensor becomes, instead of (9)

$$\begin{aligned} T^g_{uu} &= \kappa[\bar{T}^g_{uu} + \frac{\hat{\kappa}}{2\kappa}[(\partial_u \log\partial_u X^+)^2 - 2\partial_u^2 \log\partial_u X^+]] \\ T^g_{vv} &= \kappa[\bar{T}^g_{vv} + \frac{\hat{\kappa}}{2\kappa}[(\partial_v \log\partial_v X^-)^2 - 2\partial_v^2 \log\partial_v X^-]] \end{aligned} \tag{28}$$

where $\bar{T}^g$ denote the classical contributions in (9) and $\hat{\kappa} = \kappa - 1 = \frac{N-24}{24}$. The most general boundary conditions may be now derived in the gauge $X^+(u) = u$ following the same steps as in the previous sections. With the same ansatz as in (14) and the requirement that the gravity part of the energy momentum tensor is perfectly reflected at the boundary one gets instead of (16) the following equation for $F[h_-, u]$

$$\frac{\partial F}{\partial u} + \frac{\delta F}{\delta h_-}\partial_u h_- = \frac{F\partial_u h_-}{h_-} + \hat{\kappa}[(\partial_u \log h_-)^2 - \partial_u^2 \log h_-] \tag{29}$$

whose general solution is

$$F[h_-(u), u] = \beta h_-(u) - \frac{\hat{\kappa}}{\kappa}\partial_u \log h_-(u) \tag{30}$$

This shows that, in particular, the boundary conditions in [25], which is identical to that used in [7] cannot be valid unless $N = 24$ [2]. The equation satisfied by $h_-(u)$ now becomes

$$h_-^2 + \beta\partial_u h_- - \frac{\hat{\kappa}}{\kappa}\partial_u^2 \log h_- + T(u) = \frac{1}{2u^2} \tag{31}$$

In general these equations are difficult to solve exactly.

The semiclassical limit could be a good description when N is very large. However one may get some insight into quantum effects of matter for the present case of $N = 24$ where the semiclassical equations can still be solved exactly. This is because in this case the anomaly is absent and the only modifications are the inclusion of the vacuum energy and the replacement of the dilaton field by Ω in (27).

A. Semiclassical solution with shock wave for $\beta \neq 0$

To solve the constraints for a shock wave one has to solve (18) with

$$T(u) = \frac{1}{2u^2}[T\delta(u-1) - 1] \tag{32}$$

We require that the solution to the left of the shock wave is the flat minkowski space linear dilaton vacuum. This yields the following solution

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[2] S. Trivedi has informed us that he and his collaborators have also discovered that the boundary conditions $\partial_\pm \Omega = 0$ are not consistent in the RST model. [31]

$$h_-(u) = \frac{1}{2u}[(\beta + \sqrt{\Delta}) - \frac{2T\sqrt{\Delta}}{(T + (\beta\sqrt{\Delta} - T)u^{\frac{\sqrt{\Delta}}{\beta}})}\theta(u-1)] \tag{33}$$

The solution for $X^-(u)$ is

$$X^-(u) = -\frac{1}{u}[\gamma + Tu\theta(u-1) - \frac{\beta T\sqrt{\Delta}}{(T + (\beta\sqrt{\Delta} - T)u^{\frac{\sqrt{\Delta}}{\beta}})}\theta(u-1)] \tag{34}$$

where

$$\Delta = \beta^2 + 2 \qquad \gamma = \frac{1}{2}[1 + \beta(\beta + \sqrt{\Delta})] \tag{35}$$

The field $\Omega(u,v)$ is then given by

$$\begin{aligned}
\Omega(u,v) &= \gamma\frac{u}{v} - \frac{1}{2}\log(\frac{\gamma u}{v}) \qquad u,v < 1 \\
&= u(\frac{\gamma}{v} - T) + T - \frac{1}{2}\log(\frac{\gamma u}{v}) \qquad u > 1, v < 1 \\
&= \frac{\gamma u}{v} + [1 - \frac{u}{v}]\frac{\beta T\sqrt{\Delta}}{(T + (\beta\sqrt{\Delta} - T)v^{\frac{\sqrt{\Delta}}{\beta}})} \\
&\quad -\frac{1}{2}\log(\frac{\gamma u}{v}) \qquad u,v > 1
\end{aligned} \tag{36}$$

In writing down (36) we have chosen an integration constant in the solution for $\Omega(u,v)$ such that in the region before the incoming wave, one has a standard linear dilaton vacuum with $e^{-2\phi} = \frac{\gamma u}{v}$. As in the classical solution the asymptotically minkowskian coordinates are $\pm\log(\pm X^\pm)$.

From (34) it is clear that $X^-(u)$ diverges for some finite $u = u_0$ where

$$u_0 = [\frac{T}{T - \beta\sqrt{\Delta}}]^{\frac{\beta}{\sqrt{\Delta}}} \tag{37}$$

whenever $T > T_0 = \beta\sqrt{\Delta}$. Thus $\beta\sqrt{\Delta}$ is the critical value of the incoming energy beyond which the boundary runs away.

Before looking for singularities let us look for the presence of apparent horizons. This is given by the curve along which $\partial_u\Omega = 0$ (since $\partial_u\phi = 0$ implies $\partial_u\Omega = 0$). It is trivial to see that there is no apparent horizon in the region $u, v < 1$. In the region $u > 1, v < 1$ the equation for the apparent horizon is

$$\frac{\gamma}{v} - T = \frac{1}{2u} \tag{38}$$

If present, this is always a timelike line (since as u increases, so does v). For large values of u the apparent horizon asymptotes to the null line $v = \frac{\gamma}{T}$. It is then easy to see that the curve (38) does lie in the region $u > 1, v < 1$ unless

$$T > \gamma - \frac{1}{2} \tag{39}$$

For the region $u, v > 1$ the equation for the apparent horizon is given by

$$\frac{1}{2u} + \frac{A(v) - \gamma}{v} = 0 \tag{40}$$

where we have defined

$$A(v) \equiv \frac{\beta T \sqrt{\Delta}}{(T + (\beta\sqrt{\Delta} - T)v^{\frac{\sqrt{\Delta}}{\beta}})} \tag{41}$$

The singularities correspond to the zeroes of $\partial_\phi \Omega(u, v)$. This means that $\Omega = \Omega_{cr} = \frac{1}{2}(1 + \log 2)$. From the above solution it may be seen that in the region $u, v < 1$ there are no singularities for $\beta \neq 0$. (For $\beta = 0$ the boundary coincides with the critical line in this region as we will see later).

In the region $u > 1, v < 1$ the solution is exactly like the solution of the RST model. In the RST model a singularity is formed whenever the *energy density* exceeds a critical value. Thus for a delta function shock wave a spacelike singularity is always formed. In this case, however, there is a critical value of the *total energy* beyond which a singularity is formed. For a shock wave one must have $T > T_c$, where T_c may be determined as follows. The singularity, if present must intersect the $u = 1$ line or the $v = 1$ line. It is easy to see from (36) that there are no zeroes of $\Omega(u, v)$ along the $u = 1$ line. Along $v = 1$ one has $\Omega = \Omega_{cr}$ at a value of $u = u_0$ which is determined by solving the equation

$$u_0 = \frac{\log(u_0) + 2(\Phi_0 - T)}{2(\gamma - T)} \tag{42}$$

where

$$\Phi_0 = \frac{1}{2}(1 + \log(2\gamma)) \tag{43}$$

Note that $\frac{1}{2} < \Phi_0 < \gamma$. When $T > \gamma$ this always has a solution. In this regime the singularity starts out as spacelike, intersects the apparent horizon where it turns time-like and continues all the way to $u = \infty$ as a timelike naked singularity, just as in the RST model. However the solution should not be continued beyond the point of intersection of the sinmgularity and the apparent horizon which is given by (u_s, v_s) where

$$\begin{aligned} u_s &= \frac{1}{2T}[e^{2T} - 1] \\ v_s &= \frac{\gamma}{T}[1 - e^{-2T}] \end{aligned} \tag{44}$$

As noted in [7], Ω attains vaccum values along the line $v = v_s$. This is the case in our model with $\beta \neq 0$ as well. Consequently one may impose boundary conditions corresponding to a "thunderpop" such that the solution in the region $v > v_s$ is the linear dilaton vaccum. In our case this turns out to be

$$e^{-2\phi} = \kappa u(\frac{\gamma}{v} - T) \tag{45}$$

In the region $v > v_s$ the critical line $\Omega = \Omega_{cr}$ coincides with the apparent horizon. On this line the curvature is finite so there is no naked singularity. One then has a picture where the black hole has evaporated completely and information is lost in this semiclassical model.

For $T < \gamma$, (42) has a solution for $T > T_c$ where T_c is value of T for which the straight line represented by the left hand side of (42) is tangential to the curve on the right hand side for some value of u. This means that T_c is determined by the equation

$$\gamma = \frac{T_c}{(1 - e^{-2T_c})} \tag{46}$$

Note that $T_c < \gamma$ as required. A plot of the critical energy T_c as a function of β is shown in Fig. 3. Note that for $T_c < T < \gamma$ the equation (42) has two solutions so that before imposition of an additional boundary condition at (u_s, v_s) the singularity meets the outgoing pulse at a finite value of u. For $T > T_c$ the fate of the singularity in this region is identical to that for $T > \gamma$ which is described above.

The interesting point is that T_c is always greater $T_0 = \beta\sqrt{\Delta}$ which is the critical value of T beyond which the boundary starts receeding with infinite speed. A plot of the quantity $\frac{T_c}{T_0}$ is shown in Fig. 4. It also follows from the definition of γ that $\gamma - \frac{1}{2} < \beta\sqrt{\Delta} < T_c$.

In the region $u, v > 1$ there are no singularities for $T < \beta\sqrt{\Delta}$. For $\beta\sqrt{\Delta} < T < T_c$ there is a singularity which starts out at the boundary at $u = v = u_0$ along a timelike direction. It then turns spacelike and meets the apparent horizon after a finite proper time at the verge of turning timelike [3]. This happens at the point (u_s, v_s) which is obtained by solving the following implicit equations

$$u_s = \frac{v_s}{2\gamma\alpha} \quad A(v_s) = -\frac{1}{2}\log(\alpha) \tag{47}$$

where $A(v)$ has been defined above in (41) and α is the nontrivial solution of

$$\gamma\alpha - \frac{1}{2}\log\alpha = \gamma \tag{48}$$

As before one can impose boundary conditions here such the region $v > v_s$ is a linear dilaton vacuum which is given in this case by

$$e^{-2\phi} = \kappa\gamma\alpha\frac{u}{v} \tag{49}$$

Recall that u_0 is the value of v for which $X^-(v)$ blows up. Thus, just as in the classical solution, the region $v > u_0$ is beyond the singularity and not present in the spacetime defined by the semiclassical solution. For $T > T_c$ the singularity in the $u, v > 1$ region starts off as usual at the boundary, but crosses the reflected wave at some finite value of v where it joins the singularity of the $u > 1, v < 1$ region as discussed above. In the Kruskal coordinates the singularity is asymptotic to the runaway part of the boundary, similar to that in the classical solution.

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[3] We would like to thank M. O'Laughlin for pointing us a mistake about the nature of the singularity in this region in an earlier version of the paper

B. Solutions with $\beta = 0$

We now mention briefly the features of the semiclassical solution for $\beta = 0$ [4]. We will thus solve the equation (18) with $\beta = 0$ with $T(u) \to T(u) - \frac{1}{2u^2}$ to account for the vacuum energy. It is immediately clear that in a region where $T(u) - \frac{1}{2u^2} > 0$ the quantity $h_-^2 < 0$ which means that $\partial_v X^-(v)$ is negative. This means that the physical coordinates which are asymptotically minkowskian are double valued as a function of v and the boundary becomes spacelike. For a shock wave given by (32) this always happens since a shock wave corresponds to an infinite energy density. For such a shock wave the solution for $X^-(v)$ is given by

$$X^-(v) = -\frac{1}{2v} + T\theta(1-v) \tag{50}$$

Note that unlike $\beta \neq 0$ the coordinate field $X^-(v)$ is discontinuous at the location of the pulse, with a *finite* discontinuity. For a spread-out pulse this discontinuity is absent, but corresponds to a region where X^- runs backward, corresponding to the boundary becoming spacelike.

In this case a singularity is present for any positive value of T. Now one has

$$\begin{aligned}\Omega(u,v) = &\frac{u}{2v} + T(u-1)[\theta(v-1) - \theta(u-1)] \\ &-\frac{1}{2}\log(\frac{u}{2v})\end{aligned} \tag{51}$$

The boundary $u = v$ is precisely the location of $\Omega = \Omega_{cr}$. For the region $u > 1, v < 1$ the singularity forms at the wake of the incoming pulse and has a fate similar to the RST model. In fact the case $\beta = 0$ is almost identical to the RST model.

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[4]This section summarizes results obtained in collaboration with E. Martinec.

C. Information loss

Whenever parameters in the theory are in a range such that a singularity and a horizon is formed there is the standard information loss at the semiclassical level. At the semiclassical level the "in" and "out" modes of the matter fields f^i are defined as

$$a_\nu = \int du \; [X^+(u)]^{i\nu} \partial_u f^i$$
$$b_\nu = \int du \; [-X^-(u)]^{-i\nu} \partial_u f^i \tag{52}$$

since the asymptotically Minkowskian coordinates are $\pm\log(\pm X^\pm)$ in all the regions. In the $X^+ = u$ gauge all the nontriviality of the mode expansions lie in the form of $X^-(u)$. If there are no horizons $X^-(u)$ would vanish only at $u \to \infty$ and both the integrals in (52) would be over the full range of u. However in the presence of a horizon, the second integral in (52) goes from $-\infty$ to $-v_h$ where $v = -v_h$ is the location of the horizon. The resulting n_ν no longer form a complete set and one has a standard story. A reasonable definition of the S-matrix in the problem would be given by

$$< 0, out| \prod_\nu b_\nu \prod_\nu a_\nu |0, in > \tag{53}$$

and would be nonunitary semiclassically.

Normally one would get a unitary evolution if one took into account the states inside the horizon as well, i.e. if one integrates over the full range of v. Here, however, the $X^-(v)$ is a nonmonotonic function of v. As a result, the modes b_ν are not complete, all combinations of the modes a_ν cannot be expressed as combinations of b_ν, though the converse is certainly true. Thus the S-matrix satisfies $SS^\dagger = 1$ but $S^\dagger S \neq 1$.

V. QUANTUM FLUCTUATIONS AND COMPLEMENTARITY

We now consider aome aspects of the quantum fluctuations of the graviton dilaton degrees of freedom. In this model these degrees of freedom are represented by the chiral fields $X^+(u), X^-(v), Y^-(v), Y^+(u)$. The model is essentially a free field theory in these variables and one may hope to address exact quantum questions.

A. Light cone quantization

We will quantize the model around the classical solution with a shock wave described in section 2. It is sufficient to consider the set of fields $X^+(u)$, $Y^+(u)$ and $f^i(u)$ since the boundary conditions ensure an identical situation in the other chiral sector. We thus expand the operators as

$$\begin{aligned} X^+(u) &= X^+_{cl}(u) + X^+_{qu}(u) \\ Y^-(u) &= Y^-_{cl}(u) + Y^-_{qu}(u) \\ f^i(u) &= f^i_{cl}(u) + f^i_{qu}(u) \end{aligned} \tag{54}$$

where subscripts (cl) and (qu) denote classical and quantum pieces. There is a constraint

$$\partial_u X^+(u)\partial_u Y^-(u) + \frac{1}{2}\partial_u f^i(u)\partial_u f^i(u) = 0 \tag{55}$$

In the quantum theory the right hand side has to be replaced by a suitable normal ordering term $\frac{1}{2u^2}$. Our model is almost identical to critical bosonic string theory. However, as emphasized in [25] the expansion of the field in terms of the physical modes are rather different.

We will quantize the theory in a light cone gauge $X^+(u) = p^+u$. We will also put the system in a box of size $2\pi\tilde{L}$ to keep track of infrared behaviour. Using standard mode expansions

$$\partial_u f^i_{qu}(u) = \frac{1}{\sqrt{2\pi}\tilde{L}}\sum_m \tilde{f}^i_m \; e^{-im\frac{u}{\tilde{L}}} \tag{56}$$

(and similarly for the other fields) one may then solve the constraints to solve for the nonzero modes of $Y^-(u)$ in terms of the $\tilde{f}^i_m$. The remaining dynamical variables are the canonically conjugate pair of zero modes q^-, p^+ and the matter oscillators. The zero mode q^- represents the overall translational degree of freedom and decouples from the dynamics. At the classical level this is the arbitrary integration constant which is chosen to ensure that $\pm\log[\pm X^\pm]$ are good asymptotic coordinates.

The mode expansions of the type (56) are, however, not very useful in this problem. In the quantization around the specific solution representing the formation and evaporation of black hole, the asymptotic "in" coordinates are $y^+ = \log u$ rather than u. Thus the oscillators $\tilde{f}^i_m$ do not annahilate the "in" vacuum. In the region $u > 0$ one should thus expand in terms of the appropriate modes

$$\partial_{y^+} f^i_{qu}(y^+) = \frac{1}{\sqrt{2\pi L}} \sum_m f^i_m \; e^{-im\frac{y^+}{L}} \tag{57}$$

In the region $u < 0$ one has a similar modes in terms of coordinates $\log(-u)$. We will also introduce coordinates $y^- = \log(v)$. The semiclassical region of interest has $v < 1$ and hence $y^- < 0$. The boundary in terms of these coordinates is $y^+ = y^-$.

In the following we shall deal exclusively with the region $u > 0$. However to examine questions like unitarity etc. one has to deal with the oscillators corresponding to the other region as well [5].

Let us now solve the constraints in this gauge. The classical solution for $Y^-(u)$ is given by

$$Y^-_{cl}(u) = \beta^2 a\theta(-y^+) \tag{58}$$

while f^i_{cl} is nonzero only along $y^+ = 0$. Then the quantum part of $Y^-(u)$ may be solved to be, for $y^+ > 0$

$$\partial_{y^+} Y^-(y^+) = \frac{e^{-y^+}}{2\pi L^2} \sum_m L^{tr}_m e^{-im\frac{y^+}{L}} \tag{59}$$

where L^{tr} denotes the virasoro modes of the matter energy momentum tensor

$$L^{tr}_m = \frac{1}{2}[\sum_n : f_{m-n} f_n : -2\pi L^2] \tag{60}$$

[5] In the covariant quantization in [25] one has a similar doubling of states corresponding to the two signs of the zero modes of xp. The vacuum sector S-matrix is unitary only if the full space of states are taken into account.

and we have included the standard intercept coming from normal ordering. The ”in” vacuum is now defined by

$$f_n|0, in >= 0 \qquad n \geq 0 \tag{61}$$

The Virasoro generators L_m^{tr} act on the vacuum as follows

$$\begin{aligned} L_m^{tr}|0, in > &= 0 \qquad m > 0 \\ L_0^{tr}|0, in > &= -2\pi L^2|0, in > \end{aligned} \tag{62}$$

B. Fluctuations of the boundary

In terms of the coordinates y^+, y^- the invariant line element on the boundary is given by ds_B where

$$ds_B^2 = e^{2\phi_0}[\partial_{y^+}X^+(y^+)\partial_{y^-}X^-(y^-)]_{y^+=y^-} \tag{63}$$

where ϕ_0 is the constant value of the dilaton field at the boundary. (Recall that the dilaton field is maintained to be a constant along the boundary in the quantum theory so that there are no quantum fluctuations of this quantity). For the classical solution we are expanding around, $e^{2\phi_0} = \frac{1}{\beta^2}$. We will now compute the dispersion of the quantity ds_B^2 along the boundary in the region $y^+, y^- < 0$.

In a quantum theory of gravity correlations of local operators do not make much sense, since one is integrating over the metric. The meaning of the dispersions we are about to calculate is as follows. We are expanding around a specific classical solution corresponding to a dynamical black hole formation with some incoming matter. We are interested in getting an idea of the strength of quantum fluctuations of space-time quantities, one example of which is the line element on the boundary. This may be done by computing the corresponding local quantities *using the classical metric as the reference metric.* These quantities have a perfectly good meaning for an asymptotic observer. In the asymptotic region the fluctuations of the metric are weak and the asymptotic observer may thus use the classical metric to make measurements. The correlations of these local operators have physical meaning *only* for these asymptotic observers.

The correlations of ds_B require mode expansions for $\partial_{y^-}X^-(y^-)$. Unlike $\partial_{y^+}Y^-$ these are difficult to compute exactly. We shall obtain these for small fluctuations around the classical solution. It is most convenient to consider the equation (18) in appropriate coordinates, as the full quantum equation and expand this around the classical solution. To lowest order one gets in the region $y^- < 0$, where $h_-^{cl} = \beta e^{-y^-}$,

$$\beta[\partial_{y^-}h_-^q + 2h_-^q] = e^{-y^-}[1 - \frac{1}{2}\partial_{y^-}f_{qu}\partial_{y^-}f_{qu}] \tag{64}$$

Here h_-^q denotes the quantum part of h_-. Defining modes as

$$h_-^q(y^-) = \frac{e^{-y^-}}{2\pi L^2}\sum_m h_m e^{-im\frac{y^-}{L}} \tag{65}$$

one has to this order

$$h_m = \frac{L_m^{tr}}{\beta(i\frac{m}{L} - 1)} \tag{66}$$

From this expression one may obtain the modes of $\partial_{y^-}X^-(y^-)$. The correlator

$$< 0, in|ds_B^2(y^-)ds_B^2(y^{-'})|0, in > \tag{67}$$

may be now calculated using the Virasoro algebra. For $y^- \to y^{-'}$ this goes as

$$< 0, in|ds_B^2(y^-)ds_B^2(y^{-'})|0, in > \sim \frac{1}{\beta^2(y^- - y^{-'})^2} \tag{68}$$

A freely falling observer will make measurements with a cutoff which keeps the invariant local distance fixed to some value and would replace the right hand side with

$$\frac{1}{(y^- - y^{-'})^2 + b^2 e^{-2\rho_{cl}[\frac{1}{2}(y^- + y^{-'})]}} \tag{69}$$

where ρ_{cl} is the conformal mode of the classical metric. Here b is the geodesic cutoff. The quantity $e^{-2\rho_{cl}(y^-)}$ along the boundary is small near the singularity, but not near the horizon.

Thus the freely falling observer perceives boundary fluctuations which are large only at scales smaller than the cutoff b. In particular he/she does not perceive any strong fluctuation near the horizon. This is consistent with the fact that the freely falling observer does not see anything special at the horizon.

An asymptotic observer at future null infinity would make measurements with a cutoff which has some fixed value in terms of the aymptotic coordinates. Recall that the asymptotic coordinates in the classical solution are

$$w^- \equiv -\log[-X^-(v)] = -\log[e^{-y^-} - a] \tag{70}$$

Since

$$\delta y^- = \delta w^-[1 - ae^{y^-}] \tag{71}$$

the quantity (68) is

$$< 0, in|ds_B^2(y^-)ds_B^2(y^{-'})|0, in > \sim \frac{1}{\beta^2(\delta w^-)^2[1 - ae^{y^-}]^2} \tag{72}$$

The quantum dispersion at scales larger than the cutoff of the aymptotic observer, $\tilde{b}$ is obtained by setting $\delta w^- = \tilde{b}$.

For $a < 1$ the fluctuations of the boundary increase as one gets closer to the point of impact at $y^- = 0$, but never become infinite. For $a > 1$ the fluctuations become strong at the location of the horizon regardless of the value of the cutoff. We thus conclude that as the incident energy approaches the critical value and a horizon starts to form, the asymptotic obsever measures very large fluctuations near the horizon. The basic reason behind this phenomenon is the large redshift between the horizon and infinity. Our result is in fact a concrete illustration of the contention of 't Hooft.

Quantum dispersions of other quantities show a similar behaviour. For example the correlation of the rescaled curvature $\tilde{R}$ introduced in (11) may be calculated to be (in the region $u > 1, v < 1$)

$$\begin{aligned} &< 0, in|\tilde{R}(y^+, y^-)\tilde{R}(y^{+'}, y^{-'})|0, in >= \\ &\qquad (1 + e^{(y^- - y^+)})^2[\frac{1}{(\delta y^+)^2} + \frac{1}{(\delta y^-)^2}] \end{aligned} \tag{73}$$

By the same reasoning as above, curvature fluctuations as measured by the aymptotic observer grow very large near the horizon.

VI. IN LIEU OF A CONCLUSION

The real question is : does these large fluctuations of space-time as perceived by the asymptotic observer invalidate the semiclassical approximation regardless of the mass of the black hole ? The question is confused by the fact that these fluctuations are large only for an asymptotic observer and one might argue that to examine the validity of the semiclassical approximation one should use a local geodesic cutoff. It is nevertheless clear that the asymptotic observer will have to account for large space-time fluctuations in his or her description of the Hawking process.

One way to obtain a definitive answer to this important question is to calculate quantities which have unambiguous meaning in quantum gravity, for example correlations of integrated operators. These "susceptibilities" may be studied as a function of the parameter a of the classical solution and one may be able to see whether quantum corrections grow large as a approaches the critical value 1. In this regard the present model with nonzero β could be useful since this has a critical value of a necessary for black hole formation and one could study quantum properties of the model as $a \rightarrow 1$ from below.

ACKNOWLEDGMENTS

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FIGURES

Fig. 1.: Classical Solution for $a < 1$. The incident shock wave reflects off the boundary and proceeds to future null infinity. No singularities are formed

Fig. 2.: Classicial solution for $a > 1$. The boundary runs away for a value of u which is *greater* than the value of u at which the shock wave hits the boundary. A singularity is formed which is null-like in the region above the reflected wave and space-like in the region below the reflected wave. The space-like portion of the singularity asymptotes to the global event horizon

Fig.3.: The critical energy T_c beyond which there is a singularity in region below the reflected wave plotted as a function of β

Fig.4. : The ratio of T_c to the minimum energy for black hole formation T_0 as a function of β

List of changes

The Editor Physical Review D

Re: Manuscript DP5003

Dear Sir,

We are sending separately a REVTEX version of our paper entitled "Black Hole Formation and Space-time fluctuations in two dimensional dilton gravity and complementarity" (Manuscript No. DP5003) which has been accepted fot publication in Physical Review. The figures are sent separately as an unencoded compressed tar file.

As suggested by you in an earlier communication, we are detailing below the minor changes we have made in the manuscript. In the following the equations refer to the version of the manuscript which we are sending now. Of course, these modifications do not change the main content and conclusions of the paper.

1. The equation (1) has been corrected for some signs and $e^{-\phi}$ has been replaced by $e^{-2\phi}$.

2. At the end of the paragraph following equation (1) we had added the definitions of u and v used in the paper.

3. A typing mistake in equation (26) has been corrected. In the second line of the equation we have replaced $u > 0$ by $u > 1$.

4. A mistake in equation (27) has been corrected (there should be an overall factor of $\frac{1}{\kappa}$ which was missed earlier.

5. An overall factor of κ which was missed in (28) has been suppplied.

6. In equations (30) and (31) $\hat{\kappa}$ has been replaced by $\frac{\hat{\kappa}}{\kappa}$.

7. A factor of half was missing in equation (32) which has now been inserted.

8. In equation (34) $Tu\theta(u-1)$ was wrongly typed as $Tu\theta(1-u)$ earlier. This has been corrected.

9. In equation (36) $v^{\frac{\sqrt{\Delta}}{\beta}}$ was typed as $u^{\frac{\sqrt{\Delta}}{\beta}}$. This has been corrected.

10. In the line following equation (38) we have added a line which states the asymptotic behaviour of the apparent horizon.

11. After equation (39) we have given the equation for the apparent horizon in the region $u, v > 1$. This is equation (39-40) and have been added for the sake of completeness.

12. In the paragraph after equation (43) we have added another equation (eqn. 44) which gives the values of u and v where the apparent horizon meets the singularity.

13. At the end of the same paragraph we have added, for completeness, the form of the dilaton field in the region $v > v_s$in eqn. (45). We have also added a line which states the behaviour of the critical line in this region.

14. In the last paragraph of section IV A, we have added three equations, two (equations 47 and 48) denoting the point of intersection of the apparent horizon and the singularity and the other (eqn. (49)) which gives the dilaton field in the region above this intersection points.

15. In equation (63) a factor of $e^{2\phi_0}$ was missing which has been inserted.

16. In equation (66) a typing mistake has been corrected. "2" should be replaced by "1".

17. In equation (68) a factor of $\frac{1}{\beta^2}$ was missing. This has been now inserted.

18. Some of tyhe references have been updated (journal references have been given wherever possible.)

19. We have added Figure captions, as required by the submission guidelines.

20. We have modified the labels of Figs. 3 and 4 since the phrase "critical energy" could be a bit misleading since there are several critical values of energy in the problem. We have made them precise using the notation used in the text.

We shall appreciate if you could let us know the publication date of the paper.

Thank you.

Yours sincerely,

Sumit Ranjan Das Sudipta Mukherji

This figure "fig1-1.png" is available in "png" format from:

http://arXiv.org/ps/hep-th/9401102v2

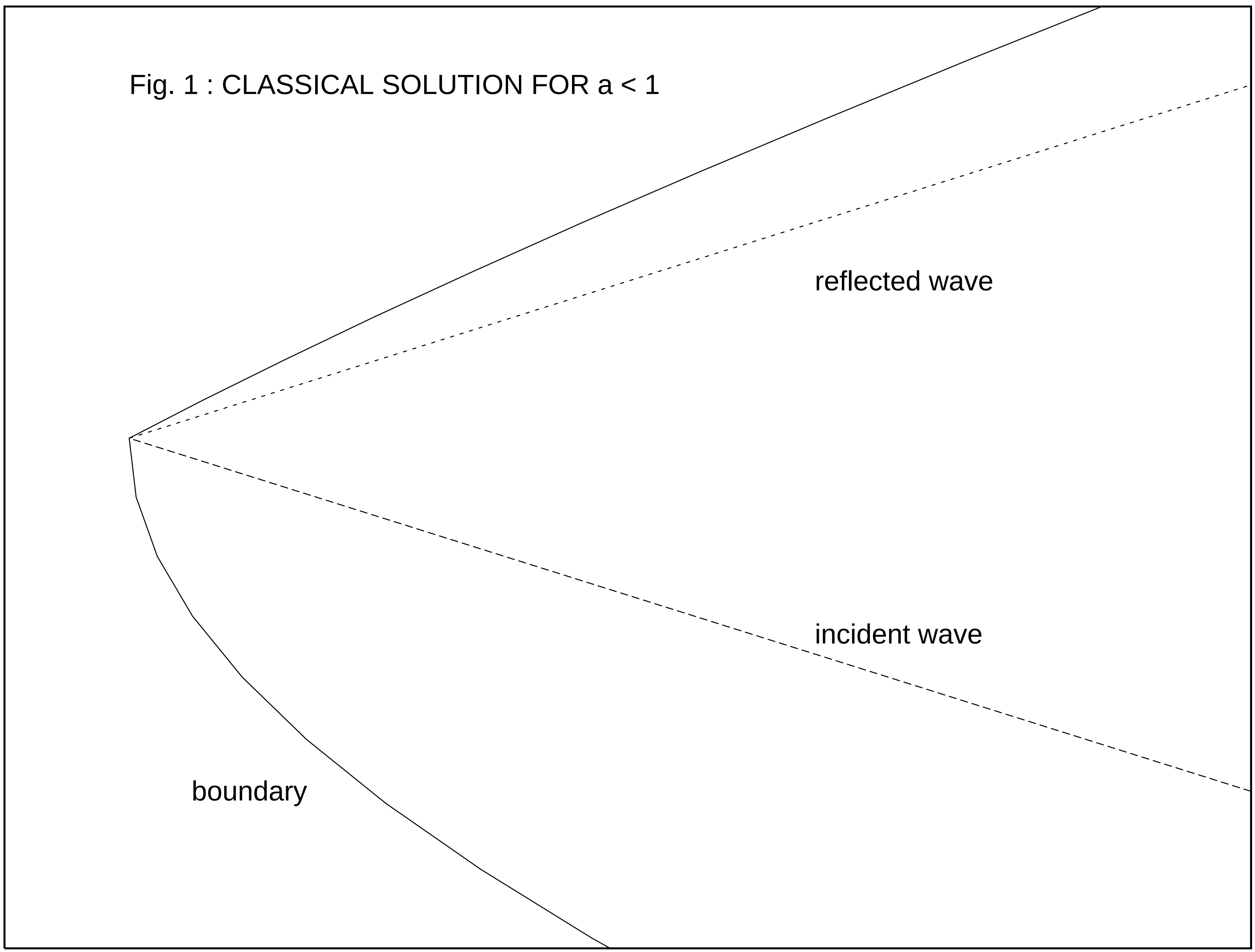
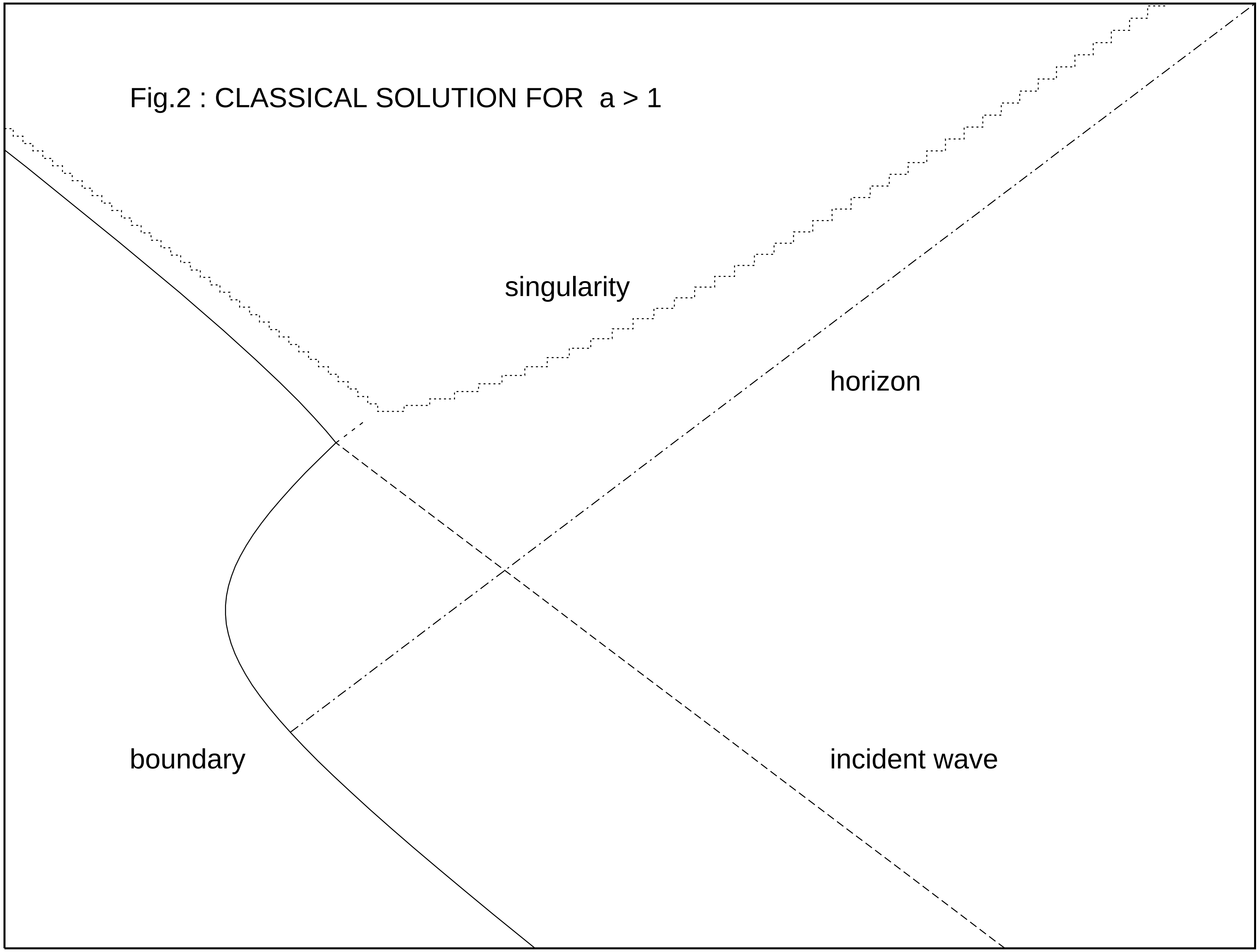


Fig. 1 : CLASSICAL SOLUTION FOR a < 1
reflected wave
incident wave
boundary

This figure "fig1-2.png" is available in "png" format from:

http://arXiv.org/ps/hep-th/9401102v2

Fig.2 : CLASSICAL SOLUTION FOR a > 1



This figure "fig1-3.png" is available in "png" format from:

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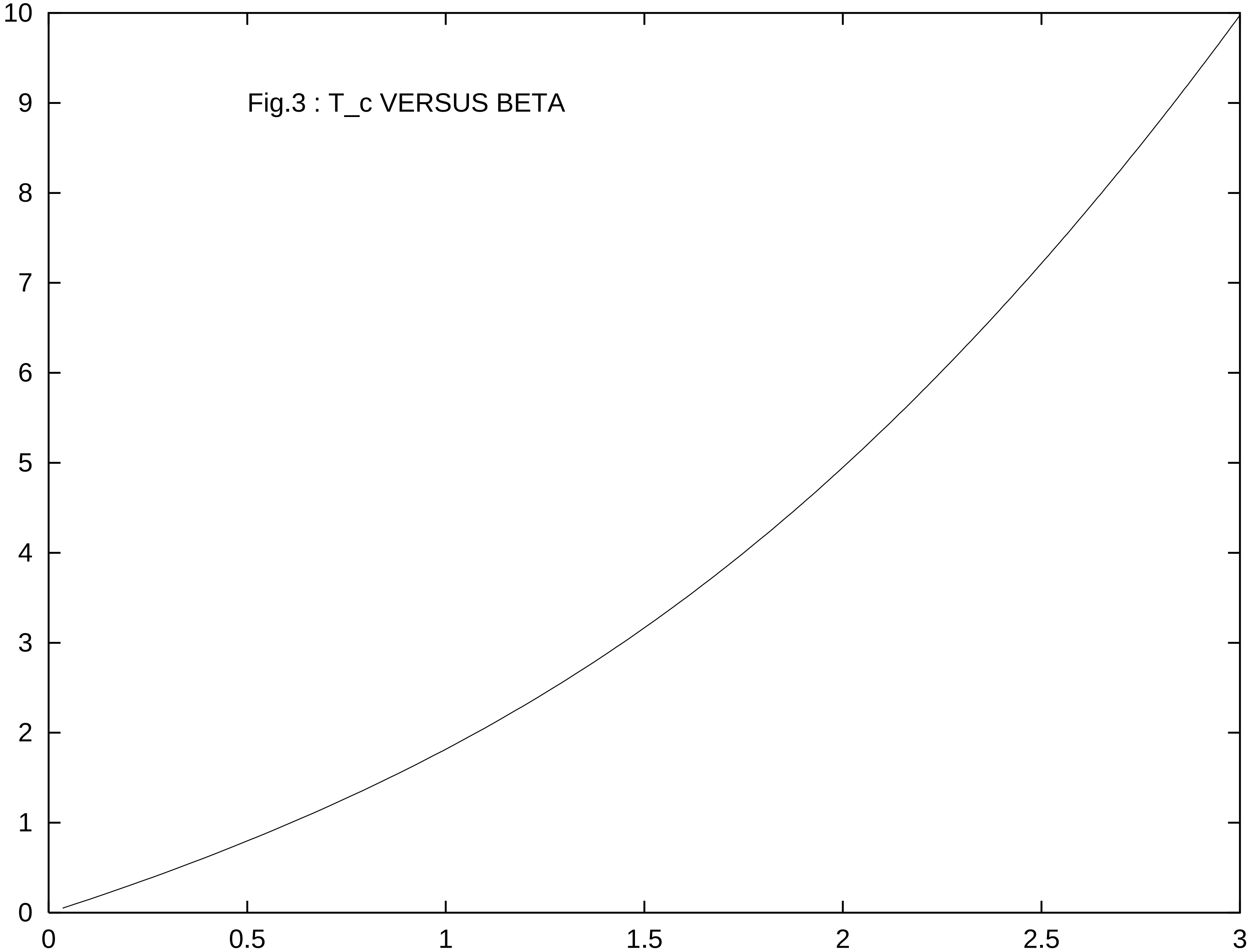
Fig.3 : T_c VERSUS BETA
10
9
8
7
6
5
4
3
2
1
0
0
0.5
1
1.5
2
2.5
3

This figure "fig1-4.png" is available in "png" format from:

http://arXiv.org/ps/hep-th/9401102v2

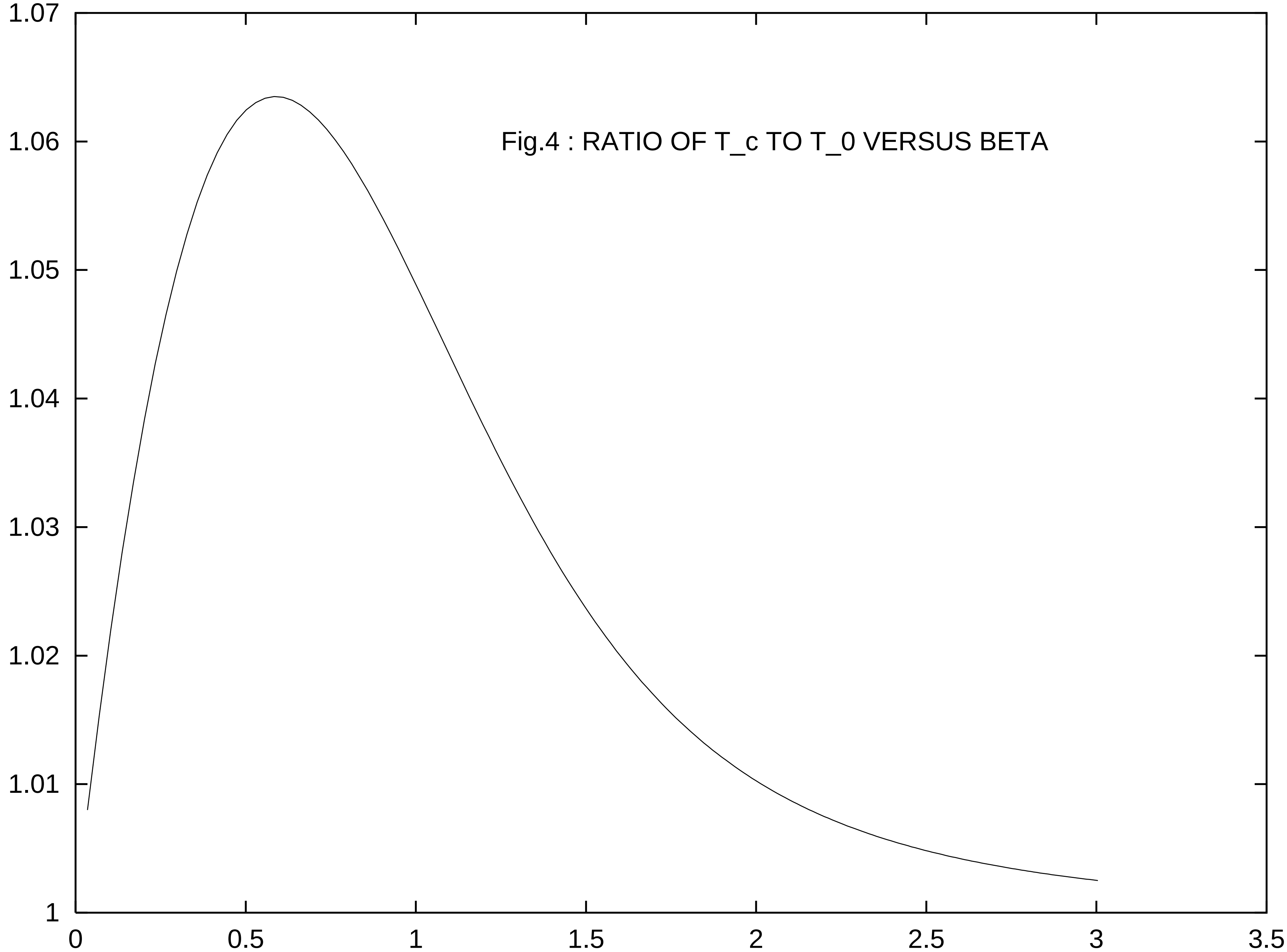
Fig.4 : RATIO OF T_c TO T_0 VERSUS BETA
1.07
1.06
1.05
1.04
1.03
1.02
1.01
1
0
0.5
1
1.5
2
2.5
3
3.5