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Time Dependent Cosmologies and Their Duals

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Abstract

We construct a family of solutions in IIB supergravity theory. These are time dependent or depend on a light-like coordinate and can be thought of as deformations of $AdS_5 \times S^5$. Several of the solutions have singularities. The light-like solutions preserve 8 supersymmetries. We argue that these solutions are dual to the $\mathcal{N} = 4$ gauge theory in a $3+1$ dimensional spacetime with a metric and a gauge coupling that is varying with time or the light-like direction respectively. This identification allows us to map the question of singularity resolution to the dual gauge theory.

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I. INTRODUCTION

Time dependent phenomena are poorly understood in string theory. It is important to understand them better. This could lead to an improved understanding of the big-bang and black-hole singularities and a better connection between string theory and observational cosmology. It could also reveal how time originates from a more fundamental description. Earlier work on time dependent phenomena includes the large body of results on stringy backgrounds unstable to tachyon condensation. Some previous attempts at studying time dependent phenomena pertaining to cosmological singularities in string theory include *e.g.* [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31].

In this paper, in part inspired by [16], we take a modest step in trying to understand some time dependent backgrounds in string theory. We find a family of time dependent and null backgrounds[1] in Type IIB string theory. These solutions are deformations of the $AdS_5 \times S^5$ background. The solutions have singularities which are space-like or null. In several cases the dilaton is weakly coupled at the singularity.

We argue that these backgrounds have a dual interpretation in terms of turning on sources in the $\mathcal{N} = 4$ gauge theory. The sources are time dependent or dependent on the light-like coordinate respectively. This dual interpretation allows us to map the question of the resolution of the singularity to the gauge theory. If the gauge theory is non-singular in some cases, it should provide an answer to how the singularity is resolved.

We have not been able to settle this important question in this paper and postpone a more detailed analysis of it for the future [32]. It is worth mentioning that the null

[1] We will loosely call backgrounds which depend on a light-like coordinate (instead of a time-like coordinate) as null backgrounds in this paper.

backgrounds are especially interesting in this context. These solutions preserve 8 of the 16 supersymmetries left unbroken by a D3 brane. In some cases the supergravity background corresponds to turning on sources in the gauge theory which become asymptotically constant, as $X^+ \to \pm\infty$. The supergravity solution also corresponds in the gauge theory to starting, as $X^+ \to -\infty$, in a state which is the $\mathcal{N} = 4$ vacuum. In these cases it is possible that a careful analysis shows that the gauge theory is non-singular and in a well defined state in the far future, as $X^+ \to \infty$, as well.

The solutions we find are also of interest from the point of view of determining the response of the $\mathcal{N} = 4$ gauge theory to time dependent sources. For this purpose even bulk singular solutions which cannot be resolved might be interesting. Such singular solutions are dual to turning on sources in the gauge theory which became singular at some moment of time. Prior to this moment it is still valid to ask about the response of the gauge theory to the source and this information is contained in the supergravity solution.

Using the ideas of this paper similar solutions can also be obtained in other AdS spaces. Particularly interesting is the $AdS_3 \times S^3$ case. Here it might be possible to analyse some backgrounds, which have a singularity with a weakly coupled dilaton, using the world sheet conformal field theory.

While this paper was being written, [33] appeared. It contains substantial overlap with the results presented here.

II. SUPERGRAVITY SOLUTIONS WITH COSMOLOGICAL SINGULARITIES

We will consider Type IIB supergravity and work in 10-dimensional Einstein frame. We are interested in solutions in which the metric, five form, F_5, and dilaton, ϕ, are excited. Our main result is that any background with metric and 5-form

$$\begin{aligned} ds^2 &= Z^{-1/2}(x)\tilde{g}_{\mu\nu}dx^\mu dx^\nu + Z^{1/2}(x)\tilde{g}_{mn}dx^m dx^n, \\ F_{(5)} &= -\frac{1}{4\cdot 4!}\tilde{\epsilon}_{\mu\nu\rho\sigma}\frac{\partial_m Z(x)}{Z(x)^2}dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma \wedge dx^m \\ &\qquad + \frac{1}{4\cdot 5!}\tilde{\epsilon}_{m_1 m_2 m_3 m_4 m_5}{}^{m_6}\partial_{m_6} Z(x) dx^{m_1} \wedge dx^{m_2} \wedge dx^{m_3} \wedge dx^{m_4} \wedge dx^{m_5}, \end{aligned} \tag{2.1}$$

is a solution of the equations of motion, as long as $Z(x)$ is a *harmonic* function on the flat, six dimensional tranverse space with coordinates x^m; $\tilde{g}_{mn}$ is Ricci-flat and depends only on the x^m; and $\tilde{g}_{\mu\nu}$ and the dilaton ϕ are only dependent on the 4-coordinates x^μ, and satisfy the conditions,

$$\tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi, \tag{2.2a}$$

$$\partial_\mu(\sqrt{-\det(\tilde{g})}\ \tilde{g}^{\mu\nu}\partial_\nu\phi) = 0. \tag{2.2b}$$

Upon taking the near horizon limit with a flat transverse space, this reduces to

$$ds^2 = (\frac{r^2}{R^2})\tilde{g}_{\mu\nu}dx^\mu dx^\nu + (\frac{R^2}{r^2})dr^2 + R^2 d\Omega_5^2\ , \qquad F_{(5)} = R^4(\omega_5 + *_{10}\omega_5)\ , \tag{2.3}$$

which is the case we will typically consider. Here it is important to emphasize that $\tilde{R}_{\mu\nu}$ is the Ricci curvature made from the metric $\tilde{g}_{\mu\nu}$ alone (without the $\frac{r^2}{R^2}$ warp factor in front). In Eq. (2.3) $d\Omega_5^2$ is the volume element and ω_5 is the volume form of the unit five-sphere. In other words, as long as $\tilde{g}_{\mu\nu}, \phi$, together solve the equations of 4-dimensional Einstein gravity coupled to a free scalar field in (2.2), we have a solution to the original 10-dimensional problem.

Let us see how this result is obtained. It is easy to see that self-duality for the 5-form means that $F^2 \equiv F_{ABCDE}F^{ABCDE} = 0$, where $A, B \ldots$ take values in ten dimensions. The Einstein equations then take the form

$$R_{AB} = \frac{1}{6}F_{AA_1A_2A_3A_4}F_B{}^{A_1A_2A_3A_4} + \frac{1}{2}\partial_A\phi\partial_B\phi \ . \tag{2.4}$$

Now for the background (2.3) it is clear that this equation with components along the S^5 directions is satisfied, since the dilaton does not depend on the angular coordinates of the S^5. The problem then reduces to studying this equation with components along the $t, x^1, \cdots x^3, r$ directions.

The 5-dimensional metric along these directions can be written as follows:

$$ds^2 = \frac{R^2}{z^2}(\tilde{g}_{\mu\nu}dx^\mu dx^\nu + dz^2) \tag{2.5}$$

where $z = R^2/r$. Now using the standard rules relating the Ricci curvature tensor for two metrics related by a conformal transformation (see *e.g.* [34], Appendix D) we then get that

$$R_{\mu\nu} = \tilde{R}_{\mu\nu} - \frac{4}{R^2}g_{\mu\nu} \ , \qquad R_{zz} = -\frac{4}{R^2}g_{zz} \ . \tag{2.6}$$

It is easy to see that the term $-\frac{4}{R^2}g_{\mu\nu}$ in the first equation is canceled by the 5-form contribution (which in effects provides a negative cosmological constant in 5-dimensions). Thus we see that as long as $\tilde{R}_{\mu\nu}$ meets the condition in (2.2), the Einstein equations with components along μ, ν directions are satisfied. The Einstein equation component along the zz directions is met because the R_{zz} contribution is again balanced by the 5-form flux. Finally, the dilaton equation in (2.2) then follows by noting that it satisfies the massless free-field equation in 10 dimensions and is independent of z.

Let us end this subsection by noting that one can obtain $AdS_5 \times S^5$ as a special case of the solutions Eqs. (2.3) and (2.2) by taking ϕ to be constant and $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$. There are also linearised small fluctuations about this solution which are included in Eqs. (2.3) and (2.2). Perturbations of the form, $\phi = \phi_0 + \delta\phi(x^\mu)$, $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$, where ϕ_0 is constant and $\delta\phi(x^\mu)$ satisfies Eq. (2.2), is of this type. Similarly included are metric perturbations of the form, $\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, with constant dilaton and vanishing linearised Ricci tensor $\tilde{R}_{\mu\nu}$.

A. Time-dependent cosmological singularities

We now turn to exploring various solutions. In this subsection we construct time-dependent solutions.

As an example consider the spacetime,

$$\tilde{g}_{\mu\nu}dx^{\mu}dx^{\nu} = -dt^2 + \sum_{i=1}^{3} t^{(2p_i)}dx^i dx^i. \tag{2.7}$$

It has the non-zero Ricci components $\tilde{R}_{tt} = (\sum_i p_i - \sum_i p_i^2)/t^2$, $\tilde{R}_{ii} = p_i(\sum_j p_j - 1)t^{2p_i}/t^2$. The conditions, (2.2) are solved if

$$\phi = \alpha \log t \ , \qquad \sum_i p_i = 1 \ , \qquad \frac{\alpha^2}{2} = 1 - \sum_i p_i^2 \ . \tag{2.8}$$

One simple set of examples are the Kasner geometries where the dilaton is constant and $\sum_i p_i = \sum_i p_i^2 = 1$. Another more symmetric example is $p_i = \frac{1}{3}$ and $\alpha = \frac{2}{\sqrt{3}}$. Note that in this case the metric $\tilde{g}_{\mu\nu}$ can be made conformally flat after a redefinition of the time coordinate. Starting with a metric of the form,

$$d\tilde{s}^2 = e^{f(T)}(-2dT^2 + e^{H_1(T)}dx_1^2 + e^{H_2(T)}dx_2^2 + e^{H_3(T)}dx_3^2) \ , \tag{2.9}$$

one can show that the metric, Eq. (2.7), meeting conditions Eq. (2.8), is the most general solution for the conditions, Eq. (2.2).

Note that the solutions, Eqs. (2.7) and (2.8), have a curvature singularity at $t = 0$.[2] The string coupling is given by $g_s = e^{\phi}$. Depending on the sign chosen for the dilaton in Eq. (2.8) the string coupling blows up or goes to zero at the singularity. It is easy to verify that the curvature singularity is reached in finite proper time for a time-like geodesic.

The class of time-dependent solutions allowed by Eq. (2.2) is in fact much larger. With constant dilaton, $\tilde{g}_{\mu\nu}$ can be the metric of a gravitational wave, or that of the Schwarzschild black hole. In the latter case the region inside the horizon gives a time-dependent geometry with a big-bang or big-crunch. Other solutions include $\tilde{g}_{\mu\nu}$ being a homogeneous FRW metric sourced by an appropriate time-dependent dilaton. These again have big-bang and big-crunch singularities.

With Euclidean signature again several solutions are possible. For example, with constant dilaton, any Ricci flat metric, $\tilde{g}_{\mu\nu}$, is allowed.

We started with an S^5 in Eq. (2.3), but analogous solutions can be obtained by replacing it with an constant curvature compact five manifold. An example is the base of the conifold, $T^{1,1}$. Additional solutions can also be obtained by using duality. For example using the S-duality of the IIB theory, one can obtain solutions where the axion is also turned on.[3]

The time-dependent solutions in this section of course do not preserve any supersymmetries.

[2] In fact in this case both the Einstein frame and string frame metrics have a singularity where the Ricci scalar blows up at $t = 0$.

[3] We mention here the papers [7, 25, 35] that find families of supergravity solutions in M-theory with dependences on either timelike or lightlike time coordinates: it would be interesting to explore the relations between these and our solutions here in the Type IIB context, and generalize them.

B. Null cosmological singularities

Next we turn to null solutions. These depend on one light-like coordinate which we call X^+.

The spacetime takes the form[4]

$$\begin{aligned} d\tilde{s}^2 &= e^{f(X^+)}(-2dX^+dX^- + e^{H(X^+)}(dX^+)^2 + e^{h_2(X^+)}dx_2^2 + e^{h_3(X^+)}dx_3^2) , \\ \phi &= \phi(X^+). \end{aligned} \tag{2.10}$$

The only non-vanishing component of the Ricci tensor is $\tilde{R}_{++}$. We should also note that $\tilde{R}_{++}$ is independent of $H(X^+)$. The background, Eq. (2.10), is a solution if it meets the condition,

$$\frac{1}{2}(\partial_+\phi)^2 = \tilde{R}_{++} = \frac{1}{2}(f')^2 - f'' - \frac{1}{4}[(h_1')^2 + (h_2')^2] - \frac{1}{2}(h_1'' + h_2'') , \tag{2.11}$$

where $h_1' = \frac{dh_1}{dX^+}$ etc. This is a large three function-parameter family of solutions. By a coordinate transformation we can set $f(X^+) = 1$. For any choice of $H(X^+), h_2(X^+), h_3(X^+)$ we can then get a solution by choosing a dilaton which satisfies Eq. (2.11).

For simplicity, let us focus on spacetimes conformal to flat space: thus we set $g_{++} = e^H = 0$ and $h_2 = h_3 = 0$ and the background becomes

$$d\tilde{s}^2 = e^{f(X^+)}(-2dX^+dX^- + dx_2^2 + dx_3^2) , \qquad \phi = \phi(X^+) . \tag{2.12}$$

This is a solution if

$$\frac{1}{2}(f')^2 - f'' = \frac{1}{2}(\partial_+\phi)^2 . \tag{2.13}$$

The 10D Einstein frame background (2.3) has the curvatures

$$R = 0 , \qquad R_{++} = \frac{1}{2}(f')^2 - f'' , \qquad -R_{+-} = R_{22} = R_{33} = \frac{4e^{f(X^+)}r^2}{R^4} , \tag{2.14}$$

besides the ones with components on S^5. Note that if $e^f \to 0$ the metric components, Eq. (2.12), shrink to zero. The conditions for the existence of a singularity in such a situation are made more precise below.

Null geodesics in the spacetime (2.12) at constant X^-, x^2, x^3, *i.e.* trajectories moving only along X^+, satisfy the condition,

$$\frac{d^2X^+}{d\lambda^2} + \Gamma^+_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = \frac{d^2X^+}{d\lambda^2} + f'\left(\frac{dX^+}{d\lambda}\right)^2 = 0 , \tag{2.15}$$

[4] There is some redundancy in this choice of metric components, but we keep it because it will be useful in the subsequent discussion.

since the only nonzero $\Gamma^+_{\alpha\beta}$ is $\Gamma^+_{++} = f'$. Here λ is the affine parameter for the geodesic. This can be solved to give λ in terms of X^+,

$$\lambda = \text{const.} \int e^{f(X^+)} dX^+ \ . \tag{2.16}$$

The curvature component,

$$R_{\lambda\lambda} = R_{++} \left(\frac{dX^+}{d\lambda} \right)^2 = \left(\frac{1}{2}(f')^2 - f'' \right) e^{-2f} \ . \tag{2.17}$$

For a suitably chosen f this can blow up when $e^f \to 0$. As we will see in the examples below this can occur at a finite value of λ.

Our first example is obtained by taking $f(X^+) = -QX^+$. Then the dilaton takes the form, $\phi = \pm QX^+$. We find from Eq. (2.17) that $R_{\lambda\lambda} = \frac{1}{2}Q^2 e^{2QX^+}$. This blows up at $X^+ \to \infty$, showing that there is a big crunch curvature singularity. The singularity occurs at finite affine parameter, $\lambda = e^{-QX^+} \to 0$, as $X^+ \to \infty$. This spacetime, with a curvature singularity at finite affine parameter, is thus geodesically incomplete. Choosing the dilaton to be $\phi = -QX^+$ we find that the string coupling constant vanishes at the singularity.[5]

Our next example is obtained by taking $e^{f(X^+)} = \tanh^2 X^+$. This gives,

$$d\tilde{s}^2 = \tanh^2 X^+(-2dX^+dX^- + dx_2^2 + dx_3^2) \ , \qquad e^\phi = g_s \left(\tanh \frac{X^+}{2} \right)^{\sqrt{8}} \ , \tag{2.18}$$

The example has been engineered so that the spacetime becomes flat in the far past and future (with constant dilaton) but exhibits interesting behaviour in the intermediate region. We have $R_{++} = \frac{4}{\sinh^2 X^+}$, so that $R_{\lambda\lambda} = \frac{4}{\sinh^2 X^+ \tanh^4 X^+}$ showing a curvature singularity at $X^+ \to 0$, with e^ϕ becoming arbitrarily small there.[6] It is easy to see from Eq. (2.16) that the singularity occurs at finite value of the affine parameter.

We end with two comments. First, it is worth pointing out that the only solutions to (2.11) with a constant constant dilaton is flat space. With $\phi = $ const, (2.11) leads to $e^f = \frac{1}{(X^+)^2}$. After a coordinate transformation this can be put in the form of the standard flat space metric.

Secondly, introduction of a nontrivial $\tilde{g}_{\mu\nu}(x^\nu)$ in (2.3) typically introduces curvature singularities at the Poincare horizon at $r = 0$. However it turns out for the null solutions considered above there is no such singularity. This is yet another reason why we focus on the null solutions.

1. *Supersymmetry of the null solutions*

In this subsection we explore the supersymmetry of the null solutions. For simplicity, we restrict ourselves to the solutions, Eqs. (2.3) and (2.12).

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[5] $R_{\lambda\lambda}$ in the string frame also blow up at the singularity in this case.

[6] The string frame curvature, $R_{\lambda\lambda}$ blows up at the singularity in this case as well.

We are considering a Type IIB background with Einstein metric g_{MN}, dilaton Φ and 5-form F_{MNPQR}, using the notation of [36] and the earlier [37]. Since $\tau = ie^{-\phi}$, the quantity $B = \frac{1+i\tau}{1-i\tau}$ is real and the quantity $Q_M = (1-BB^*)^{-1}\mathrm{Im}(B\partial_M B^*) = 0$. Then the supersymmetry variations are

$$\delta\lambda = \frac{i}{\kappa}\gamma^M(1-B^2)^{-1}\partial_M B\varepsilon^* = 0, \tag{2.19a}$$

$$\delta\psi_M = \frac{1}{\kappa}D_M\varepsilon + \frac{i}{480}\gamma^{M_1\ldots M_5}F_{M_1\ldots M_5}\gamma_M\varepsilon = 0, \tag{2.19b}$$

for the complex Weyl dilatino λ ($\gamma^{11}\lambda = \lambda$) and the complex Weyl gravitino ψ_M ($\gamma^{11}\psi = -\psi$). Also the supersymmetry parameter ε is a complex Weyl spinor with $\gamma^{11}\varepsilon = -\varepsilon$. The covariant derivative is $D_M = \partial_M + \frac{1}{4}\omega_M^{ab}\Gamma_{ab}$, with $\Gamma_{ab} = \frac{1}{2}[\Gamma_a, \Gamma_b]$, using flat space Γ-matrices and curved space γ-matrices which satisfy $\gamma_M = e_M^a\Gamma_a$. The spin connection ω^{ab} satisfies $de^a + \omega^a{}_b \wedge e^b = 0$, the e^a being an orthonormal frame.

The null background, Eqs. (2.3) and (2.12), take the form,

$$ds_E^2 = Z^{-1/2}e^f(-2dX^+dX^- + dx^i dx^i) + Z^{1/2}dx^m dx^m, \tag{2.20a}$$

$$F_{0123m} = \frac{1}{4\kappa}\frac{1}{Ze^{-2f}}\partial_m \log Z, \qquad F_{m_1m_2m_3m_4m_5} = \frac{1}{4}\epsilon_{m_1m_2m_3m_4m_5}{}^{m_6}\partial_{m_6}Z, \qquad \phi = \phi(X^+), \tag{2.20b}$$

where $Z = Z(x^m)$ is a harmonic function in the flat transverse space with coordinates x^m, $f = f(X^+)$, and the $\epsilon_{(6)}$ is the flat one. In our notation the indices $i = 1, 2$, refer to two directions parallel to the D3 brane, and the indices $m = 1, \cdots, 6$ refer to the six directions transverse to the D3 brane. We choose the obvious, diagonal frame $e^+ = Z^{-1/4}e^{f/2}dx^+$, etc.

The spin connection in this background, with the above choice of frame, is,

$$\begin{aligned} \omega_{-+} &= -\frac{1}{2}f'dX^+, \quad \omega_{-m} = \frac{1}{4}Z^{-1/2}e^{f/2}\partial_m \log Z dX^+, \quad \omega_{mn} = \frac{1}{4}(\partial_n \log Z dx^m - \partial_m \log Z dx^n), \\ \omega_{i+} &= \frac{1}{2}f'dx^i, \qquad \omega_{+m} = \frac{1}{4}Z^{-1/2}e^{f/2}\partial_m \log Z dX^-, \quad \omega_{im} = -\frac{1}{4}Z^{-1/2}e^{f/2}\partial_m \log Z dx^i. \end{aligned} \tag{2.21}$$

It is easy to see that for the background (2.3), (2.12), the dilatino variation gives the condition,

$$\gamma^+\epsilon = 0. \tag{2.22}$$

The gravitino variations take the form,

$$\kappa\delta\psi_i = \partial_i\epsilon - \frac{1}{8}\gamma_i\gamma_\omega(1-\Gamma^4)\epsilon - \frac{1}{4}f'\gamma^i\gamma_-\epsilon \tag{2.23a}$$

$$\kappa\delta\psi_m = \partial_m\epsilon + \frac{1}{8}\epsilon\,\omega_m - \frac{1}{8}\gamma_\omega\gamma_m(1-\Gamma^4)\epsilon \tag{2.23b}$$

$$\kappa\delta\psi_- = \partial_-\epsilon - \frac{1}{8}\gamma_-\gamma_\omega(1-\Gamma^4)\epsilon \tag{2.23c}$$

$$\kappa\delta\psi_+ = \partial_+\epsilon - \frac{1}{8}\gamma_+\gamma_\omega(1-\Gamma^4)\epsilon - \frac{1}{4}f'\epsilon + \frac{1}{4}f'\gamma^-\gamma_-\epsilon, \tag{2.23d}$$

where $\omega_m = \partial_m \ln Z, \gamma_\omega = \gamma^m \omega_m$, and $\Gamma^4 = i\Gamma^{0123}$.

It is easy to see that all the conditions are satisfied if and only if

$$\Gamma^4 \epsilon = \epsilon, \qquad \gamma^+ \epsilon = 0, \qquad \epsilon = Z^{-1/8} e^{f/4} \eta, \tag{2.24}$$

for η a constant spinor. (Equivalently, $\gamma^+\eta = 0$ and $i\gamma^{23}\eta = \eta$.) The first condition in Eq. (2.24) is the standard one for supersymmetry in the presence of a D3 brane and gives rise to 16 supersymmetries. We see that the second condition in Eq. (2.24) is the same as Eq. (2.22) and breaks the supersymmetry further by half giving rise to a total of 8 supersymmetries. Thus all backgrounds of the form, Eq. (2.12) preserve 8 supersymmetries.

III. SOME COMMENTS ON THE DUAL GAUGE THEORY

Here we take a few preliminary steps in constructing and analysing the gauge theory duals to the supergravity backgrounds discussed above.

We would like to claim that the backgrounds discussed in section II, Eqs. (2.3) and (2.2), are dual to an $\mathcal{N} = 4$ gauge theory with a gauge coupling $g_{YM}^2 = e^\phi$, in a four dimensional spacetime with metric, $\tilde{g}_{\mu\nu}$. We now give some supporting evidence for this claim.

The $AdS_5 \times S^5$ background is a special case of Eq. (2.3) with $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$, and a constant dilaton. Small fluctuations around this background give rise to supergravity modes. These can be mapped to operators using the AdS/CFT dictionary. The family of solutions, Eqs. (2.3) and Eq. (2.2), include some of these supergravity modes as well. For example, as was discussed in section 2, solutions where the dilaton varies satisfying Eq. (2.2), with the metric $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$, is a solution of the linearised equations obtained from Eqs. (2.3) and (2.2). This mode is dual to the operator $\mathrm{Tr}F^2$ in the Yang -Mills theory. Similarly the mode where $\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x^\mu)$ with constant ϕ, where $h_{\mu\nu}$ satisfied the linearised Ricci flatness condition, $\tilde{R}_{\mu\nu} = 0$, is dual to the stress energy tensor in the gauge theory, $T_{\mu\nu}$. A general linearised solution, $\delta\phi(x^\mu), h_{\mu\nu}(x^\mu)$, of the type, Eq. (2.3), Eq. (2.2), then is dual to turning on sources in the gauge theory which couple to $\mathrm{Tr}F^2, T_{\mu\nu}$,

$$S_{source} = \int d^4x [\delta\phi(x^\mu) \mathrm{Tr}F^2 + h_{\mu\nu} T^{\mu\nu}], \tag{3.1}$$

which is in agreement with the claim above.

Most solutions, especially the interesting ones we have found above, are of course not small fluctuations. Since the identification of the dilaton and $\tilde{g}_{\mu\nu}$ with the gauge coupling and metric of the gauge theory works for the small perturbations, it seems reasonable to assert that this is true for these solutions as well.

One additional piece of evidence here is to consider a single D3 brane moving in the background of such a solution. It is easy to see from the DBI action of this brane that it has a gauge coupling $e^{\phi/2}$, and metric, $\tilde{g}_{\mu\nu}$. Note that it is only for D3 branes that the excitations perceive the Einstein metric $\tilde{g}_{\mu\nu}$.

Now actually for small fluctuations, each supergravity mode has two solutions which fall off differently as $r \to \infty$. These correspond to the non-normalisable and normalisable modes [38, 39] and determine the source coupling to the dual operator and the expectation value of the operator in the dual theory respectively. It is easy to see that the linearised perturbations which lie in the class of solutions, Eqs. (2.3) and (2.2), correspond to only exciting the non-normalisable modes, as may be seen from the positive power of r in these perturbations. Thus, in their case the dual gauge theory continues to be in the $\mathcal{N} = 4$ vacuum with the sources mentioned above turned on. What happens for solutions which are not small fluctuations is less clear. Solutions which approach $AdS_5 \times S^5$ for early times, when $t \to -\infty$ or $X^+ \to -\infty$ must be dual to the gauge theory starting in the $\mathcal{N} = 4$ vacuum at early times. The subsequent state of the gauge theory would then be determined by the sources which are turned on.

Once we have accepted the identification proposed above we can analyse the gauge theory description in some more detail. The most interesting question is whether the gauge theory dual to a singular spacetime is itself singular or not. In the time dependent solutions we analysed, of Kasner type, Eq. (2.8), at the cosmological singularity in the bulk the four dimensional metric $\tilde{g}_{\mu\nu}$ is also singular. This suggests that the gauge theory living in such a singular spacetime would itself be singular and ill-behaved. One case worth examining separately is when $\tilde{g}_{\mu\nu}$ is conformally flat. An example is the solution, Eq. (2.8), with $p_i = \frac{1}{3}, \alpha = \frac{2}{\sqrt{3}}$. Since the gauge theory is conformally invariant it might seem at first glance that it is oblivious of the shrinking conformal factor. This is of course not true. The conformal anomaly in 4 dimensions [40, 41, 42, 43, 44] (see also *e.g.* [45, 46, 47, 48] in the holographic context of $AdS_5 \times S^5$) tells us that

$$T_\mu{}^\mu = \frac{c}{16\pi^2}(C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}) - \frac{a}{16\pi^2}(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2) \;\propto\; -R_{\alpha\beta}R^{\alpha\beta} + \frac{1}{3}R^2 \;, \quad (3.2)$$

where in the last expression we have used $a = c = \frac{N^2-1}{4}$ for the $SU(N)$ $\mathcal{N}$=4 super Yang Mills theory. Note that in the first expression, the first term involves the Weyl tensor and vanishes in a conformally flat space-time. Focussing on the $\mathcal{N}$=4 theory at hand, the terms in the last expression give $T_\mu{}^\mu \propto 1/t^4$ in the example above. Thus we see that the stress energy tensor blows up at the singularity in this example signalling that the gauge theory is probably ill behaved.

The null solutions are more promising in this respect. These solutions preserve 8 supercharges, as we discussed above. In the conformally flat cases, Eq. (2.12), since R_{++} is the only non-vanishing component of the stress tensor, the conformal anomaly vanishes.

Consider in particular the solution discussed in Eq. (2.18). In this case the solution approaches $AdS_5 \times S^5$ as $X^+ \to \pm\infty$. Thus the sources in the gauge theory are turned off at $X^+ = \pm\infty$ where it becomes the $\mathcal{N} = 4$ Yang Mills theory. We also learn, as was mentioned above, that the gauge theory is in the $\mathcal{N} = 4$ vacuum as $X^+ \to -\infty$. We expect that the deformed gauge theory inherits the supersymmetries of the bulk since it is basically the theory on the branes which themselves give rise to the background and one might hope that this may be useful in drawing conclusions about the nature of the state at finite X^+. At $X^+ \to -\infty$, the number of supersymmetries are enhanced to the maximal number and the gauge theory dual should be in the state annihilated by all the supercharges. At finite time one may naively think that the state continues to be annihilated by eight of the supercharges. However, all these supersymmetry parameters obey $\gamma^+\epsilon = 0$. The corresponding supercharges do not commute with the hamiltonian and therefore a state which preserves these supersymmetries at some time alone do not generically preseve this at later times. In fact the anticommutator of the corresponding supercharges is proportional to P_- rather than the hamiltonian.

We have not been able to conclusively establish yet whether the gauge theory continues to be non-singular as one approaches $X^+ = 0$ and leave this for further study [32]. If true, we should be able to answer whether the solution of the form, Eq. (2.18), is also valid in the far future, as $X^+ \to \infty$, or what is its appropriate continuation in that region.

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