

Vibration modes of giant gravitons

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We examine the spectrum of small vibrations of giant gravitons, when the gravitons expand in the anti-de-Sitter space and when they expand on the sphere. For any given angular harmonic, the modes are found to have frequencies related to the curvature length scale of the background; these frequencies are independent of radius (and hence angular momentum) of the brane itself. This implies that the holographic dual theory must have, in a given R charge sector, low-lying non-BPS excitations with level spacings independent of the R charge.

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1. Introduction

One of the interesting recent discoveries in string theory is the fact that objects that are naively pointlike may in fact be extended branes. The Myers effect [1] implies that in the presence of field strengths of the gauge fields in string theory, certain branes can expand into higher dimensional branes. McGreevy, Susskind and Toumbas [2] considered the behavior of gravitons in the near horizon geometries produced by branes. They argued that while these gravitons may appear pointlike at low angular momentum, one could (at a classical level) find extended brane states - called “giant gravitons” - carrying the same energy and angular momentum, and these extended objects could be the correct picture of the graviton at large angular momenta. For $AdS_m \times S^n$ spacetimes these can be a $(n - 2)$ -brane wrapped around a S^{n-2} contained in S^n [2] or a $(m - 2)$ -brane wrapped around a S^{m-2} contained in AdS_m [3][4]. The equilibrium configuration consists of the brane rotating rigidly without a change of its size, and saturates a BPS bound for the energy E for a given angular momentum P_ϕ , $E \geq P_\phi/L$ where L is the radius of the S^n [5]. The angular velocity is *independent* of the angular momentum and depends only on L . When embedded in a supersymmetric theory these configurations respect half of the supersymmetries of the background [3],[4] and therefore the BPS bound follows from supersymmetry. The structure of the final quantum state representing the gravitons is not yet clear, as there can be tunneling between these various classical configurations [3],[4],[5], and also to multiple brane states with the same quantum numbers [5]. Interestingly, brane states with energies equal to gravitons with the same angular momentum have been also found to occur in spacetimes other than $AdS_m \times S^n$ [6]. In some of these cases there are also extended branes with nonzero $D0$ brane charge which are degenerate with pointlike $D0$ branes and in this situation the phenomenon may be understood quantitatively in terms of the magnetic analog of Myers’ effect [6].

An important issue related to these ‘giant graviton’ states is whether they can provide an interpretation of the stringy exclusion principle [7] in terms of the limits on the allowed size of the expanded brane configurations [2]. It has been argued that the stringy exclusion principle implies that the supergravity should live on a noncommutative spacetime, e.g. quantum deformations of $AdS_m \times S^n$ [8]. Structures similar to expanded branes can also be obtained using the non-abelian interactions of multi-particle systems in string theory [9][10][11][12][13] where a noncommutative structure arises. Indeed for the giant gravitons there are hints of such a noncommutativity of space emerging [5],[14].

Extended objects possess a set of low energy excitations arising from small vibrations about their equilibrium configuration. Such modes have been very important in string theory in the study of black holes. If we regard a black hole as a point singularity surrounded by empty space then we cannot account for its microscopic degrees of freedom. But a string theory description of charged holes replaces the pointlike matter by extended branes, and vibration modes of these branes can be used to derive the microscopic entropy [15] and unitary low energy Hawking radiation for these black holes [16].

There are several reasons why we would like to study the vibration spectrum of giant gravitons. First, if there is a family of solutions with the same energy and angular momentum, then it would show up as a corresponding mode of the vibration spectrum. Thus looking at such modes would provide a way of checking whether the spherical brane ansatz used in [2] captures all the states that are classically BPS. Secondly, the branes in AdS space with angular momentum greater than the exclusion principle bound pose a puzzle [3]; we would like to check if these configurations are unstable to some harmonic of vibration (we do not in fact find such an instability). Thirdly, such vibration modes give a closely spaced set of low energy motions of the graviton that could be excited in interactions - thus these modes appear to be essential to finding the inclusive cross section for the interaction of two gravitons. In particular, one finds the following aspect of the stringy exclusion principle in the $D1 - D5$ system. One can compute the 3-point function of chiral primaries in the boundary CFT, for operators that are dual to supergravity modes in the bulk theory [17] [18]. It was found in [18] that the three point function starts dropping significantly below the naive supergravity expectation when the angular momenta are in fact much smaller than the allowed upper bound. If the giant graviton picture is right, then it could shed some light on this drop in the 3-point function.

In this paper we study the vibration spectrum of giant gravitons in $AdS_m \times S^n$. We focus on the excitations arising from motion of the branes in spacetime, and thus do not consider here the excitations that arise from fermionic modes, or from any gauge fields that may live on the brane representing the giant graviton. The embedding geometry has the structure of an anti-de-Sitter (AdS) spacetime times a sphere. We consider both the case where the brane expands in the AdS spacetime, and the case where it expands on the sphere. In the former case the radius of the brane can be much larger than the curvature scale of the embedding spacetime, while in the latter case the radius of the brane is limited by the size of the sphere.

One of the questions that we shall focus on is the scale which gives the vibration frequencies. There are several scales in the problem: the microscopic string scale or planck scale, the radius of the brane, and the curvature scale of the spacetime (the AdS spacetime and the sphere have curvatures of the same order). A priori the frequency of vibration can be any combination of these scales which has the right dimension. Since we are extremising the classical action we will not encounter the string or planck scales, but it is not immediately obvious which of the other scales should give the vibration frequencies.

We find that the frequencies are all real and, somewhat surprisingly, *independent of the size of brane*. They depend only on the AdS scale \tilde{L} and the radius of the sphere L . The system thus has low energy excitations with spacings independent of the angular momentum of the brane. This gives a prediction for the holographic dual: for a given R-charge sector of the dual theory, one should find a non-BPS spectrum of excitations with level spacings independent of the R charge.

The plan of this paper is the following. In section 2 we set up notation and describe the equilibrium configurations about which we will study the vibrations. In section 3 we find the spectrum for the branes that expand in the AdS spacetime, and in section 4 we repeat this calculation for branes that expand on the sphere. Section 5 studies some aspects of the frequencies and section 6 is a general discussion of the implications of the spectrum.

2. The equilibrium configurations

We will for the most part follow the notation of [3]. The spacetime will have the form $AdS_m \times S^n$. In particular we have the $D = 11$ supergravity backgrounds with $(m, n) = (4, 7)$ and $(m, n) = (7, 4)$, and the $D = 10$ background of IIB supergravity with $(m, n) = (5, 5)$. In the first two cases the graviton can expand into 2-branes and 5-branes of M-theory, while in the last case it can expand into the D-3 brane of type IIB string theory. In addition we have $(m, n) = (3, 3)$ for the cases $AdS_3 \times S^3 \times M^4$, though this case is more involved since since the 1-branes can be various combinations of strings and 5-branes wrapped on the compact 4-manifold (T^4 or K3).

2.1. Branes expanding in AdS spacetime

We will find it convenient to use slightly different coordinates when looking at branes expanding in the AdS and when looking at branes expanding on the sphere. The AdS space and the sphere are orthogonal to each other

$$ds^2 = ds_{AdS}^2 + ds_{sph}^2 \quad (2.1)$$

(In the case of $AdS_3 \times S^3 \times M^4$ the M^4 is orthogonal to the other two components as well.)

For branes in AdS, we write the AdS metric in global coordinates as

$$ds_{AdS}^2 = -(1 + \frac{r^2}{\tilde{L}^2})dt^2 + \frac{dr^2}{(1 + \frac{r^2}{\tilde{L}^2})} + r^2 d\Omega_{m-2}^2 \quad (2.2)$$

We can further write the metric $d\Omega_{m-2}^2$ as

$$d\Omega_{m-2}^2 = d\alpha_1^2 + \sin^2 \alpha_1 (d\alpha_2^2 + \sin^2 \alpha_2 (\dots + \sin^2 \alpha_{m-3} d\alpha_{m-2}^2)) \quad (2.3)$$

The sphere S^n has radius L . We write

$$ds_{sph}^2 = L^2 d\Omega_n^2 \quad (2.4)$$

where $d\Omega_n^2$ is the metric on the unit n sphere. We further describe this unit sphere by using coordinates $z_1, z_2, y_i, i = 1 \dots n - 1$

$$z_1^2 + z_2^2 + y_1^2 + \dots + y_{n-1}^2 = 1 \quad (2.5)$$

$$z_1^2 + z_2^2 = \cos^2 \theta, \quad \sum_{k=1}^{n-1} y_k^2 = \sin^2 \theta \quad (2.6)$$

$$z_1 = \cos \theta \cos \phi, \quad z_2 = \cos \theta \sin \phi \quad (2.7)$$

A complete set of coordinates for S^n is ϕ, y_k . In these coordinates the metric for S^n is

$$ds_{sph}^2 = L^2 [(1 - \sum_k y_k^2) d\phi^2 + (\delta_{ij} + \frac{y_i y_j}{1 - \sum_k y_k^2}) dy_i dy_j] \quad (2.8)$$

The brane in its unexcited configuration moves at $\theta = 0$, which gives $y_k = 0$. Further,

$$\phi = \omega_0 t, \quad \omega_0 = \frac{1}{L} \quad (2.9)$$

$$r = r_0 \tag{2.10}$$

The worldsheet extends for all t , and wraps the $m - 2$ sphere in (2.2), covering all values of the coordinates $\alpha_i, i = 1 \dots m - 2$. The angular momentum P_ϕ is given by

$$P_\phi = \tilde{N} \left(\frac{r_0}{\tilde{L}} \right)^{m-3} \tag{2.11}$$

where

$$\tilde{N} = V_{m-2} T_{m-2} L \tilde{L}^{m-2} \tag{2.12}$$

and V_{m-2} is the volume of the unit S^{m-2} and T_{m-2} is the tension of the $(m - 2)$ -brane. The energy of this state is given by

$$E = \frac{P_\phi}{L} \tag{2.13}$$

2.2. Branes expanding on the sphere

For this case we must essentially interchange the roles of the AdS and sphere metrics in the above discussion, and so we adopt the following coordinates. We describe an AdS_m spacetime of unit radius through

$$-(u_1^2 + u_2^2) + (v_1^2 + \dots + v_{n-1}^2) = -1 \tag{2.14}$$

$$u_1^2 + u_2^2 = \cosh^2 \mu, \quad \sum_{k=1}^{m-1} v_k^2 = \sinh^2 \mu \tag{2.15}$$

$$u_1 = \cosh \mu \cos \frac{t}{\tilde{L}}, \quad u_2 = \cosh \mu \sin \frac{t}{\tilde{L}} \tag{2.16}$$

A complete set of coordinates for the AdS space is t, v_k . In these coordinates the metric for AdS_m is

$$ds_{AdS}^2 = -\left(1 + \sum_k v_k^2\right) dt^2 + \tilde{L}^2 \left(\delta_{ij} + \frac{v_i v_j}{1 + \sum_k v_k^2} \right) dv_i dv_j \tag{2.17}$$

(We pass to the covering space where t runs over $(-\infty, \infty)$ instead of $(0, 2\pi)$ to recover the complete AdS spacetime.)

The metric of the sphere S^n is

$$ds_{sph}^2 = L^2 [d\theta^2 + \cos^2 \theta d\phi^2 + \sin^2 \theta d\Omega_{n-2}^2] \tag{2.18}$$

Here θ, ϕ are the same coordinates as introduced in (2.6), (2.7) and where we can write

$$d\Omega_{n-2}^2 = d\chi_1^2 + \sin^2 \chi_1 (d\chi_2^2 + \sin^2 \chi_2 (\dots + \sin^2 \chi_{n-3} d\chi_{n-2}^2)) \tag{2.19}$$

This time the unexcited brane will have a fixed value of θ between 0 and π . We write

$$\sin \theta = \frac{q}{L} \quad (2.20)$$

Then the metric on S^n becomes

$$ds_{sph}^2 = L^2 \left(1 - \frac{q^2}{L^2}\right) d\phi^2 + \frac{dq^2}{1 - \frac{q^2}{L^2}} + q^2 d\Omega_{n-2}^2 \quad (2.21)$$

which resembles the form (2.2) of the AdS metric. The brane has a radius determined by

$$q = q_0. \quad (2.22)$$

$\phi(t)$ is again given by (2.9), and the brane extends over the coordinate t , and over the coordinates χ_i in (2.19). The angular momentum is now

$$P_\phi = N \left(\frac{q_0}{L}\right)^{n-3} \quad (2.23)$$

where

$$N = L^{n-1} V_{n-2} T_{n-2} \quad (2.24)$$

is the quantized flux of the n -form field strength. The energy is still given by (2.13).

The length scales of the AdS and the sphere are related by

$$\frac{\tilde{L}}{L} = \frac{m-1}{n-1} \quad (2.25)$$

3. Vibration spectrum - branes in AdS

The action for the brane is

$$S = S_{DBI} + S_{CS} \quad (3.1)$$

where S_{DBI} is the Dirac-Born-Infeld action, and S_{CS} is the Chern-Simons term.

For a p brane, S_{CS} is

$$S_{CS} = T_p \int P[A^{(p+1)}] \quad (3.2)$$

where P denotes the pullback of the $p+1$ form gauge potential onto the brane worldvolume.

We will consider a $m-2$ brane. The gauge potential in spacetime gives a constant field

strength on AdS_m and its dual (constant) field strength on S^n . The potential on AdS_m is¹

$$A_{t\alpha_1\dots\alpha_{m-2}}^{(m-1)} = \frac{r^{m-1}}{\tilde{L}} \sqrt{g_\alpha} \quad (3.3)$$

where $\sqrt{g_\alpha} d\alpha_1 \dots d\alpha_{m-2}$ is the volume element on the $m-2$ sphere. Thus the field strength is

$$F_{rt\alpha_1\dots\alpha_{m-2}}^{(m)} = (m-1) \frac{r^{m-2}}{\tilde{L}} \sqrt{g_\alpha} \quad (3.4)$$

In a local orthonormal frame the value of F is

$$F_{\hat{r}\hat{t}\hat{\alpha}_1\dots\hat{\alpha}_{m-2}}^{(m)} = \frac{(m-1)}{\tilde{L}} = \text{constant} \quad (3.5)$$

The potential on the sphere is

$$A_{\phi\chi_1\dots\chi_{n-2}}^{(n-1)} = L^{n-1} \sin^{n-1} \theta \sqrt{g_\chi} = q^{n-1} \sqrt{g_\chi} \quad (3.6)$$

where $\sqrt{g_\chi} d\chi_1 \dots d\chi_{n-2}$ is the volume element on the $n-2$ sphere (2.19), and q was defined in (2.20). The field strength is

$$F_{q\phi\chi_1\dots\chi_{n-2}}^{(n)} = (n-1) q^{n-2} \sqrt{g_\chi} \quad (3.7)$$

In a unit orthonormal frame we get

$$F_{\hat{q}\hat{\phi}\hat{\chi}_1\dots\hat{\chi}_{n-2}}^{(n)} = \frac{(n-1)}{L} \quad (3.8)$$

Using (2.25) we see that (3.5) and (3.8) are dual forms.

3.1. The action

The configuration of the brane is described by giving the spacetime coordinates as a function of the worldsheet coordinates $\tau, \sigma_1, \dots, \sigma_{m-2}$. We choose the static gauge, where

$$t = \tau \quad (3.9)$$

$$\alpha_i = \sigma_i, \quad i = 1 \dots m-2 \quad (3.10)$$

The remaining coordinates are given by

$$r = r_0 + \epsilon \delta r(\tau, \sigma_1, \dots, \sigma_{m-2}) \quad (3.11)$$

¹ We choose signs of the gauge potentials that are different from those in [3].

$$\phi = \omega_0 \tau + \epsilon \delta\phi(\tau, \sigma_1, \dots, \sigma_{m-2}) \quad (3.12)$$

$$y_k = \epsilon \delta y_k(\tau, \sigma_1, \dots, \sigma_{m-2}), \quad k = 1 \dots n-1 \quad (3.13)$$

Recall that we are describing AdS_m by $t, r, \alpha_1, \dots, \alpha_{m-2}$ and S^n by $\phi, y_1, \dots, y_{n-1}$.

Let G_{MN} be the metric of $AdS_m \times S^n$, and let g_{ij} be the induced metric on the brane

$$g_{ij} = \frac{\partial X^M}{\partial \xi^i} \frac{\partial X^N}{\partial \xi^j} G_{MN} \quad (3.14)$$

where X^M are coordinates on $AdS_m \times S^n$ and ξ^i are coordinates on the worldsheet. The DBI action is

$$S_{DBI} = -T_{m-2} \int \sqrt{-g} d\tau d\sigma_1 \dots d\sigma_{m-2} \quad (3.15)$$

The Chern-Simmons term has two possible contributions, from the two nonvanishing components (3.3) and (3.6); we call them S_{CS1} and S_{CS2} respectively. We are interested only in the action to quadratic order in the fluctuations, and we will find that S_{CS2} does not contribute if $n > 3$, since its contribution becomes higher order than quadratic in the fluctuation. Thus the case $n = 3$ will have to be treated separately when computing the vibration frequencies. (We do not consider $n < 3$ or $m < 3$.)

The pullback of the gauge field is

$$P[A^{(m-1)}]_{\tau\sigma_1\dots\sigma_{m-2}} = A_{M_1\dots M_{m-1}}^{(m-1)} \frac{\partial X^{M_1}}{\partial \tau} \frac{\partial X^{M_2}}{\partial \sigma_1} \dots \frac{\partial X^{M_{m-1}}}{\partial \sigma_{m-2}} \quad (3.16)$$

Using (3.9) - (3.13) we get

$$\begin{aligned} S_{CS1} &= T_{m-2} \int A_{t\alpha_1\dots\alpha_{m-2}}^{(m-1)} d\tau d\sigma_1 \dots d\sigma_{m-2} \\ &= T_{m-2} \int \frac{1}{L} (r_0 + \epsilon \delta r(\tau, \sigma_1, \dots, \sigma_{m-2}))^{m-1} \sqrt{g_\sigma} d\tau d\sigma_1 \dots d\sigma_{m-2} \end{aligned} \quad (3.17)$$

where $\sqrt{g_\sigma} d\sigma_1 \dots d\sigma_{m-2}$ is the volume element on a constant τ hypersurface on the worldsheet.

For S_{CS2} we write down only the terms that contribute to lowest nonzero order in ϵ . For small values of q, y_k we see from (3.8) that we can write the gauge potential on S^n as

$$A_{\phi y_2 \dots y_{n-1}}^{(n-1)} \approx L^{n-1} (n-1) y_1 \quad (3.18)$$

Then we get

$$\begin{aligned}
S_{CS2} &\approx T_{m-2} L^{n-1} \int (n-1) y_1 \frac{\partial \phi}{\partial \tau} \frac{1}{(m-2)!} \epsilon_{i_1 \dots i_{m-2}} \frac{\partial y_2}{\partial \sigma_{i_1}} \dots \frac{\partial y_{m-1}}{\partial \sigma_{i_{m-2}}} d\tau d\sigma_1 \dots d\sigma_{m-2} \\
&\approx T_{m-2} L^{n-1} \omega_0 (n-1) \int y_1 \frac{1}{(m-2)!} \epsilon_{i_1 \dots i_{m-2}} \frac{\partial y_2}{\partial \sigma_{i_1}} \dots \frac{\partial y_{m-1}}{\partial \sigma_{i_{m-2}}} d\tau d\sigma_1 \dots d\sigma_{m-2}
\end{aligned} \tag{3.19}$$

For this to be nonvanishing, we need $n = m$. But further, the order of this term is $m - 1$ in the perturbation, so for $m > 3$ it is not relevant for the linearised perturbation analysis. It *would* be relevant for $AdS_3 \times S^3$, though this case is more involved since the 1-branes can be various combinations of strings and 5-branes wrapped on the compact 4-manifold (T^4 or K3). We will analyze these aspects of the $AdS_3 \times S^3$ case elsewhere, but for completeness work out here the frequencies that follow from an action of the form $S = -T_1 \int \sqrt{-g} + T_1 \int P[A^{(2)}]$.

3.2. Linearised equations for $m > 3$

First we look at the linear term in ϵ in the action (3.1). A straightforward calculation gives

$$\begin{aligned}
S_{DBI} \pm S_{CS1} &= -\epsilon T_{m-2} \int d\tau d\sigma_1 \dots d\sigma_{m-2} \sqrt{g_\sigma} \\
&\quad \frac{r_0^{m-3}}{\tilde{L}} \left[\left\{ \frac{(m-1)r_0^2 + (m-2)\tilde{L}^2(1-L^2\omega_0^2)}{\sqrt{r_0^2 + \tilde{L}^2(1-L^2\omega_0^2)}} \mp r_0(m-1) \right\} \delta r \right. \\
&\quad \left. - \frac{L^2 \tilde{L}^2 \omega_0 r_0}{\sqrt{r_0^2 + \tilde{L}^2(1-L^2\omega_0^2)}} \frac{\partial \delta \phi}{\partial \tau} \right]
\end{aligned} \tag{3.20}$$

As is clear from this formula, the action of the equilibrium configuration vanishes.

The coefficient of the term $\frac{\partial \delta \phi}{\partial \tau}$ is a constant, and so this term gives no contribution to the variation of the action with fixed boundary values. The coefficient of the term δr vanishes if we take

$$\omega_0 = \pm \frac{1}{L} \tag{3.21}$$

and the + sign on the LHS of (3.20). We choose the positive sign in (3.21) for concreteness; the frequencies we find are independent of this choice.

Looking at the linear order variation (3.20) we see that we also get the coefficient of δr to vanish if we choose

$$\omega_0^2 = \frac{1}{L^2} \left[1 + \frac{r_0^2}{\tilde{L}^2} \frac{(m-1)(m-3)}{(m-2)^2} \right] \tag{3.22}$$

These solutions should correspond to the maxima of the potential in [3][4], and thus describe unstable configurations. We will not consider perturbations around these configurations.

With the choices given in (3.21) (and immediately following that equation) we find that the zeroth order term in ϵ vanishes, while the second order term in ϵ is

$$\begin{aligned}
S = & \epsilon^2 T_{m-2} r_0^{m-3} \int d\tau d\sigma_1 \dots d\sigma_{m-2} \sqrt{g_\sigma} \\
& \left[\frac{\tilde{L}^3}{2(r_0^2 + \tilde{L}^2)} \left(\frac{\partial \delta r}{\partial \tau} \right)^2 - \frac{\tilde{L}}{2(r_0^2 + \tilde{L}^2)} \frac{\partial \delta r}{\partial \sigma_i} \frac{\partial \delta r}{\partial \sigma_i} g^{\sigma_i \sigma_j} \right. \\
& + \frac{L^2 \tilde{L} (r_0^2 + \tilde{L}^2)}{2r_0^2} \left(\frac{\partial \delta \phi}{\partial \tau} \right)^2 - \frac{L^2 (r_0^2 + \tilde{L}^2)}{2\tilde{L} r_0^2} \frac{\partial \delta \phi}{\partial \sigma_i} \frac{\partial \delta \phi}{\partial \sigma_i} g^{\sigma_i \sigma_j} \\
& + \frac{L\tilde{L}(m-3)}{r_0} \delta r \frac{\partial \delta \phi}{\partial \tau} \\
& \left. + \frac{L^2 \tilde{L}}{2} \frac{\partial \delta y_k}{\partial \tau} \frac{\partial \delta y_k}{\partial \tau} - \frac{L^2}{2\tilde{L}} \frac{\partial \delta y_k}{\partial \sigma_i} \frac{\partial \delta y_k}{\partial \sigma_i} g^{\sigma_i \sigma_j} - \frac{\tilde{L}}{2} y_k y_k \right]
\end{aligned} \tag{3.23}$$

Let Y_l be spherical harmonics on the unit $m-2$ sphere

$$g^{\sigma_i \sigma_j} \frac{\partial}{\partial \sigma_i} \frac{\partial}{\partial \sigma_j} Y_l(\sigma_1 \dots \sigma_{m-2}) = -Q_l Y_l(\sigma_1 \dots \sigma_{m-2}) \tag{3.24}$$

For example, on a 2-sphere we have $Q_l = l(l+1)$.

We expand the perturbations as

$$\begin{aligned}
\delta r(\tau, \sigma_1 \dots \sigma_{m-2}) &= \tilde{\delta} r e^{-i\omega\tau} Y_l(\sigma_1 \dots \sigma_{m-2}) \\
\delta \phi(\tau, \sigma_1 \dots \sigma_{m-2}) &= \tilde{\delta} \phi e^{-i\omega\tau} Y_l(\sigma_1 \dots \sigma_{m-2}) \\
\delta y_k(\tau, \sigma_1 \dots \sigma_{m-2}) &= \tilde{\delta} y_k e^{-i\omega\tau} Y_l(\sigma_1 \dots \sigma_{m-2})
\end{aligned} \tag{3.25}$$

We see that the δy_k perturbations decouple from $\delta r, \delta \phi$, and have frequencies given by

$$\omega^2 = \frac{Q_l}{\tilde{L}^2} + \frac{1}{L^2} \tag{3.26}$$

The $\delta r, \delta \phi$ perturbations are seen to be coupled. The resulting frequencies are given by the equation

$$\begin{pmatrix} \frac{\tilde{L}}{(r_0^2 + \tilde{L}^2)} (-\omega^2 \tilde{L}^2 + Q_l) & i\omega(m-3) \frac{L\tilde{L}}{r_0} \\ -i\omega(m-3) \frac{L\tilde{L}}{r_0} & \frac{L^2(r_0^2 + \tilde{L}^2)}{\tilde{L}r_0^2} (-\omega^2 \tilde{L}^2 + Q_l) \end{pmatrix} \begin{pmatrix} \tilde{\delta} r \\ \tilde{\delta} \phi \end{pmatrix} = 0 \tag{3.27}$$

which yields

$$\omega_\pm^2 = \frac{1}{\tilde{L}^2} \left[Q_l + \frac{(m-3)^2}{2} \pm (m-3) \sqrt{Q_l + \frac{(m-3)^2}{4}} \right] \tag{3.28}$$

3.3. The case $n = 3, m = 3$

In this case the $m - 2$ sphere is just a circle, which is parametrized by only one coordinate $\sigma_1 \equiv \sigma$. Thus we have

$$Y_l(\sigma) = e^{il\sigma}, \quad Q_l = l^2 \quad (3.29)$$

From (3.19) we get an additional contribution to the action

$$S_{CS2} = 2T_1 L^2 \omega_0 \int y_1 \frac{\partial y_2}{\partial \sigma} d\tau d\sigma \quad (3.30)$$

The $\delta r, \delta\phi$ perturbations are unaffected by S_{CS2} , and so we get the equation (3.27) with $m = 3$. Thus the δr and $\delta\phi$ perturbations decouple, and each has a frequency given by

$$\omega^2 = \frac{Q_l}{\tilde{L}^2} = \frac{l^2}{\tilde{L}^2} \quad (3.31)$$

There are two y coordinates, y_1 and y_2 . The frequencies of their fluctuations are given by

$$\begin{pmatrix} \frac{L^2}{\tilde{L}}(-\omega^2 \tilde{L}^2 + l^2) + \tilde{L} & -2iLl \\ 2iLl & \frac{L^2}{\tilde{L}}(-\omega^2 \tilde{L}^2 + l^2) + \tilde{L} \end{pmatrix} \begin{pmatrix} \tilde{\delta}y_1 \\ \tilde{\delta}y_2 \end{pmatrix} = 0 \quad (3.32)$$

which gives the frequencies

$$\omega_{\pm}^2 = \frac{1}{\tilde{L}^2} \left(l \pm \frac{\tilde{L}}{L} \right)^2 = \frac{1}{\tilde{L}^2} (l \pm 1)^2 \quad (3.33)$$

where in the last step we have used the fact that by (2.25) $L = \tilde{L}$ in this case.

4. Vibration spectrum - branes on the sphere

In this case the brane worldsheet has dimension $n - 1$, and we describe it by coordinates $\tau, \sigma_1, \dots, \sigma_{n-2}$. We choose the static gauge

$$t = \tau \quad (4.1)$$

$$\chi_i = \sigma_i, \quad i = 1 \dots n - 2 \quad (4.2)$$

The remaining coordinates are given by

$$q = q_0 + \epsilon \delta q(\tau, \sigma_1, \dots, \sigma_{n-2}) \quad (4.3)$$

$$\phi = \omega_0 \tau + \epsilon \delta \phi(\tau, \sigma_1 \dots \sigma_{n-2}) \quad (4.4)$$

$$v_k = \epsilon \delta v_k(\tau, \sigma_1 \dots \sigma_{n-2}), \quad k = 1 \dots m-1 \quad (4.5)$$

Recall that now AdS_m is described by the oordinates t, v_1, \dots, v_{m-1} and S^n is described by $\phi, q, \chi_1, \dots, \chi_{n-2}$.

The DBI action is

$$S_{DBI} = -T_{n-2} \int \sqrt{-g} d\tau d\sigma_1 \dots d\sigma_{n-2} \quad (4.6)$$

The gauge field again can give two kinds of terms. This time we will call S_{CS1} the term arising from the gauge field on the sphere S^n , and S_{CS2} the term from the gauge field on AdS_m . Then

$$\begin{aligned} S_{CS1} &= T_{n-2} \int A_{\phi\chi_1 \dots \chi_{n-2}} \frac{\partial \phi}{\partial \tau} \frac{\partial \chi_1}{\partial \sigma_1} \dots \frac{\partial \chi_{n-2}}{\partial \sigma_{n-2}} d\tau d\sigma_1 \dots d\sigma_{n-2} \\ &= T_{n-2} \int (\omega_0 + \epsilon \frac{\partial \delta \phi(\tau, \sigma_1 \dots \sigma_{n-2})}{\partial \tau}) A_{\phi\chi_1 \dots \chi_{n-2}} d\tau d\sigma_1 \dots d\sigma_{n-2} \\ &= T_{n-2} \int (\omega_0 + \epsilon \frac{\partial \delta \phi}{\partial \tau}) (q_0 + \delta q)^{n-1} \sqrt{g_\sigma} d\tau d\sigma_1 \dots d\sigma_{n-2} \end{aligned} \quad (4.7)$$

To compute S_{CS2} we write the form of the gauge potential on AdS_m for small v_k

$$A_{\tau v_2 \dots v_{m-1}}^{(m-1)} \approx \tilde{L}^{m-2} (m-1) v_1 \quad (4.8)$$

We write the lowest order term for S_{CS2}

$$S_{CS2} \approx T_{n-2} \tilde{L}^{m-2} (m-1) \int v_1 \frac{1}{(n-2)!} \epsilon_{i_1 \dots i_{n-2}} \frac{\partial v_2}{\partial \sigma_{i_1}} \dots \frac{\partial v_{n-1}}{\partial \sigma_{i_{n-2}}} d\tau d\sigma_1 \dots d\sigma_{n-2} \quad (4.9)$$

Again, this term is nonvanishing only for $n = m$, and is of order higher than quadratic in ϵ if $n > 3$.

4.1. Linearised equations for $n > 3$

Expanding the action to the linear order term in ϵ we get

$$\begin{aligned} S_{DBI} \pm S_{CS1} &= -\epsilon T_{n-2} \int d\tau d\sigma_1 \dots d\sigma_{n-2} \sqrt{g_\sigma} \\ & q_0^{n-3} \left[\left\{ \frac{(n-1)q_0^2 \omega_0^2 + (n-2)(1-L^2 \omega_0^2)}{\sqrt{1-L^2 \omega_0^2 + q_0^2 \omega_0^2}} \mp (n-1)q_0 \omega_0 \right\} \delta q \right. \\ & \left. + \left\{ \frac{(q_0^2 - L^2)q_0 \omega_0}{\sqrt{1-L^2 \omega_0^2 + q_0^2 \omega_0^2}} \pm q_0^2 \right\} \frac{\partial \delta \phi}{\partial \tau} \right] \end{aligned} \quad (4.10)$$

The coefficient of the term $\frac{\partial \delta \phi}{\partial \tau}$ is a constant as before, and so this term gives no contribution to the variation of the action with fixed boundary values. The coefficient of the term δq vanishes if we take

$$\omega_0 = \pm \frac{1}{L} \quad (4.11)$$

and the + sign on the LHS of (4.10). We again choose the positive sign in (4.11) for concreteness; the frequencies will be independent of this choice. With these choices the zeroth order term in ϵ vanishes, while the second order term in ϵ is

$$\begin{aligned} S = & \epsilon^2 T_{n-2} q_0^{n-3} \int d\tau d\sigma_1 \dots d\sigma_{n-2} \sqrt{g_\sigma} \\ & \left[\frac{L^3}{2(L^2 - q_0^2)} \left(\frac{\partial \delta q}{\partial \tau} \right)^2 - \frac{L}{2(L^2 - q_0^2)} \frac{\partial \delta q}{\partial \sigma_i} \frac{\partial \delta q}{\partial \sigma_i} g^{\sigma_i \sigma_j} \right. \\ & + \frac{L^3(L^2 - q_0^2)}{2q_0^2} \left(\frac{\partial \delta \phi}{\partial \tau} \right)^2 - \frac{L(L^2 - q_0^2)}{2q_0^2} \frac{\partial \delta \phi}{\partial \sigma_i} \frac{\partial \delta \phi}{\partial \sigma_i} g^{\sigma_i \sigma_j} \\ & + \frac{L^2(n-3)}{q_0} \delta q \frac{\partial \delta \phi}{\partial \tau} \\ & \left. + \frac{L\tilde{L}^2}{2} \frac{\partial \delta v_k}{\partial \tau} \frac{\partial \delta v_k}{\partial \tau} - \frac{\tilde{L}^2}{2L} \frac{\partial \delta v_k}{\partial \sigma_i} \frac{\partial \delta v_k}{\partial \sigma_i} g^{\sigma_i \sigma_j} - \frac{L}{2} v_k v_k \right] \end{aligned} \quad (4.12)$$

Let Y_l be spherical harmonics on the unit $n-2$ sphere

$$g^{\sigma_i \sigma_j} \frac{\partial}{\partial \sigma_i} \frac{\partial}{\partial \sigma_j} Y_l(\sigma_1 \dots \sigma_{n-2}) = -Q_l Y_l(\sigma_1 \dots \sigma_{n-2}) \quad (4.13)$$

We expand the perturbations as

$$\begin{aligned} \delta q(\tau, \sigma_1 \dots \sigma_{n-2}) &= \tilde{\delta} r e^{-i\omega\tau} Y_l(\sigma_1 \dots \sigma_{n-2}) \\ \delta \phi(\tau, \sigma_1 \dots \sigma_{n-2}) &= \tilde{\delta} \phi e^{-i\omega\tau} Y_l(\sigma_1 \dots \sigma_{n-2}) \\ \delta v_k(\tau, \sigma_1 \dots \sigma_{n-2}) &= \tilde{\delta} v_k e^{-i\omega\tau} Y_l(\sigma_1 \dots \sigma_{n-2}) \end{aligned} \quad (4.14)$$

The v_k perturbations decouple from $\delta q, \delta \phi$, and have frequencies given by

$$\omega^2 = \frac{Q_l}{L^2} + \frac{1}{\tilde{L}^2} \quad (4.15)$$

The $\delta q, \delta \phi$ perturbations are coupled. The resulting frequencies are given by the equation

$$\begin{pmatrix} \frac{L}{(L^2 - q_0^2)} (-\omega^2 L^2 + Q_l) & i\omega(n-3) \frac{L^2}{q_0} \\ -i\omega(n-3) \frac{L^2}{q_0} & \frac{L(L^2 - q_0^2)}{q_0^2} (-\omega^2 L^2 + Q_l) \end{pmatrix} \begin{pmatrix} \tilde{\delta} q \\ \tilde{\delta} \phi \end{pmatrix} = 0 \quad (4.16)$$

which yields

$$\omega_\pm^2 = \frac{1}{L^2} \left[Q_l + \frac{(n-3)^2}{2} \pm (n-3) \sqrt{Q_l + \frac{(n-3)^2}{4}} \right] \quad (4.17)$$

4.2. The case $n = 3, m = 3$

The spherical harmonics are again given by (3.29). We get an additional contribution to the action

$$S_{CS2} = 2T_1 \tilde{L} \int v_1 \frac{\partial v_2}{\partial \sigma} d\tau d\sigma \quad (4.18)$$

The $\delta r, \delta \phi$ perturbations are unaffected by S_{CS2} , and setting $n = 3$ we see that they decouple from each other. Each of these perturbations has a frequency given by

$$\omega^2 = \frac{Q_l}{L^2} = \frac{l^2}{L^2} \quad (4.19)$$

There are two v coordinates, v_1 and v_2 . The frequencies of their fluctuations are given by

$$\begin{pmatrix} \frac{\tilde{L}^2}{L}(-\omega^2 L^2 + l^2) + L & -2i\tilde{L}l \\ 2i\tilde{L}l & \frac{\tilde{L}^2}{L}(-\omega^2 L^2 + l^2)_+ \end{pmatrix} \begin{pmatrix} \tilde{\delta}v_1 \\ \tilde{\delta}v_2 \end{pmatrix} = 0 \quad (4.20)$$

which gives the frequencies

$$\omega_{\pm}^2 = \frac{1}{L^2} \left(l \pm \frac{L}{\tilde{L}} \right)^2 = \frac{1}{L^2} (l \pm 1)^2 \quad (4.21)$$

5. Comments on the excitation spectrum

In this section we discuss some aspects of the vibration modes. We do not discuss however the case $m = 3, n = 3$ since as mentioned above there is a richer set of issues in that case and we will discuss those details elsewhere.

5.1. Qualitative comments on the frequency spectrum

At first glance one might think that the larger the radius r_0 of the brane, the lower would be the frequency of its normal modes of vibration. But we have seen that these frequencies (measured in the coordinate τ) are in fact independent of r_0 . The reason for such behavior can be traced to the following. Consider first the case of branes in AdS . If the graviton were pointlike it would be placed at $r = 0$, where $|g^{\tau\tau}| = 1$, but because of its size the surface of the brane is near $r = r_0$ where $|g^{\tau\tau}| \approx (1 + \frac{r^2}{L^2})^{-1}$. Thus if we look at large r_0 we will have $|g^{\tau\tau}| \sim r_0^{-2}$, which is the same as the behavior $g^{\alpha\alpha} \sim r_0^{-2}$. Thus the frequencies, which see the ratio of tension to density of any extended object, become independent of r_0 in this limit. When we take into account the motion in the ϕ direction as well in the complete analysis, then this rough statement in fact becomes exactly true.

The induced metric for the equilibrium configuration also includes a contribution from the angular velocity which in fact precisely cancels the 1 and leaves us with a result which is simply $g^{\tau\tau} = -\frac{\tilde{L}^2}{r_0^2}$. Since the contribution from the angular momenta of the fluctuations to the action also scale as $\frac{1}{r_0^2}$, the curvature scale of the background determines all the frequencies of the giant graviton. For branes in S^n there is a similar effect. Now the contribution of the angular velocity to the induced $g_{\tau\tau}$ is $(1 - \frac{q_0^2}{L^2})$ and this combines with the usual contribution from the target space metric to lead to a final $g^{\tau\tau} = -\frac{L^2}{q_0^2}$. In fact it may be easily verified that by a suitable rescaling of the fluctuation fields all the r_0 or q_0 dependences can be completely scaled out of the small fluctuations action.

5.2. Modes arising from shift of BPS configuration

We do not find any unstable modes in the system at this quadratic order in the analysis, as all the ω^2 are real and nonnegative. In the formulae for ω^2 the Q_l are nonnegative numbers. Further, $Q_l = 0$ for $l = 0$ (which is the mode constant over the σ coordinates), and $Q_l > 0$ for $l > 0$.

We can see in the spectrum the consequence of the fact that we have several parameters that can be varied in the equilibrium configuration. Let us examine such modes in turn.

(a) From (3.28) and (4.17) we see that $\omega^2 = 0$ is one of the solutions when $Q_l=0$, in the $\delta r, \delta\phi$ system and in the $\delta q, \delta\phi$ system. This zero mode corresponds to the fact that the radius r_0 or q_0 of the equilibrium configuration can be taken to have any chosen value allowed by the geometry. Different values of r_0 and q_0 have different energies, but the same value of the action, viz. zero.

(b) Consider branes expanding in the AdS. We have taken the brane to move along the ϕ direction at $\theta = 0$ on the sphere S^n . But we could let the brane rotate along some other great circle on S^n . We can achieve this by a rotation

$$z'_1 = z_1 \cos \alpha - y_1 \sin \alpha, \quad y'_1 = z_1 \sin \alpha + y_1 \cos \alpha \quad (5.1)$$

with α a constant. For small α we find that starting with the configuration with $z_1 = \cos \theta \cos \phi = \cos \phi = \cos(\omega_0\tau)$ (see eq. (2.7)) and $y_1 = 0$ we get a configuration with

$$z'_1 \approx z_1, \quad y'_1 \approx \alpha \cos(\omega_0\tau) = \alpha \cos\left(\frac{1}{L}\tau\right) \quad (5.2)$$

This perturbation has $l = 0$ (since the deformation is independent of the σ_i) and agrees with the frequency $\omega^2 = 1/L^2$ obtained from (3.26) for the y_i vibrations with $l = 0$.

(c) By a similar analysis, if we study the branes on the sphere and look at v_i perturbations with $l = 0$ we find the frequency $\omega^2 = 1/\tilde{L}^2$, in agreement with (4.15). We interpret these modes as ‘boosts’ in the AdS space, and the frequency corresponds to the natural period in time t of this space.

Thus we have found all the expected families of solutions: the different values of r_0 or q_0 (corresponding to different P_ϕ), the different orientations of angular momenta and the possible motions in the AdS space of the center of mass of the brane.

(Note that we have not discussed here the details of the case $m = 3, n = 3$, where there are additional zero modes seen at $l = \pm 1$ in (3.33) and (4.21).)

5.3. Excitation spectrum in a Hamiltonian analysis

Now let us ask the question: What would be the excitation energies of the brane that would result from a quantization of the small vibrations around the configuration with $E = |P_\phi| = J$? When there are continuous family of equilibrium solutions as in this case, one should do the following. Fix the values of conserved quantities, and for any choice of these quantities write down the classical Hamiltonian. If fixing the the conserved quantities gives a unique lowest energy state, then the Hamiltonian for small perturbations will be a quadratic form in the coordinates and momenta describing the perturbation. Then one does the usual diagonalisation of quadratic forms and extracts the classical frequencies of oscillation ω . If we then consider the quantum problem, the energy of an excited configuration will be

$$E \approx E_0 + \hbar\omega = J + \hbar\omega \tag{5.3}$$

The value of $E_0 = J$ will be itself quantised too, since the angular momentum operator in quantum mechanics has a discrete spectrum.

Our analysis has been Lagrangian rather than Hamiltonian, and we did not fix the values of conserved quantities; we found *all* solutions near the equilibrium configuration. Of course if there is an oscillation with frequency ω found from the Hamiltonian with fixed values of the conserved quantities, then this oscillation and its frequency will be found among our Lagrangian solutions. But some of the solutions found in the Lagrangian method will not appear in the Hamiltonian analysis, since they will not hold fixed the values of the conserved quantities. After we locate these latter modes and remove them from the spectrum, we will be left with the modes that will correspond to the excitation levels (5.3).

We will now see that the modes to be removed are precisely those that we looked at in the last subsection. The conserved quantities to be held fixed are all the components of the angular momentum J_{ab} , and the conserved quantities corresponding to generators of the isometries of AdS_m (which we loosely call "momentum"). The mode of type (a) in the above subsection leads to a change in the magnitude of J (without changing its direction). The mode in (b) gives a nonzero value of the angular momentum in the $z_1 - y_1$ plane, (while the original angular momentum was in the $z_1 - z_2$ plane); thus it also does not hold the angular momentum fixed. Similarly the mode of type (c) leads to a change of 'momentum' in the AdS space. Thus these modes will not appear in the analysis of the Hamiltonian with fixed values of conserved quantities.

All the above modes had $l = 0$. Note that if $l \neq 0$, then there will be no change in the conserved quantities when we excite the mode. This is because these conserved quantities appear as an integral over a $\tau = \text{constant}$ hypersurface of the worldvolume, with the integrand being for example the angular momentum density. If $l \neq 0$ then we get no contribution to the conserved quantity at linear order in the perturbation.²

Thus the only modes that we could lose when working with fixed values of the conserved quantities are modes with $l = 0$. Apart from the modes (a)-(c) of the above subsection, there is only one such mode with $l = 0$ for the brane expanding in AdS space and one for the brane expanding on the sphere. Consider for concreteness the case of the brane expanding in AdS space. From (3.28) we see that this mode has frequency

$$\omega = \pm \frac{m - 3}{\tilde{L}} \quad (5.4)$$

But a short calculation reveals that under this mode the shift of the value of $P_\phi = J$ is in fact zero. In this mode the value of r changes with time, but the value of $\dot{\phi}$ changes as well, so that the net change in angular momentum ends up being zero. To verify this we first find the eigenvector for this eigenvalue from (3.27); we choose $\omega = (m - 3)/\tilde{L}$ from (5.4) for concreteness. Then we get

$$\frac{\tilde{\delta}r}{\tilde{\delta}\phi} = i \frac{L}{\tilde{L}} \frac{r_0^2 + \tilde{L}^2}{r_0} \quad (5.5)$$

² We may still get a contribution of order ϵ^2 to the value of a conserved quantity like J (under the perturbation of order ϵ), but we can undo this change by an order ϵ^2 change in the equilibrium value of r_0 , and so can regard the perturbation as one that gives no change in the conserved quantity.

But we have for the angular momentum density on the brane

$$p_\phi = T_{m-2} \frac{L^2 r^{m-2} \dot{\phi}}{\sqrt{1 + \frac{r^2}{L^2} - L^2 \dot{\phi}^2}} \quad (5.6)$$

where we have dropped terms like \dot{r}^2 and \dot{y}^2 from the denominator that will vanish to linear order around the equilibrium configuration.

Using (5.5) we find that under the perturbation,

$$\delta p_\phi = \frac{\partial p_\phi}{\partial r} \delta r + \frac{\partial p_\phi}{\partial \dot{\phi}} \delta \dot{\phi} = 0 \quad (5.7)$$

Thus the angular momentum does not change, under this perturbation, and the perturbation will survive in the Hamiltonian analysis that gives the vibration spectrum. Due to the symmetry between the cases of branes on the AdS space and branes on the sphere, we expect that a similar conclusion will hold for the $l = 0$ mode for branes expanding on the sphere; this mode has $\omega = \pm \frac{n-3}{L}$ from (4.17).

Thus we conclude that the vibration spectrum at fixed values of angular momenta and AdS momenta is given by all the modes found in the Lagrangian analysis with the exception of those discussed in (a)-(c) in the previous subsection.

6. Discussion

We have not found any families of BPS solutions besides the known ones, so the ansatz using spherical branes for BPS configurations appears to be an adequate one. The excitation modes all have real positive ω^2 , so we have not found any instabilities, for any size of the brane. Thus this analysis does not shed light on the issue of whether BPS branes can exist in the AdS space with arbitrarily large angular momentum. Note however that when branes expand in the AdS space then we use very little knowledge of the sphere in the spacetime - we just use the motion around the equator of the sphere. Thus if the compactification had a torus in the internal space (as in the case $AdS_3 \times S^3 \times T^4$) then we could let the brane move along a circle on the torus instead. Then the question of whether we should have an upper bound to the brane size would be related to whether we expect a limit on the total U(1) charge of the graviton state.

In [6] it was found that the giant graviton phenomenon occurs in spacetimes other than $AdS \times S$. In particular, consider p -branes wrapping the transverse S^{p+2} of the near

horizon geometry of $D(6-p)$ branes (both extremal and near-extremal) in string theory. In Poincare coordinates there is now an equilibrium solution where the brane has a fixed size, carries angular momentum on the S^{p+2} and moves in the radial direction transverse to the $D(6-p)$ brane. The energy of this system as a function of the radial and angular momenta is exactly the same as that of a graviton moving in this geometry. Note that generically these states are not BPS, though they are the lowest energy states for a given angular momentum. It would be interesting to examine the issue of small fluctuations around these configurations.

Let us comment on a significant implication of our result. The gravitons with high angular momentum correspond to chiral operators with high scaling dimensions in the dual CFT: $\Delta \sim E$, where E is the energy (and angular momentum) of the graviton (measured in units of the curvature length scale). The fact that the excitation spectrum of this graviton has spacings of order the curvature scale means that, if the giant graviton picture is right, these chiral operators have a set of associated nonchiral operators with dimensions increasing in steps of order unity. It would be interesting to look for such a spectrum explicitly in a strongly coupled CFT, for example in the $d = 4$ super-Yang-Mills theory.

In [18] it was found that in the orbifold CFT corresponding to a D1-D5 system, the correlation function of three chiral primaries drops significantly below the naive supergravity expectation when the spin of the chiral primaries becomes comparable to the square root of $N = Q1Q5$. This phenomenon is a manifestation of the stringy exclusion principle, which truncates the spectrum of chiral primaries at spin equal to N . Let us assume for the moment that in the supergravity limit of the theory, this decrease in coupling continues to be valid. One possible explanation for the drop in coupling could be that when the energy of two interacting supergravity quanta becomes very high, we produce particles other than supergravity modes, and thus the amplitude to produce just a third BPS graviton by colliding two gravitons becomes very small.³

If the giant graviton picture describes supergravity quanta at high energy, then we see that some of the energy of interaction may in fact go into exciting the vibration modes of the gravitons. Since the spacing between excitations of the graviton excitations is comparatively small, a large number of modes are available to be excited, and the effect of exciting these modes can be quite significant. (A typical graviton mode in AdS space has

³ We thank E. Martinec for a discussion on this point.

a frequency that is of the order of the AdS scale or higher, so interactions will generally be able to excite these vibration modes.)

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