UK/06-02
hep-th/0602099

Matrix Membrane Big Bangs and D-brane Production

Sumit R. Das* and Jeremy Michelson†

Department of Physics and Astronomy
University of Kentucky,
Lexington, KY 40506 U.S.A.

Abstract

We construct Matrix Membrane theory in *pp* wave backgrounds that have a null linear dilaton in Type IIB string theory. Such backgrounds can serve as toy models of big bang cosmologies. At late times only abelian degrees of freedom survive, and if the Kaluza-Klein modes along one of the directions of the membrane decouple, standard perturbative strings emerge. Near the "big bang", non-abelian configurations of fuzzy *ellipsoids* are present, as in the Type IIA theories. A generic configuration of these shrink to zero volume at late times. However, the Kaluza Klein modes (which can be thought of as states of (p, q) strings in the original IIB theory) can be generically produced in pairs in both *pp* wave and flat backgrounds in the presence of time dependence. Indeed, if we require that at late times the theory evolves to the perturbative string vacuum, these modes must be prepared in a squeezed state with a thermal distribution at early times.

*Electronic address: das@pa,uky,edu
†Electronic address: jeremy@pa,uky,edu

Contents

I. INTRODUCTION

It has been always difficult to address questions involving strongly time dependent backgrounds in string theory, particularly those which involve an apparent "beginning" or "end" of time. Recently there have been several attempts to investigate such backgrounds which have holographic duals in the form of open string theories. These include nontrivial solutions corresponding to closed string tachyon condensation of two dimensional non-critical string theory [1, 2]; certain simple backgrounds of critical string theory which admit a Matrix Theory or a Matrix String Theory formulation [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]; as well as models with tachyon condensation in critical string theory [15, 16, 17, 18, 19, 20, 21, 22, 23, 24]. The first two sets of examples have a common feature: the underlying quantum mechanics is always defined in the open string formulation which has a "time" and this open string time runs over the full range of values, while the "time" which *emerges* in the closed string interpretation appears to have a "beginning" or an "end".

In the Matrix Theory models, the Yang-Mills theory of D-branes provide a non-perturbative formulation of string theory in a time dependent dilaton background with some fixed quantized momentum N along a null direction. Consider for example the case where the string theory is weakly coupled in the future and strongly coupled in the past. Then, in the holographic Matrix string theory description (obtained using [25, 26, 27, 28][1]

[1] It was shown in [29] that one can obtain a supersymmetric quantum mechanics via dimensional reduction of $\mathcal{N} = 1, D = 10$ SYM. We thank M. Halpern for bringing this to our attention.

and the arguments of [30, 31]) the reverse happens—i.e. the $SU(N)$ Yang-Mills coupling is strong in the future and weak in the past. This means that at late times, the fields of the Yang-Mills theory are constrained to lie in the Cartan subalgebra and become the coordinates of the emerging "space"—the Yang-Mills action reduces to the worldsheet action of multiple strings (in the light cone gauge) in this "space". At early times, all the non-abelian degrees of freedom are important and there is no interpretation in terms of strings any more. Equivalently, the closed string which emerges from the model may be thought to live in the future quadrant of the Milne universe[3][2]. Thus, while the time of the open string theory, i.e. Matrix theory, runs over a full range, the time as perceived in the closed string theory may appear to begin at a finite point.

In [7] we found solutions of Type IIA string theory in a *pp* wave background which have a null linear dilaton, and we constructed Matrix String Theory in this background starting with the BMN Matrix theory of [48], compatifying [49], and following [50, 51, 52, 53, 54, 55, 56, 57, 58, 59] . The presence of a second length scale (the background flux of the *pp* wave) allowed us to analyze the model in a regime of parameters where a specific class of non-abelian configurations—fuzzy spheres [60]—are relevant. We looked at the dynamics of these fuzzy spheres and found that with *generic initial conditions*, these oscillate in size, but the maximum size vanishes exponentially fast as the background evolves from the big bang. This leaves only perturbative closed strings at late times. For large N these fuzzy spheres become spherical D2 branes. This model therefore provides a concrete example in which D-branes proliferate a typical state of the theory at early times and tame what would appear as an initial singularity from the point of view of perturbative closed strings.

In this note, we consider a *pp* wave background in IIB string theory with a null linear dilaton, with two compact directions, one of which is null. In the absence of a dilaton background this theory has a holographic description in terms of the large R-charge sector of a 3+1 dimensional N=4 Yang-Mills theory [48], or more precisely as a version of a quiver gauge theory constructed along the lines of [61]. However, now we have a second dual description as well. Following the standard procedure in flat space [26, 27, 62] a sector of the theory with some given momentum along the null direction should be dual to a *Matrix Membrane Theory*—a 2+1 dimensional Yang-Mills theory on a torus [63]. This theory was explicitly constructed in [64, 65]. The 2+1 dimensional Yang-Mills theory lives on a torus, the ratio of the two sides being the IIB string coupling. In the absence of a background dilaton, and at weak string coupling only abelian configurations survive. Furthermore, one of the sides becomes very small and the Kaluza-Klein modes along this direction decouple. The resulting theory then becomes the lightcone action for a fundamental IIB string in the *pp* wave background.

Here we construct the Matrix Membrane theory in the presence of a linear null dilaton.

[2] Perturbative strings in Milne universes have been investigated in [32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47]

At late times this reduces to the worldsheet action of the appropriate IIB string. However at early times, nonabelian configurations are important, in particular *fuzzy ellipsoids*. In a way similar to [7] these ellipsoids generically shrink to zero at late times. However now we have a new phenomenon. The Kaluza-Klein modes, which are states of (p, q) strings in the original IIB theory, now become important at early times. We find that if we require that the state at late times is the vacuum of perturbative string theory, the initial state must be a squeezed state of these (p, q) strings with no net winding number. This model therefore throws light on the question of initial conditions.[3]

II. IIB *PP* WAVES WITH NULL DILATON

The string frame metric, RR field strengths and the dilaton Φ are given by

$$\begin{aligned} ds^2 &= 2dx^+dx^- - 4\mu^2[(x^1)^2 + \cdots (x^6)^2](dx^+)^2 - 8\mu x^7 dx^8 dx^+ + [(dx^1)^2 + \cdots (dx^8)^2], \\ F_{+1234} &= F_{+5678} = \mu\ e^{Qx^+}, \qquad \Phi = -Qx^+. \end{aligned} \tag{2.1}$$

It is easy to check that this solves the low energy equations of motion. These solve the equations of motion for $\mu = \mu(x^+)$, and not just constant μ; however, for most of the paper we will consider just $\mu =$ constant, although we suspect that $\mu = \mu_0 e^{Qx^+}$ would be a very interesting case, as in [7]. The coordinates x^- and x^8 are compact

$$x^- \sim x^- + 2\pi R, \qquad x^8 \sim x^8 + 2\pi R_B. \tag{2.2}$$

We will denote the string coupling of this Type IIB theory by g_B and the string length by l_B.

T-dualizing along x^8 yields a IIA background with a string coupling g_A and an x^8 radius given by R_A,

$$g_A = g_B \frac{l_B}{R_B}, \qquad R_A = \frac{l_B^2}{R_B}. \tag{2.3}$$

This may be further lifted to M-theory by introducing another compact direction x^9. This M-theory background is given by

$$\begin{aligned} ds^2 &= e^{\frac{2Qx^+}{3}}\{2dx^+dx^- - 4\mu^2[(x^1)^2 + \cdots (x^6)^2 + 4(x^7)^2](dx^+)^2 + [(dx^1)^2 + \cdots (dx^8)^2]\} \\ &\quad + e^{-\frac{4Qx^+}{3}}(dx^9)^2, \\ F_{+789} &= -4\mu, \qquad F_{+567} = 8\mu e^{Qx^+}. \end{aligned} \tag{2.4}$$

Again, we could have taken $\mu = \mu(x^+)$ in this background, as for the dual background (2.1). The various factors of e^{Qx^+} may be understood as follows. Typically a NS-NS gauge field will not acquire any such factor, whereas an RR gauge field will [7]. This is why F_{+789} does

[3] The fact that models of this type can be used to address issues of initial conditions was suggested to us by S. Trivedi.

not have such a factor but F_{+567} does. In terms of the IIB quantities, the radius of the x^9 direction is R_9 and the Planck length of the M-theory is l_p, where

$$R_9 = g_B \frac{l_B^2}{R_B}, \qquad l_p^3 = g_B \frac{l_B^4}{R_B}. \tag{2.5}$$

III. MATRIX MEMBRANE THEORY

Consider a sector of the IIB theory with momentum $p_- = J/R$ along the x^- direction. If we treat x^- as the Kaluza-Klein direction of the M-theory background (2.4) we have a IIA theory living on *two* compact directions x^8 and x^9, with radii R_A, R_9 respectively, and a net D0 brane charge equal to J. T-dualizing along x^8 and x^9 then leads to a IIA theory with D2 brane charge J living on a torus with sides

$$\tilde{R}_8 = g_B \frac{l_B^2}{R}, \qquad \tilde{R}_9 = \frac{l_B^2}{R}. \tag{3.1}$$

Matrix membrane theory is the $SU(J)$ supersymmetric $2+1$ dimensional Yang-Mills theory of these J D2 branes. The dimensionful coupling constant of the YM theory is

$$G_{\rm YM}^2 = \frac{R}{R_9 R_A} = \frac{R R_B^2}{g_B l_B^4}. \tag{3.2}$$

In flat space and a constant dilaton, this Matrix Membrane theory was constructed in [26, 27, 62]. The $2+1$ Yang-Mills lives on a torus: the ratio of the two radii is equal to the string coupling of the original theory [66]. Therefore, for small string coupling, the Kaluza-Klein modes for the smaller circle decouple, and at the same time the potential restricts the fields to lie in a Cartan subalgebra; the resulting two dimensional theory becomes the usual light cone superstring after a suitable dualization[62] of the gauge field strength. For time-independent *pp* waves, several physical aspects expected from such a theory were considered in [63], and the theory was explicitly constructed in [64, 65].

The construction of Matrix membrane theory for the background given in (2.4) follows the procedure of [64, 65].[4] The bosonic terms of the action for J D0 branes may be written down following [57, 58, 60, 69, 70].

The light cone lagrangian of these J D0 branes is given by

$$L = \mathrm{Tr}\{\frac{1}{2Rl_p} G^{+-} G_{IJ} D_\tau X^I D_\tau X^J - \frac{G^{--} l_p}{2G^{+-}R} + \frac{R}{4G^{+-} l_p^5} G_{IK} G_{JL} [X^I, X^J][X^K, X^L] - \frac{i}{2l_p^2} A_{+IJ} X^I X^J\}, \tag{3.3}$$

[4] Bonelli [67] has found this type of supersymmetric action by dimensional reduction from a deformed ten dimensional gauge theory. However, Kim *et. al.* [68] have shown that, for example, the M-theory *pp* wave quantum mechanics follows from dimensional reduction of $\mathcal{N} = 4, d = 4$ SYM on an S^3.

where the indices $I, J, K = 1 \cdots 9$ and τ is a dimensionless light cone time. Using a gauge in which the M theory gauge potentials are given by

$$A_{+89} = 4\mu x^7, \qquad A_{+56} = -8\mu e^{Qx^+} x^7, \tag{3.4}$$

the explicit form of the bosonic part of the Matrix theory lagrangian is

$$\begin{aligned} L = \mathrm{Tr}\ \{ & \frac{1}{2Rl_p}(D_\tau X^i)^2 + \frac{1}{2Rl_p} e^{-2Q\tau}(D_\tau X^9)^2 - \frac{2l_p\mu^2}{R}[(X^1)^2 + \cdots (X^6)^2 + 4(X^7)^2] \\ & + \frac{R}{2l_p^5}[X^9, X^i]^2 + \frac{R}{4l_p^5} e^{2Q\tau}[X^j, X^i]^2 \\ & + \frac{4\mu i}{l_p^2} X^7\ [X^8, X^9] - \frac{8\mu i}{l_p^2} e^{Q\tau} X^7\ [X^5, X^6]\}, \end{aligned} \tag{3.5}$$

where the indices $i, j = 1 \cdots 8$.

To obtain the Matrix membrane theory we need to substitute

$$X^8 \to -iR_A D_\rho, \qquad X^9 \to -iR_9 D_\sigma, \qquad \mathrm{Tr} \to \mathrm{Tr} \int_0^{2\pi} d\sigma \int_0^{2\pi} d\rho, \tag{3.6}$$

where $D_\alpha, \alpha = \tau, \sigma, \rho$ are covariant derivatives with a $SU(J)$ gauge connection A_α. We will also rescale the coordinates τ, σ, ρ, the fields $X^a, a = 1 \cdots 7$ and A_α as follows

$$\begin{aligned} & \tau \to l_p \tau, \qquad \sigma \to \frac{l_p^3}{RR_9}\sigma, \quad \rho \to \frac{l_p^3}{RR_A}\rho, \\ & X^a \to \frac{\sqrt{RR_AR_9}}{l_p^3} X^a, \quad A_\tau \to \frac{1}{l_p} A_\tau, \quad A_\sigma \to \frac{RR_9}{l_p^3} A_\sigma, \quad A_\rho \to \frac{RR_A}{l_p^3} A_\rho. \end{aligned} \tag{3.7}$$

The $2+1$ Yang-Mills theory action for the Matrix Membrane then becomes

$$S = \int d\tau \int_0^{2\pi \frac{l_p^3}{RR_9}} d\sigma \int_0^{2\pi \frac{l_p^3}{RR_A}} d\rho\ \mathcal{L}, \tag{3.8}$$

where

$$\begin{aligned} \mathcal{L} = \mathrm{Tr}\ \{ & \frac{1}{2}[(D_\tau X^a)^2 - (D_\sigma X^a)^2 - e^{2Q\tau}(D_\rho X^a)^2] + \frac{1}{2(G_{\mathrm{YM}} e^{Q\tau})^2}[F_{\sigma\tau}^2 + e^{2Q\tau}(F_{\rho\tau}^2 - F_{\rho\sigma}^2)] \\ & - 2\mu^2[(X^1)^2 + \cdots (X^6)^2 + 4(X^7)^2] + \frac{(G_{\mathrm{YM}} e^{Q\tau})^2}{4}[X^a, X^b]^2 \\ & - \frac{4\mu}{(G_{\mathrm{YM}} e^{Q\tau})} e^{Q\tau} X^7\ F_{\rho\sigma} - 8\mu i (G_{\mathrm{YM}} e^{Q\tau}) X^7\ [X^5, X^6]\ \}, \end{aligned} \tag{3.9}$$

with

$$G_{\mathrm{YM}} = \sqrt{\frac{R}{R_A R_9}}. \tag{3.10}$$

In the above $a, b = 1 \cdots 7$. Using the relations (2.5) the radii of the σ and ρ directions in (3.8) become $\tilde{R}_9$ and $\tilde{R}_8$ of (3.1) respectively.

The action (3.9) has two important features:

1. Each factor of $G_{\rm YM}$ is accompanied by a factor of $e^{Q\tau}$.

2. Each factor of ∂_ρ or A_ρ is accompanied by a factor of $e^{Q\tau}$.

It is straightforward to see that one could rescale the fields and the coordinates to write the action S in the form

$$S = \frac{\mu}{G_{\rm YM}^2} S(\mu = 1, G_{\rm YM} = 1). \tag{3.11}$$

In terms of these new coordinates, the range of σ and ρ become

$$0 \leq \sigma \leq \frac{\mu l_B^2}{R}, \qquad\qquad 0 \leq \rho \leq \frac{\mu l_B^2 g_B}{R}. \tag{3.12}$$

The dimensionless coupling which controls the physics is then

$$\lambda = \frac{\mu}{G_{\rm YM}^2} = \frac{\mu g_B l_B^4}{R R_B^2}. \tag{3.13}$$

In the rest of the paper, however, we will stick to the choice of coordinates and fields in which the action is given by (3.9).

IV. MATRIX MEMBRANE THEORY ON CURVED SPACE

For the IIA big bang [3, 7], the corresponding Matrix String Theory could be written as an ordinary gauge theory—albeit with time-dependent masses [7]—on Milne space, rather than Minkowski space. We will see a similar phenomenon occur here.

The first step is to perform a change of variables, $\rho = \rho' e^{Q\tau}$. Although this inserts τ-dependence into the range of ρ', this is natural in that the size of the ρ direction should be related to the string coupling—more precisely, the ratio of the radii of ρ and σ circles is g_s, *cf.* Eq. (3.12)—which is indeed τ-dependent. The resulting action is

$$S = \int d\tau \int_0^{2\pi \frac{l_p^3}{R R_9}} d\sigma \int_0^{2\pi \frac{l_p^3}{R R_A} e^{-Q\tau}} d\rho' \; \mathcal{L}', \tag{4.1}$$

where

$$\mathcal{L}' = {\rm Tr} \left\{ -\frac{1}{2} e^{Q\tau} (D_\mu X^a)^2 - \frac{1}{4 G_{\rm YM}^2} e^{-Q\tau} F_{\mu\nu}^2 - 2\mu^2 e^{Q\tau} [(X^1)^2 + \cdots (X^6)^2 + 4 (X^7)^2] \right.$$
$$\left. + \frac{G_{\rm YM}^2}{4} e^{3Q\tau} \left[X^a, X^b\right]^2 - \frac{4\mu}{G_{\rm YM}} X^7 \; F_{\rho'\sigma} - 8 i \mu G_{\rm YM} e^{2Q\tau} X^7 \; [X^5, X^6] \right\}. \tag{4.2}$$

Upon—as in [7]—defining $\mu(\tau) = \mu e^{-Q\tau}$, this action—except for the funny, non-Lorentz-invariant $X^7 F_{\rho\sigma}$ term, the consistency of which will be explained in section V (see also [65])—can be reinterpreted as a standard Yang-Mills Lagrangian on a space with metric

$$ds^2 = e^{2Q\tau} \left[-d\tau^2 + d\sigma^2 + d\rho'^2 \right]. \tag{4.3}$$

That is,

$$S = \mathrm{Tr} \int d\tau \int_0^{2\pi \frac{l_p^3}{RR_9}} d\sigma \int_0^{2\pi \frac{l_p^3}{RR_A} e^{-Q\tau}} d\rho' \sqrt{-g} \left\{ -\frac{1}{2} g^{\mu\nu} D_\mu X^a D_\nu X^a \right.$$
$$- \frac{1}{4G_{\rm YM}^2} F_{\mu\nu} F^{\mu\nu} - 2\mu^2(\tau)[(X^1)^2 + \cdots (X^6)^2 + 4(X^7)^2]$$
$$\left. + \frac{G_{\rm YM}^2}{4} \left[X^a, X^b\right]^2 - \frac{4\mu(\tau)}{G_{\rm YM}} e^{-2Q\tau} X^7 \; F_{\rho'\sigma} - 8i\mu(\tau) G_{\rm YM} X^7 \; \left[X^5, X^6\right] \right\}, \quad (4.4)$$

with the metric (4.3).

Interestingly, the metric (4.3) has a curvature singularity at $\tau = -\infty$, which geodesics can reach in finite affine parameter. In fact, this singularity is a worldvolume, spacelike big bang singularity. This differs from the IIA big bang, in which this metric is that of the flat two-dimensional Milne space. That said, although the ρ'-direction appears to be shrinking as $\tau \to -\infty$, its coordinate distance is growing as $\tau \to \infty$. and thus the total physical size of the ρ'-direction is actually constant in time. (This is most easily seen by undoing the coordinate transformation to ρ', to obtain $ds^2 = e^{2Q\tau}(-d\tau^2 + d\sigma^2) + (d\rho - Q\rho d\tau)^2$.) Nevertheless, this global picture does not change the local picture that gives the big bang singularity at $\tau = -\infty$.

Since the worldvolume theory is nongravitational, the worldvolume big bang cannot be resolved by quantum gravity (a.k.a. stringy) effects. Thus, unlike the IIA big bang [3], one cannot attempt to extrapolate through the $\tau = -\infty$ singularity to a pre-big bang scenario.

V. BEHAVIOR FOR $Q = 0$

Let us first review some aspects of the physics for usual time independent pp waves following [65]. When the original IIB theory is weakly coupled, $g_B \ll 1$ with $\frac{\mu l_B^4}{RR_B^2} \sim O(1)$, the effective coupling constant of this YM theory becomes strong. The potential terms then restrict the fields $X^a, F_{\mu\nu}$ to lie in the Cartan subalgebra and can be therefore chosen to be diagonal. This also has the effect that the covariant derivatives become ordinary derivatives. We now perform a duality transformation followed by a redefinition of fields:

1. Introduce an auxillary field ϕ and add a term to the action $\frac{1}{2}\epsilon^{\mu\nu\lambda}\partial_\mu\phi F_{\nu\lambda}$.

2. Integrate out the gauge fields to obtain a lagrangian density of the form

$$\mathcal{L}' = -\frac{1}{2}[\sum_{a=1}^{7}(\partial_\mu X^a)^2 + G_{\rm YM}^2(\partial_\mu\phi)^2] - 2\mu^2[\sum_{i=1}^{6}(X^i)^2 + 4(X^7)^2] + 4G_{\rm YM}\mu \; X^7\partial_\tau\phi. \quad (5.1)$$

3. Make a redefinition of the fields $(X^a, \phi) \to Y^I$

$$\begin{aligned} X^i &= Y^i, \qquad i = 1, \cdots, 6, \\ X^7 &= Y^7 \; \cos(2\mu\tau) + Y^8 \; \sin(2\mu\tau), \\ G_{\rm YM}\phi &= -Y^7 \; \sin(2\mu\tau) + Y^8 \; \cos(2\mu\tau). \end{aligned} \quad (5.2)$$

This leads to the lagrangian density

$$\mathcal{L}_{diag} = -\frac{1}{2}\sum_{I=1}^{8}(\partial_\mu Y^I)^2 - 2\mu^2\sum_{I=1}^{8}(Y^I)^2. \tag{5.3}$$

In equations (5.1) and (5.3), the *index* μ runs over all three of the membrane worldvolume directions. However, in the $g_B \ll 1$ regime, the size of the ρ direction shrinks to zero which implies that the $2+1$ dimensional theory reduces to a $1+1$ dimensional theory. The lagrangian density (5.3) is then precisely the (bosonic part of the) light cone gauge fixed Green-Schwarz lagrangian density in the *pp* wave background. Allowed boundary conditions lie in the conjugacy classes of the group which split the action into pieces characterized by strips in σ space whose lengths add up to the total extent $J\tilde{R}_9$. This leads to the worldsheet description of multiple light cone strings. The fermionic terms also agree, as shown in [65].

In the limit $\lambda \gg 1$ the Yang-Mills theory becomes weakly coupled and classical solutions play a significant role. These classical solutions include the *fuzzy ellipsoids* discussed in [64, 65].

Specifically, the potential of (3.5) vanishes for the static (taking μ to be constant) configuration

$$X^5 = 2\sqrt{2}\frac{\mu l_p^3}{R}J^1, \qquad X^6 = 2\sqrt{2}\frac{\mu l_p^3}{R}J^2, \qquad X^7 = 2\frac{\mu l_p^3}{R}J^3, \tag{5.4}$$

where J^a obey the SU(2) algebra, and the remaining matrices X^i vanish. (Actually, X^8 and X^9 need only commute with $X^{5,6,7}$, corresponding to arbitrary positions of the fuzzy ellipsoids.) By virtue of the SU(2) Casimir, on irreducible representations this configuration obeys,

$$(X^5)^2 + (X^6)^2 + 2(X^7)^2 = \text{constant}, \tag{5.5}$$

and so is ellipsoidal. These vacua can be shown [64, 65] to preserve all 24 supercharges of the M-theory background.

VI. SHRINKING ELLIPSOIDS

In the presence of a $Q > 0$ the theory is again weakly coupled near the big bang singularity at $\tau = -\infty$. Therefore, in this regime there are fuzzy ellipsoids in addition to standard strings. The time evolution of these fuzzy ellipsoids generalizes that of the fuzzy spheres in IIA *pp* waves discussed in [7]; for generic initial conditions, the size of these extended non-abelian configurations oscillate at early times and then the ellipsoids degenerate as one evolves to $\tau = \infty$. However, it turns out that the ellipsoids do not shrink to zero size, but only to zero volume. In particular, the solution depicted in Fig. 1 exhibits exponential decay of what had been the major axes (along X^5, X^6) of the ellipsoid, but the diagonal matrix X^7 remains nonzero. Thus, there are (matrix) strings at late time, but they are ordinary and not giant gravitons.

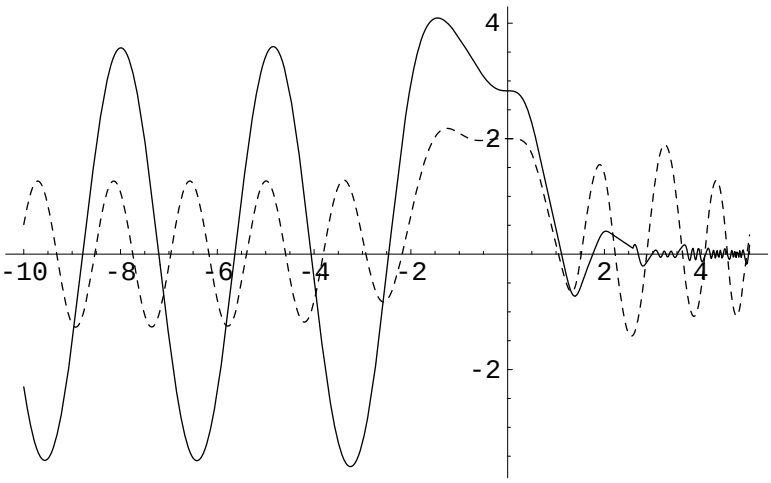

FIG. 1: The time evolution of the shape of the fuzzy ellipsoid, for $A = 1$ and a momentarily stationary fuzzy ellipsoid of the (for $Q = 0$) supersymmetric size. The solid line is R_1, the size in the $5, 6$-directions, and the dashed line is R_2, the size in the 7-direction. Thus, at late times, the fuzzy ellipsoid degenerates.

Explicitly, consider the ansatz, with all other fields zero,

$$X^5 = \frac{\mu}{G_{\rm YM}} R_1(\tau) J^1, \qquad X^6 = \frac{\mu}{G_{\rm YM}} R_1(\tau) J^2, \qquad X^7 = \frac{\mu}{G_{\rm YM}} R_2(\tau) J^3, \tag{6.1}$$

and set

$$A \equiv Q\sqrt{\frac{G_{\rm YM}}{\mu^3}}, \qquad\qquad \tau \equiv \sqrt{\frac{G_{\rm YM}}{\mu^3}}\, t. \tag{6.2}$$

For $Q = 0$, the equations of motion yield $R_1 = 2\sqrt{2}$ and $R_2 = 2$. In general, the equations of motion imply the coupled differential equations

$$0 = \ddot{R}_1 + 4R_1 + e^{2At} R_1 (R_1^2 + R_2^2) - 8 e^{At} R_1 R_2, \tag{6.3a}$$

$$0 = \ddot{R}_2 + 16 R_2 + 2 e^{2At} R_1^2 R_2 - 8 e^{At} R_1^2, \tag{6.3b}$$

where the dots denote t-derivatives. Note that setting $R_1 = 0$ satisfies the first equation, whereupon the second equation reduces to a harmonic oscillator for R_2, which does not require that R_2 vanish.

Presumably a similar situation arises for IIA. This was not seen in [7] simply because the SO(3) symmetry made it natural to start with a fuzzy sphere, whereupon the symmetry guaranteed that the evolution preserved the spherical shape, but with exponential decay of the radius. The results here suggest that it would be interesting to investigate nonspherical initial IIA configurations, which might decay into (Matrix) strings.

VII. PARTICLE PRODUCTION AND INITIAL STATES

At late times, the $2+1$ dimensional theory is strongly coupled. Therefore, as in the time-independent case discussed above, the fields can be chosen to be diagonal. Exactly the same dualization and field redefinitions as discussed above now lead to a $2+1$ dimensional lagrangian density

$$\mathcal{L}_{diag} = \frac{1}{2}[\sum_{I=1}^{8}(\partial_\tau Y^I)^2 - (\partial_\sigma Y^I)^2 - e^{2Q\tau}(\partial_\rho Y^I)^2] - 2\mu^2\sum_{I=1}^{8}(Y^I)^2. \tag{7.1}$$

As expected, there is a factor of $e^{Q\tau}$ for each factor of ∂_ρ. It is tempting to argue that as $\tau \to \infty$ the Kaluza-Klein modes in the ρ direction become infinitely massive so that the theory becomes $1+1$ dimensional and exactly identical to the Green-Schwarz string action in this background. However, this is too hasty since we have a time-dependent background here and energetic arguments do not apply.

Since the size of the ρ direction is given by $\tilde{R}_8$ given in (3.1), the mass scale associated with these Kaluza-Klein modes is $M_{KK} \sim \frac{R}{g_B l_B^2}$ while the mass scale associated with the coupling is G_{YM}^2 which is given in (3.2). Therefore when $R_B \gg l_B$ the KK modes are much lighter than the Yang-Mills scale. In our present time-dependent context, these scales become time-dependent and it follows from the coupling and the ∂_ρ terms in (3.9) that the KK modes are expected to decouple much *later* than the time when the non-abelian excitations decouple. Therefore, there is a regime where we can ignore the non-abelian excitations, but cannot ignore the KK modes. In this regime, the Matrix Membrane lagrangian density is given by (7.1). In the following we will assume that the time interval for which (7.1) is valid is long enough to be well approximated by the entire interval $(-\infty,\infty)$.

To determine the fate of these KK modes we need to find the modes of the field Y^I. The mode functions which are positive frequency at early times are

$$\varphi_{m,n}^{(\text{in})} = \{\frac{R}{8\pi^2 l_B^4 g_B}\}^{1/2}\ \Gamma(1-i\omega_m/Q)\ e^{i(\frac{mR}{l_B^2}\sigma + \frac{nR}{g_B l_B^2}\rho)}\ J_{-i\frac{\omega_m}{Q}}(\kappa_n e^{Q\tau}), \tag{7.2}$$

where

$$\omega_m^2 = 4\mu^2 + \frac{m^2R^2}{l_B^4}, \qquad \kappa_n = \frac{nR}{Qg_B l_B^2}, \tag{7.3}$$

while those which are appropriate at late times are

$$\varphi_{m,n}^{(\text{out})} = \{\frac{R}{16\pi l_B^4 g_B Q}\}^{1/2}\ e^{i(\frac{mR}{l_B^2}\sigma + \frac{nR}{g_B l_B^2}\rho)}\ H^{(2)}_{-i\frac{\omega_m}{Q}}(\kappa_n e^{Q\tau}). \tag{7.4}$$

The problem is of course equivalent to that of a bunch of two dimensional scalar fields with time-dependent masses and it is well known that such time-dependent masses lead to particle production or depletion [71, 72, 73]. Because of standard relations between the Hankel function $H_\nu^{(2)}(z)$ and the Bessel function $J_\nu(z)$ there is a non-trivial Bogoliubov

transformation between these modes which imply that the vacua defined by the in and out modes are not equivalent. Indeed one has

$$\varphi^{(\rm out)}_{m,n} = \alpha_m \varphi^{(\rm in)}_{m,n} + \beta_m (\varphi^{(\rm in)}_{-m,-n})^\star. \tag{7.5}$$

where

$$\alpha_m = \{\frac{Q}{2\pi\omega_m}\}^{1/2}\ \Gamma(1+i\omega_m/Q)\ e^{\frac{\pi\omega_m}{2Q}}, \qquad \beta_m = -\{\frac{Q}{2\pi\omega_m}\}^{1/2}\Gamma(1-i\omega_m/Q)\ e^{-\frac{\pi\omega_m}{2Q}}. \tag{7.6}$$

This means that the out vacuum $|0\rangle_{\rm out}$ is a squeezed state of the "in" particles

$$|0\rangle_{\rm out} = \prod_{n,m}\ \{(1-|\gamma_m|^2)^{1/4}\ \exp[\frac{1}{2}\gamma^\star_m a^{\dagger I,(\rm in)}_{m,n}\ a^{\dagger I,(\rm in)}_{-m,-n}]\}\ |0\rangle_{\rm in}, \tag{7.7}$$

where $a^{I,(\rm in)}_{m,n}$ is the annihilation operator of the KK mode labeled by n with m units of momentum in the σ direction. Here

$$\gamma_m = \frac{\beta^\star_m}{\alpha_m} = -ie^{-\frac{\pi\omega_m}{Q}}. \tag{7.8}$$

There is of course a similar relationship which expresses the in vacuum as a squeezed state of the out particles.

In the present context, the relation (7.7) means that *if we require that the final state at late times does not contain any of the KK modes, the initial state must be a squeezed state of these modes*. The occupation number of the in modes in this state is thermal

$${}_{\rm out}\langle 0|a^{\dagger I,(\rm in)}_{m,n} a^{I,(\rm in)}_{m,n}|0\rangle_{\rm out} = \frac{1}{e^{\frac{2\pi\omega_m}{Q}}-1}. \tag{7.9}$$

Note that the Bogoliubov coefficients and number densitites depend only on m for all $n \neq 0$. This follows from the fact that n-dependence may be removed by shifting the time τ by $\log(\kappa_n)$. However, the modes with $n = 0$ need special treatment. Indeed, in the $n \to 0$ limit the "in" modes (7.2) go over to standard positive frequency modes of the form $e^{-i\omega_m\tau}$ as expected. In this limit, however, the out modes (7.4) contain both positive and negative frequencies. This is of course a wrong choice, since for these $n = 0$ modes there is no difference between "in" and "out" states. In fact, the "out" modes (7.4) have been chosen by considering an appropriate large time property for *nonzero* n and do not apply for $n = 0$. In other words, the squeezed state (7.7) contains only the $n \neq 0$ modes.

The phenomenon we described above of course occurs even when $\mu = 0$, in which case the IIB string frame metric is flat. In this case, it is useful to consider the lagrangian as a sum of the lagrangian densities of an infinite number of 1+1 dimensional scalar and fermion fields with time-dependent masses

$$M^2_n(\tau) = [\frac{nR}{g_B l^2_B}]^2\ e^{2Q\tau}. \tag{7.10}$$

The resulting action may be viewed as that of fields with time *independent* masses (and their fermionic partners) in a Milne universe. In a way similar to the IIA background of [3] the theory for each n has a conserved supercurrent. However the periodicity in σ leads to a breaking of supersymmetry due to boundary conditions. Indeed if σ were non-compact, one could make a coordinate transformation to "Minkowski" coordinates in which the supersymmetry is obvious. The "Minkowski" vacuum is supersymmetric and there is no particle production. The periodicity of σ destroys this property, and there is no supersymmetric vacuum since there is no supersymmetry. However, as $\tau \to \infty$ the periodicity of σ becomes less important and constant τ slices approach slices of constant "Minkowski" time. Indeed the "out" vacuum defined above is identical to the "Minkowski" vacuum: the positive frequency modes defined with respect to these out modes are also positive frequency with respect to the "Minkowski" modes [73].

The operators $a^I_{m,n}$ in fact create states of $(p.q)$ strings in the original Type IIB theory [27]. To see this, let us recall how the light cone IIB fundamental string states arise from the $n = 0$ modes of the Matrix Membrane. In this sector the action is exactly the Green-Schwarz action. The oscillators $a^{\dagger I}_{m,0}$ defined above are in fact the world sheet oscillators and create excited states of a string. The gauge invariance of the theory allows nontrivial boundary conditions, so that m defined above can be fractional. Equivalently the boundary conditions are characterized by conjugacy classes of the gauge group. The longest cycle corresponds to a single string whose σ coordinate has an extent of $2\pi J\frac{l_B^2}{R}$ which is the same as $2\pi l_B^2 p_-$ as it should be in the light cone gauge. Shorter cycles lead to multiple strings - the sum of the lengths of the strings is always $2\pi l_B^2 p_-$, so that there could be at most J strings. Note that m is the momentum in the σ direction: a state with *net* momentum in the σ direction in fact corresponds to a fundamental IIB string wound in the x^- direction. This may be easily seen from the chain of dualities which led to the Matrix Membrane.

As shown in [27], following the arguments of [66, 74, 75], $SL(2,\mathbb{Z})$ transformations on the torus on which the Yang-Mills theory lives become the $SL(2,\mathbb{Z})$ transformations which relate (p,q) strings in the original IIB theory. In particular the oscillators $a^I_{0,n}$ create states of a D-string.

The squeezed state (7.7) is therefore a superposition of excited states of these (p,q) strings. The number of such strings depends on the choice of the conjugacy classes characterizing boundary conditions. Since each (m,n) quantum number is accompanied by a partner with $(-m,-n)$ this state does not carry any F-string or D-string winding number. Finally this squeezed state contains only $n \neq 0$ modes, *i.e.* they do not contain the states of a pure F-string. We therefore conclude that in this toy model the initial state has to be chosen as a special squeezed state of *unwound* (p,q) strings near the big bang to ensure that the late time spectrum contains only perturbative strings. It is interesting that this toy model of cosmology can address the issue of initial conditions.

In the above discussion we have ignored the effect of D-string interactions. In fact when $g_B \ll 1$ the pure D-strings described above are strongly coupled and all excited states rapidly

decay into supergravity modes. However states with $n \neq 0$ which are "almost" F-strings could be relatively long lived. For such states the above results based on free strings will continue to be relevant.[5]

Finally, in this paper we have not considered the possible generation of potentials for the fields due to quantum effects. In the absence of *pp* waves this indeed happens [83, 84] and one would expect that the same would be true in the presence of *pp* waves. However, as found in [83, 84] and emphasized in [84] the potential vanishes at early times (as expected) *as well as at late times*. This suggests that standard perturbative string physics is indeed recovered at late times. We have not yet performed a similar analysis in the *pp* wave background, but we expect similar results to hold. In our discussion of particle production, however, we implicitly assumed that the potential vanishes at an intermediate time where the non-abelian excitations have decoupled, but the Kaluza-Klein modes have not. This requires a detailed investigation. The presence of a potential will certainly change the details of particle production. However we expect that the basic fact that (p, q) strings are produced to still hold.

It would be interesting to explore the meaning of the supergravity background in the holographic dual in terms of the 3+1 dimensional gauge theory. This seems to require a deformation of $AdS_5 \times S^5$ which, in the Penrose limit, would become the solution of this paper. In [85, 86] a large class of time-dependent deformations of $AdS_5 \times S^5$ have been found for which there is a natural proposal for the dual gauge theory. Although this class does not include the one we are looking for, further investigations along these lines might lead to the answer.

Acknowledgments

We would like to thank K. Narayan and Sandip Trivedi for numerous discussions and collaboration at early stages. We also thank Samir Mathur, Alfred Shapere and Xinkai Wu for helpful conversations. S.R.D. would like to thank Tata Institute of Fundamental Research, Mumbai for hospitality. This work was supported in part by a National Science Foundation grant No. PHY-0244811 and a Department of Energy contract #DE-FG01-00ER45832.

[1] S. R. Das and J. L. Karczmarek, *Spacelike boundaries from the c = 1 matrix model*, Phys. Rev. D **71** (2005) 086006 [hep-th/0412093].

[2] S. R. Das, *Non-trivial 2d space-times from matrices*, hep-th/0503002.

[3] B. Craps, S. Sethi and E. P. Verlinde, *A matrix big bang*, hep-th/0506180.

[5] Previous work on string pair production includes [76, 77, 78, 79, 80, 81, 82].

[4] M. Li, *A class of cosmological matrix models*, hep-th/0506260.
[5] M. Li and W. Song, *Shock waves and cosmological matrix models*, hep-th/0507185.
[6] Y. Hikida, R. R. Nayak and K. L. Panigrahi, *D-branes in a big bang / big crunch universe: Misner space*, J. High Energy Phys. **09** (2005) 023; [hep-th/0508003].
[7] S. R. Das and J. Michelson, *pp wave big bangs: Matrix strings and shrinking fuzzy spheres*, Phys. Rev. D **72** (086005) 2005; [hep-th/0508068].
[8] B. Chen, *The time-dependent supersymmetric configurations in M-theory and matrix models*, hep-th/0508191.
[9] J. H. She, *A matrix model for Misner universe*, hep-th/0509067.
[10] B. Chen, Y. l. He and P. Zhang, *Exactly solvable model of superstring in plane-wave background with linear null dilaton*, hep-th/0509113.
[11] T. Ishino, H. Kodama and N. Ohta, *Time-dependent solutions with null Killing spinor in M-theory and superstrings*, hep-th/0509173.
[12] D. Robbins and S. Sethi, *A matrix model for the null-brane*, hep-th/0509204.
[13] S. Kalyana Rama, *Branes in a time dependent universe*, hep-th/0510008.
[14] Y. Hikida and T. S. Tai, *D-instantons and closed string tachyons in Misner space*, hep-th/0510129.
[15] J. McGreevy and E. Silverstein, *The tachyon at the end of the universe*, J. High Energy Phys. **08** (2005) 090 [hep-th/0506130].
[16] E. Silverstein, *Dimensional mutation and spacelike singularities*, [hep-th/0510044].
[17] M. Berkooz, Z. Komargodski, D. Reichmann and V. Shpitalnik, *Flow of geometries and instantons on the null orbifold*, hep-th/0507067.
[18] M. Pawlowski, W. Piechocki and M. Spalinski, *Winding strings in singular spacetimes*, hep-th/0507142.
[19] S. Kawai, E. Keski-Vakkuri, R. G. Leigh and S. Nowling, *Brane decay from the origin of time*, hep-th/0507163.
[20] G. Moore and A. Parnachev, *Profiling the brane drain in a nonsupersymmetric orbifold*, hep-th/0507190.
[21] B. McInnes, *The most probable size of the universe*, hep-th/0509035.
[22] S. F. Ross, *Winding tachyons in asymptotically supersymmetric black strings*, hep-th/0509066.
[23] O. Bergman and S. Hirano, *Semi-localized instability of the Kaluza-Klein linear dilaton vacuum*, hep-th/0510076.
[24] K. Narayan, *Closed string tachyons, flips and conifolds*, hep-th/0510104.
[25] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, *M theory as a matrix model: A conjecture*, Phys. Rev. D **55** (1997) 5112; [hep-th/9610043].
[26] L. Motl, *Proposals on nonperturbative superstring interactions*, hep-th/9701025.
[27] T. Banks and N. Seiberg, *Strings from matrices*,; Nucl. Phys. **B497** (1997) 41; [hep-th/9702187].
[28] R. Dijkgraaf, E. P. Verlinde and H. L. Verlinde, *Matrix string theory*, Nucl. Phys. **B500** (1997) 43; [hep-th/9703030].
[29] M. B. Halpern and M. Claudson *Supersymmetric Ground State Wave Functions*, Nucl. Phys. **B250** (1985) 689–715.
[30] A. Sen, *D0 branes on T(n) and matrix theory*, Adv. Theor. Math. Phys. **2** (1998) 51; [hep-th/9709220].
[31] N. Seiberg, *Why is the matrix model correct?*, Phys. Rev. Lett. **79** (1997) 3577;

[hep-th/9710009].

[32] G. T. Horowitz and A. R. Steif, *Space-time Singularities in String Theory*, Phys. Rev. Lett. **64** (1990) 260.

[33] V. Balasubramanian, S. F. Hassan, E. Keski-Vakkuri and A. Naqvi, *A space-time orbifold: A toy model for a cosmological singularity*, Phys. Rev. D **67** (2003) 026003; [hep-th/0202187].

[34] L. Cornalba and M. S. Costa, *A new cosmological scenario in string theory*, Phys. Rev. D **66** (2002) 066001; [hep-th/0203031].

[35] J. Simon, *The geometry of null rotation identifications*, J. High Energy Phys. **06** (2002) 001; [hep-th/0203201].

[36] H. Liu, G. Moore and N. Seiberg, *String in a time-dependent orbifold*, J. High Energy Phys. **06** (2002) 045; [hep-th/0204168].

[37] H. Liu, G. Moore and N. Seiberg, *String in time-dependent orbifolds*, J. High Energy Phys. **10** (2002) 031; [hep-th/0205288].

[38] A. J. Tolley and N. Turok, *Quantum Fields in a Big Crunch/Big Bang Spacetime*, Phys. Rev. D **66** (2002) 106005; [hep-th/0204091].

[39] B. Craps, D. Kutasov and G. Rajesh, *String propagation in the presence of cosmological singularities*, J. High Energy Phys. **06** (2002) 053; [hep-th/0205101].

[40] M. Fabinger and J. McGreevy, *On smooth time-depedent orbifolds and null singularities*, J. High Energy Phys. **06** (2003) 042; [hep-th/0206196].

[41] J. R. David, *Plane waves with weak singularities*, J. High Energy Phys. **11** (2003) 064; [hep-th/0303013].

[42] B. C. Da Cunha and E. J. Martinec, *Closed string tachyon condensation and worldsheet inflation*, Phys. Rev. D **68** (2003) 063502 [hep-th/0303087].

[43] J. G. Russo, *Cosmological string models from Milne spaces and SL(2,$\mathbb{Z}$) orbifold*, Mod. Phys. Lett. ***A*19** (2004) 421–432; [hep-th/0305032].

[44] A. Giveon, E. Rabinovici and A. Sever, *Strings in singular time-dependent backgrounds*, Fortsch. Phys. **51** (2003) 805–823; [hep-th/0305137].

[45] B. Pioline and M. Berkooz, *Strings in an electric field, and the Milne universe*, J. Cosmol. Astropart. Phys. **11** (2003) 007; [hep-th/0307280].

[46] B. Durin and B. Pioline, *Closed strings in Misner space: A toy model for a big bounce?*, LPTHE-P05-03, hep-th/0501145.

[47] M. Berkooz, B. Durin, B. Pioline and D. Reichmann, *Closed strings in Misner space: Stringy fuzziness with a twist*, J. Cosmol. Astropart. Phys. **10** (2004) 002; hep-th/0407216.

[48] D. Berenstein, J. M. Maldacena and H. Nastase, *Strings in flat space and pp waves from $N = 4$ super Yang Mills*, J. High Energy Phys. **04** (2002) 013; [hep-th/0202021].

[49] J. Michelson, *(Twisted) toroidal compactification of pp-waves*, Phys. Rev. D **66** (2002) 066002; [hep-th/0203140].

[50] K. Dasgupta, M. M. Sheikh-Jabbari and M. Van Raamsdonk, *Matrix perturbation theory for M-theory on a PP-wave*, J. High Energy Phys. **05** (2002) 056; [hep-th/0205185].

[51] K. Dasgupta, M. M. Sheikh-Jabbari and M. Van Raamsdonk, *Protected multiplets of M-theory on a plane wave*, J. High Energy Phys. **09** (2002) 021; [hep-th/0207050].

[52] N. W. Kim and J. Plefka, *On the spectrum of pp-wave matrix theory*, Nucl. Phys. **B643** (2002) 31; [hep-th/0207034].

[53] N. Kim and J. H. Park, *Superalgebra for M-theory on a pp-wave*, Phys. Rev. D **66** (2002) 106007; [hep-th/0207061].

[54] K. Sugiyama and K. Yoshida, *Type IIA string and matrix string on pp-wave*, Nucl. Phys.

B644 (2002) 128; [hep-th/0208029].

[55] S. J. Hyun and H. J. Shin, *N = (4,4) type IIA string theory on pp-wave background*, J. High Energy Phys. **10** (2002) 070; [hep-th/0208074].

[56] S. J. Hyun and H. J. Shin, *Solvable N = (4,4) type IIa string theory in plane-wave background and D-branes*, Nucl. Phys. **B654** (2003) 114; [hep-th/0210158].

[57] N. Iizuka, *Supergravity, Supermembrane and M(atrix) model on PP-Waves*, Phys. Rev. D **68** (2003) 126002; [hep-th/0211138].

[58] M. Cvetič, H. Lü and C. N. Pope, *M-theory, PP-Waves, Penrose Limits and Supernumerary Supersymmetries*, CTP-TAMU-05/02, UPR-985-T, MCTP-02-18, hep-th/0203229.

[59] S. R. Das, J. Michelson and A. D. Shapere, *Fuzzy spheres in pp-wave matrix string theory*, Phys. Rev. D **70** (2004) 026004; [hep-th/0306270].

[60] R. C. Myers, *Dielectric-branes*, J. High Energy Phys. **12** (1999) 022; [hep-th/9910053].

[61] S. Mukhi, M. Rangamani and E. P. Verlinde, *Strings from quivers, membranes from moose*, J. High Energy Phys. **05** (2002) 023; [hep-th/0204147].

[62] S. Sethi and L. Susskind, *Rotational Invariance in the M(atrix) Formulation of Type IIB Theory*, Phys. Lett. B **400** (1997) 265–268; [hep-th/9702101].

[63] R. Gopakumar, *String interactions in PP-waves*, Phys. Rev. Lett. **89** (2002) 171601; [hep-th/0205174].

[64] J. Michelson, *Matrix theory of pp waves*, in *Quantum Theory and Symmetries: Proceedings of the 3rd International Symposium on Quantum Theory and Symmetries* (P. C. Argyres, et. al., eds.) World Scientific, 2004; [hep-th/0401050].

[65] J. Michelson, *to appear*

[66] J. H. Schwarz, *The Power of M Theory*, Phys. Lett. B **367** (1996) 97–103; [hep-th/9510086].

[67] G. Bonelli, *Matrix Strings in pp-wave backgrounds from deformed Super Yang-Mills Theory*, J. High Energy Phys. **08** (2002) 022; [hep-th/0205213].

[68] N. Kim, T. Klose and J. Plefka, *Plane-wave Matrix Theory from $N = 4$ Super Yang-Mills on $\mathbb{R} \times S^3$*, Nucl. Phys. **B671** (2003) 359–382; [hep-th/0306054].

[69] W. Taylor and M. Van Raamsdonk, *Multiple Dp-branes in Weak Background Fields*, Nucl. Phys. **B573** (2000) 703–734; [hep-th/9910052].

[70] W. I. Taylor and M. Van Raamsdonk, *Multiple D0-branes in weakly curved backgrounds*, Nucl. Phys. **B558** (1999) 63; [hep-th/9904095].

[71] A. Strominger, *Open string creation by S-branes*, hep-th/0209090.

[72] N. D. Birrell and P. C. W. Davies, *Quantum Fields In Curved Space*, (Cambridge University Press, Cambridge, 1982).

[73] T. Tanaka and M. Sasaki, *Quantized gravitational waves in the Milne universe*, Phys. Rev. D **55** (1997) 6061; [gr-qc/9610060].

[74] J. H. Schwarz, *"An SL(2,Z) multiplet of type IIB superstrings*, Phys. Lett. B **360** (1995) 13, *erratum* Phys. Lett. B **364** (1995) 252; [hep-th/9508143].

[75] P. S. Aspinwall, *Some relationships between dualities in string theory*, Nucl. Phys. Proc. Suppl. **46** (1996) 30; [hep-th/9508154].

[76] A. E. Larwence and E. J. Martinec, *String field theory in curved spacetime and the resolution of spacelike singularities*, Class. Quant. Grav. **13** (1996) 63; [hep-th/9509149].

[77] S. S. Gubser, *String production at the level of effective field theory*, Phys. Rev. D **69** (2004) 123507; [hep-th/0305099].

[78] S. S. Gubser, *String Creation and Cosmology*, hep-th/0312321.

[79] J. J. Friess, S. S. Gubser and I. Mitra, *String creation in cosmologies with a varying dilaton*,

Nucl. Phys. **B689** (2004) 243; [hep-th/0402156].

[80] N. Turok, M. Perry and P. J. Steinhardt, *M-Theory Model of a Big Crunch/Big Bang Transition*, Phys. Rev. D **70** (2004) 106004, *erratum* Phys. Rev. D **71** (2005) 029901; [hep-th/0408083].

[81] A. J. Tolley and D. H. Wesley, *String Pair Production in a Time-Dependent Gravitational Field*, hep-th/0509151.

[82] S. Cremonini and S. Watson *Dilaton Dynamics from Production of Tensionless Membranes*, hep-th/0601082.

[83] M. Li and W. Song, *A one loop problem of the matrix big bang model*,; hep-th/0512335.

[84] B. Craps, A. Rajaraman and S. Sethi, *Effective dynamics of the matrix big bang*, hep-th/0601062.

[85] C. S. Chu and P. M. Ho, *Time-dependent AdS/CFT duality and null singularity*, hep-th/0602054.

[86] S.R. Das, J. Michelson, K. Narayan and S. P. Trivedi *Time Dependent Cosmologies and Their Duals*, hep-th/0602107.