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The Deconfinement Phase Transition at large N: A Study of the Hot Twisted Eguchi-Kawai Model.

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ABSTRACT

The deconfinement phase transition for SU(54) gauge theory is studied using twisted Eguchi-Kawai methods. Expectation values of the action, the Wilson line and the energy density are measured. We find evidence for deconfining phase transition and compare our data with the naive string picture.

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The deconfinement transition for SU(N) gauge theories at finite temperature has been the subject of intense study during the past several years [1-3]. As yet,the nature of the transition is well understood only for SU(2)[2] and SU(3)[3].In the SU(2) theory one has a second order transition,while in SU(3) the transition is strongly first order, in agreement with general universality arguments[4].For $N>4$,the situation is less clear:a combination of strong coupling and mean field theory techniques predicts a first order transition[5];the reliability of these arguments is,however,rather doubtful. To understand the nature of the transition,it is essential to know its nature for higher values of N.In this respect,the recent work on large N theories is of particular importance.The observation that these theories are equivalent , at $N=\infty$,to single point matrix models(Eguchi-Kawai models)[6-8] has made numerical simulations possible.The finite temperature Quenched Eguchi-Kawai (QEK) model has been studied earlier [9]:the deconfinement temperature reported was very large compared to the values at $N=2$ and $N=3$. However,it is well known that the QEK model suffers from severe finite size effects. It is more appropriate to study the twisted Eguchi-Kawai (TEK) model [8]. A finite temperature version of the TEK model which has the same Dyson- Schwinger equations and identical perturbation theory as the corresponding field theory has been constructed by Klinkhamer and van Baal [10].(There has been another attempt to construct such a model in Ref 11.It is not clear whether this construction is correct).In this letter we shall report some results of a Monte-Carlo study of the hot TEK model of Ref.10.More detailed studies,both numerical and analytical,are in progress and shall be reported in a future communication.

The TEK model is defined by the partition function:

$$Z = \int \prod_{\mu=1}^{d} dU_\mu \, \exp(-\beta S_{TEK})$$
$$S_{TEK} = -\sum_{\mu>\nu} Z_{\mu\nu} \, Tr(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger) + h.c. \quad \cdots (1)$$

where U_μ's are SU(N) matrices and $Z_{\mu\nu}$ is a constant element of Z_N:

$$Z_{\mu\nu} = \exp\left(\frac{2\pi i n_{\mu\nu}}{N}\right)$$

For a symmetric twist,i.e. $N=L^2$ and $n_{\mu\nu}=L$ for all $\nu>\mu$,the above model is equivalent to the zero temperature SU(N) gauge theory defined on a periodic box of size L [8].For this twist,the $N=\infty$ limit is also the thermodynamic limit in which both the spatial and temporal extents of the box go to infinity. However, to obtain a reduced model for a finite temperature theory one must be able to let the spatial extent go to infinity keeping a fixed temporal extent.One of the twists which accomplishes this is given by [10]:

$$n_{\mu\nu} = N_0 \begin{pmatrix} 0 & 2K(4K^2-1) & 4K(4K^2-1) & 2K(4K^2-1) \\ & 0 & 2K(2K+1) & 4K^2-1 \\ & & 0 & 2K(2K-1) \\ & & & 0 \end{pmatrix} \cdots (2)$$

where N_o is an odd integer and K is any integer. Let us define:

$$\begin{aligned} N &= 2N_o^2 K(4K^2-1) \\ N_1 &= 2N_o K(2K-1) \\ N_2 &= N_o(4K^2-1) \\ N_3 &= 2N_o K(2K+1) \end{aligned} \qquad \ldots\ldots \quad (3)$$

Then, for $K\to\infty$, N_o fixed, the TEK model with the above twist is equivalent to a SU(N) gauge theory in a periodic box of temporal extent N_o and spatial extents N_1, N_2 and N_3; i.e. a finite temperature theory with the physical temperature T given by $T=1/N_o a$, a being the lattice spacing. At extreme weak couplings the partition function (1) is dominated by the following twist-eating configuration [10]:

$$\begin{aligned} U_0 &= \Gamma_0 = Q_1^{-2} \otimes P_2^{2K(2K+1)(4K^2-1)} Q_2^{4K(1-4K^2)} \\ U_1 &= \Gamma_1 = P_1^{K+1} \otimes P_2^{2K(2K+1)(K+1)} Q_2^{-(2K+1)^2} \\ U_2 &= \Gamma_2 = P_1 \otimes P_2^{2K(2K+1)} Q_2^{-4K} \\ U_3 &= \Gamma_3 = P_1^{1-K} \otimes P_2^{(1-2K^2)(2K-1)} Q_2^{(2K-1)^2} \end{aligned} \qquad \ldots \quad (4)$$

where (P_1,Q_1) and (P_2,Q_2) are 't Hooft matrices in $SU(N_o)$ and $SU(M_2)$ respectively, where $M_2=2N_oK(4K^2-1)$. It may be easily checked that

$$\Gamma_\mu \Gamma_\nu = Z_{\nu\mu} \Gamma_\nu \Gamma_\mu$$

A link variable of the field theory $U_\mu(x)$ is related to the reduced variable U_μ by the reduction rule:

$$U_\mu(x) \to D(x)\, U_\mu\, D^\dagger(x) \quad ; \; D(x) \equiv \prod_\mu (\Gamma_\mu)^{x_\mu}$$

This also provides a prescription for obtaining averages of gauge-invariant quantities:

$$\langle f(U_\mu(x))\rangle_{\text{FIELD THEORY}} = \langle f(D(x)\,U_\mu\,D^\dagger(x))\rangle_{TEK}$$

where $\langle\ \rangle_{TEK}$ denotes averaging over the U_μ's in the ensemble defined by (1). In particular, the Wilson line (which is a product of link variables along a straight time-like line running from one end of the box to the other) is given by

$$\langle WL\rangle = \frac{1}{N}\,\mathrm{Re}\,\langle \mathrm{Tr}\, U_0^{N_0}\rangle_{TEK} \qquad \cdots (5)$$

In the standard lattice gauge theory there is a Z_N symmetry which, if unbroken, forces <WL> to be zero. This simply acts by a translation of the eigenvalues of the Wilson line operator, i.e. the untraced WL. In weak coupling these eigenvalues all cluster to the same value, thus breaking the Z_N symmetry and leading to a nonzero WL. In the reduced model one is interested in the eigenvalues of $U_o^{N_o}$. In extreme weak coupling $U_o^{N_o} = \Gamma_o^{N_o} = 1$ so that the Z_N symmetry is broken. (Note, however, that N_o is the lowest integer for which $\Gamma_o^{N_o} = 1$. This is a special case of the fact that all open lines vanish, except those which run from one end of the box to the other-which is necessary for the equivalence of the reduced model to the field theory). By standard arguments, the Z_N symmetry is restored at strong couplings. Thus one expects a phase transition.

Another good order parameter for the deconfining transition is the average energy density per plaquette [11,3]. In the hot TEK model this is given by

$$\epsilon = \frac{\beta}{3N} \left[\sum_{i>j} Z_{ij} \, Tr(U_i U_j U_i^\dagger U_j^\dagger) - \sum_i Z_{0i} \, Tr(U_0 U_i U_0^\dagger U_i^\dagger) \right] \quad \cdots (6)$$

where i,j runs from 1 to 3.

We have performed Monte Carlo simulations of the above model for $N_0=3$ and K=1. This corresponds to a SU(54) theory in a box of temporal extent 3 and spatial extents 6X9X18 lattice spacings. The updating procedure was that of Ref 13. We measured the total action density, given by :

$$\langle S \rangle = \frac{1}{N} Re \left\langle \sum_{\mu>\nu} Z_{\mu\nu} \, Tr(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger) \right\rangle_{TEK} \quad \cdots (7)$$

the Wilson line (eqn.5) and the energy density ϵ (eqn.6). Typically, the action would equilibrate after 30-50 sweeps, except in the crossover region where there is considerable critical slowing down. The Wilson line takes longer to relax, as is expected for a non-local quantity. We went through about 100 sweeps for each β in the strong and weak coupling regions. In intermediate couplings considerably longer runs were made. The average action is plotted as a function of β/N in Figure 1. The points labelled "weak coupling side" and "strong coupling side" were obtained by starting the calculation at large or small β respectively, performing approximately 200 sweeps before passing to the neighboring value of β/N and gradually working our way into the intermediate coupling region. This is a good procedure for searching for a phase transition and studying its order. The data in Fig.1 is in good agreement with the lowest order expressions in the weak and strong coupling expansions:

$$\langle S \rangle = \begin{cases} 6\left(1 - \frac{1}{8}\frac{N}{\beta}\right) & \beta \gg N \\ \frac{6\beta}{N} & \beta \ll N \end{cases} \quad \cdot\cdot (8)$$

There is a clear signal for a first order phase transition in the vicinity of $\beta/N=0.35$, in agreement with the findings of Ref.8 and 14. A typical history of the action for strong and weak coupling starts is shown in Figure 2. Figure 3 shows the average Wilson line obtained from strong and weak coupling starts. <WL> remains zero up to $\beta/N=0.44$ for strong starts. For weak starts the Wilson line jumps discontinuously from zero to about 0.3 at $\beta/N=0.34$. Such a discontinuous jump, together with the presence of hysteresis indicates a first order transition. However, in this particular case the presence of the zero temperature bulk transition at $\beta/N=0.35$ complicates the interpretation of our data. In a finite temperature large N theory the Wilson line average is related to Wilson loop averages via Dyson-Schwinger equations. The bulk transition is not deconfining, but does involve a discontinuous change in the string tension value. Therefore, if the deconfining transition lies in the strong coupling side of the bulk transition, the Wilson line would suffer a similar discontinuity at the latter. In view of the severe hysteresis we cannot pin down the coupling at which deconfinement occurs. However, if we trust the value of β/N where <WL> takes off from the weak coupling side, the Wilson line is probably influenced by the bulk transition which is rather close to this value of the critical coupling.

Figure 4 shows the energy density as a function of β/N. This quantity is usually a good guide to the finite temperature transition because it vanishes identically on a symmetric lattice and is not affected by any zero temperature bulk transition. However, our data for ε is rather noisy as typical error bars indicate. The energy density turns on at $\beta/N=0.32$ for weak coupling starts and at $\beta/N=0.44$ for strong coupling starts, i.e. at the same places where the Wilson line turns on. We cannot say with any certainty whether the change in ε is discontinuous or smooth. It may be noted that internal energy calculations required 3,000 to 5,000 sweeps in SU(3) gauge theory on a 4×8^3 lattice to measure the system's latent heat [3] This indicates that we probably require an order of magnitude more computer power to decide this question without bias. Nevertheless the magnitude of ε for very weak couplings is consistent with the standard Stefan-Boltzman value on a 3×7^3 lattice [12] which has typically the same size as our lattice.

In view of the large hysteresis loop in Figures 3 and 4 it is difficult to obtain the exact deconfinement temperature. A simple theoretical estimate follows from the observation that at large N the theory reduces to a free string theory. In the string picture the deconfinement transition occurs when the entropy of a string starts dominating over the energy. In three spatial dimensions the transition temperature is approximately given by [15] :

$$T_c = \frac{\sigma a}{\log 5} \qquad \cdots \quad (9)$$

where σ is the string tension and a is the lattice spacing. In a latiice of temporal extent N_o this corresponds to :

$$a(\beta_c)\Lambda_L = \left(\frac{\log 5}{N_o}\right)^{1/2} \frac{\Lambda_L}{\sqrt{\sigma}}$$

Using the data of Ref.14 one has $a(\beta_c)\Lambda_L \simeq 0.0026$ corresponding to a $\beta_c/N = 0.326$. This value is close to the observed value of the coupling where <WL> and ε turn on in the Monte Carlo runs which begin from the weak coupling side (see Figures 3 and 4). It might very well be that the deconfinement transition is actually in this vicinity. This, however, means that deconfinement occurs in the strong coupling side of the bulk transition, and by our previous arguments the Wilson line is grossly influenced by the latter. Furthermore, in this case the critical coupling does not lie in the scaling region and it is not possible to extract the physical temperature quantitatively from our data,.

It is also possible that the finite temperature phase transition occurs at the opposite end of the hyteresis loop, at $\beta/N \simeq 0.442$. Comparing to string tension calculations [14] a critical temperature of $T_c/\Lambda_L \simeq 1370$ follows as measured in units of the lattice Λ_L parameter

$$a\Lambda_L = \left(\frac{48\pi^2}{11}\frac{\beta}{N}\right)^{51/121} \exp\left(-\frac{24\pi^2}{11}\frac{\beta}{N}\right)$$

Since $\sqrt{\sigma}/\Lambda_L = 280 \pm 20$ [14], we would find $T_c/\sqrt{\sigma} \simeq 4.90$ - a very large temperature. If ,however, the transition occurs on the strong coupling side of the hysteresis loop at β/N = 0.34 - 0.35, then $T_c/\sqrt{\sigma} \simeq 0.60$, which is close to the SU(2) and SU(3) values. Unfortunately, this estimate is particularly uncertain because of the nearby bulk transition.

Clearly the large hysteresis loop in the Monte Carlo data limits its usefulness. As pointed out to us by G.Parisi, considerable hysteresis is expected at large N because tunneling probabilities behave as $\exp(-1/g^2)$ = $\exp(-N)$. In other words, fluctuations are suppressed at large N and metastability effects are enhanced.

One message of our work is that one requires a larger N_o to push back the critical coupling for deconfinement into the scaling region, separating it from the bulk transition and thereby obtain reliable information about the physics. In addition, longer Monte Carlo runs are necessary to deal with the tunneling suppression inherent in the large N limit.

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FIGURE CAPTIONS

Fig. 1: Total Action for strong coupling starts (squares) and weak coupling starts (crosses). The lines represent lowest order contributions in the strong and weak coupling expansions. Typical errors are +0.05.

Fig. 2: History of the total action for $\beta/N = 0.37$. Strong coupling starts are represented by dots, weak coupling starts by crosses.

Fig. 3: Wilson line average for strong (dots) and weak (crosses) coupling starts. The dashed lines are not exact fits - they are drawn to guide the eye.

Fig. 4: Internal energy density ε for strong (circles) and weak (triangles) coupling starts. Typical error bars are shown. The line at $\varepsilon = 0.135$ corresponds to the Stefan-Bolzmann value on a 3×7^3 lattice.

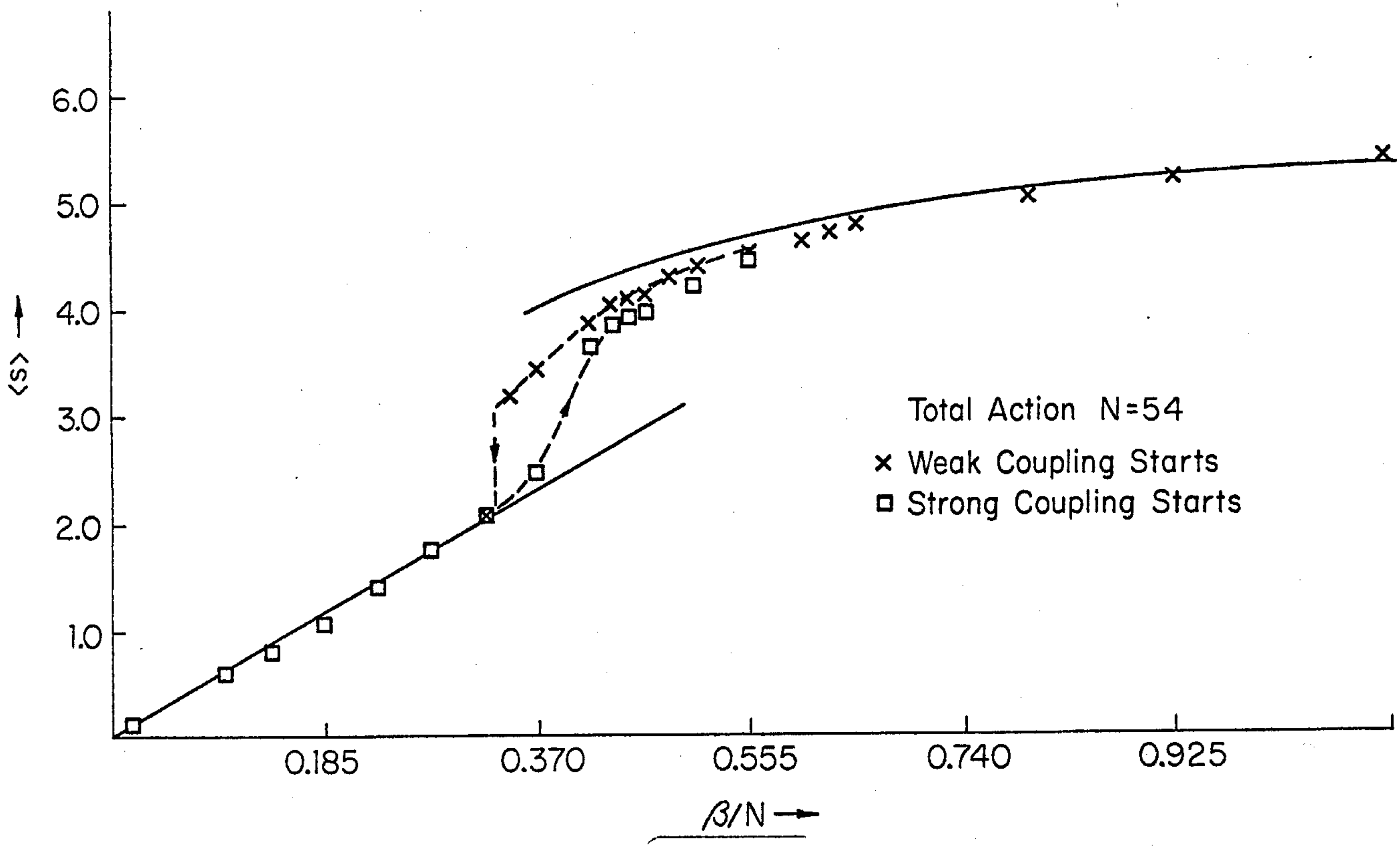

6.0
5.0
4.0
3.0
2.0
1.0
⟨s⟩
0.185
0.370
0.555
0.740
0.925
β/N
Total Action N=54
× Weak Coupling Starts
□ Strong Coupling Starts

Fig.1.

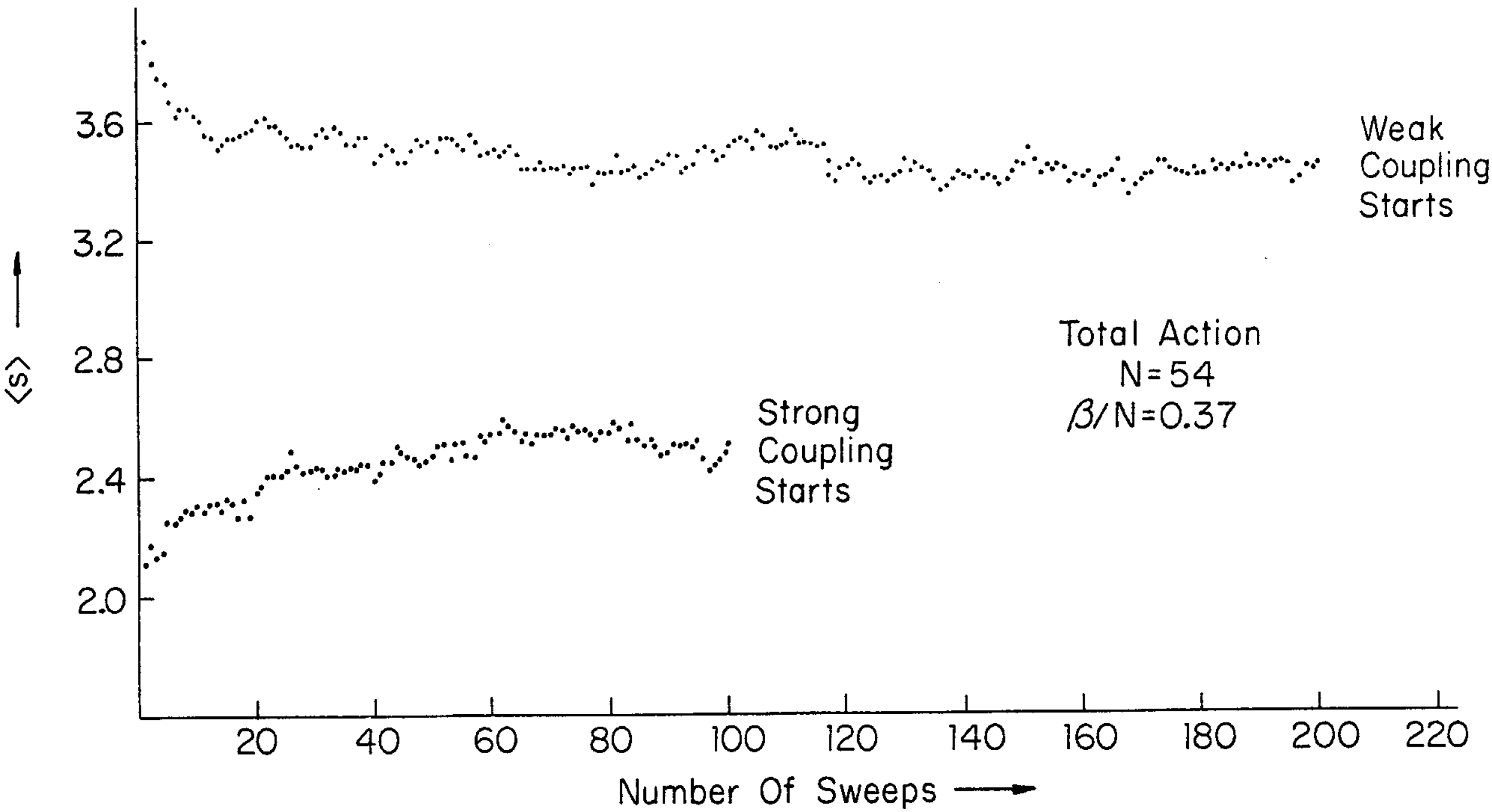

3.6
3.2
2.8
2.4
2.0
⟨s⟩
Weak Coupling Starts
Total Action
N=54
β/N=0.37
Strong Coupling Starts
20
40
60
80
100
120
140
160
180
200
220
Number Of Sweeps

Fig. 2

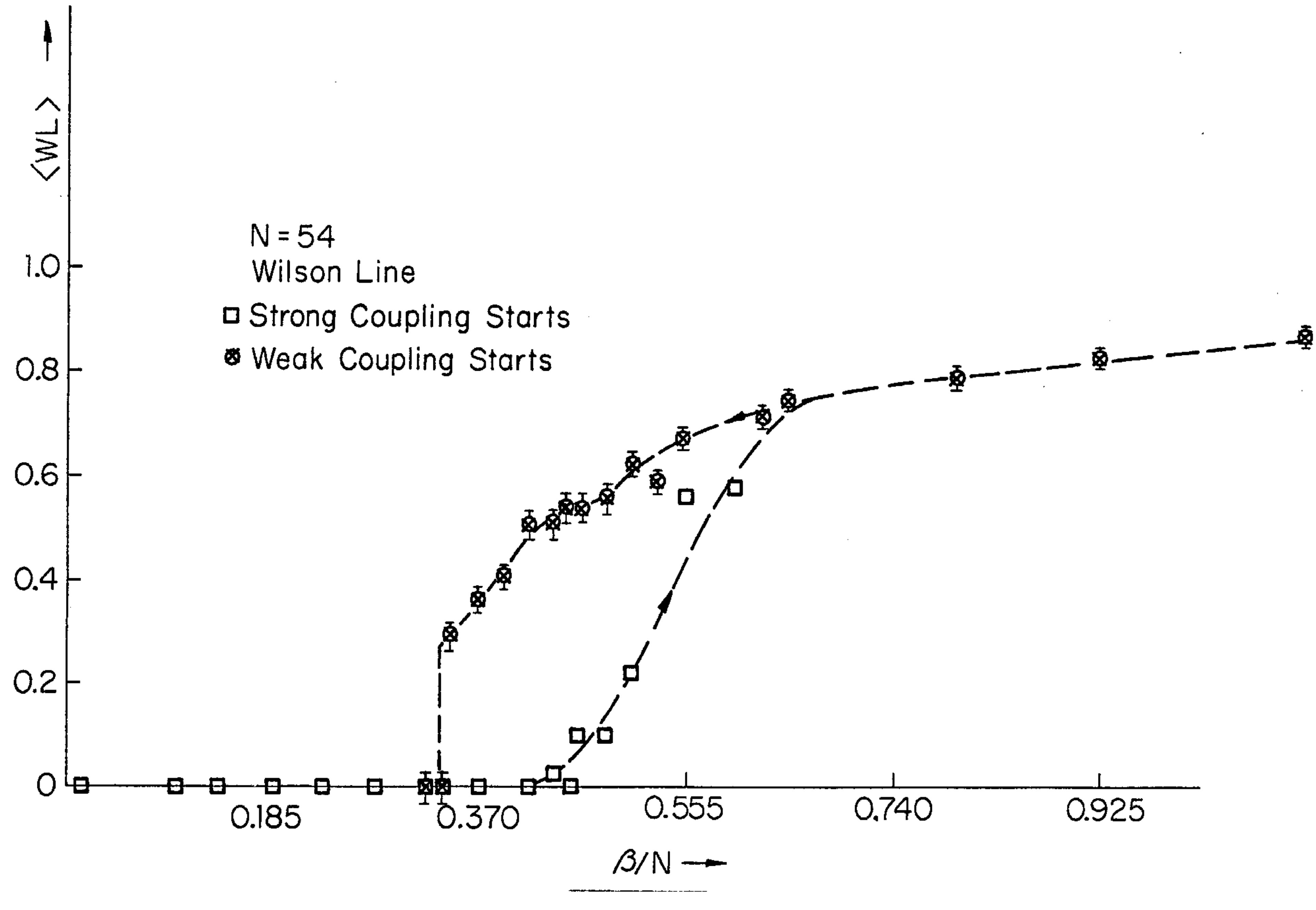
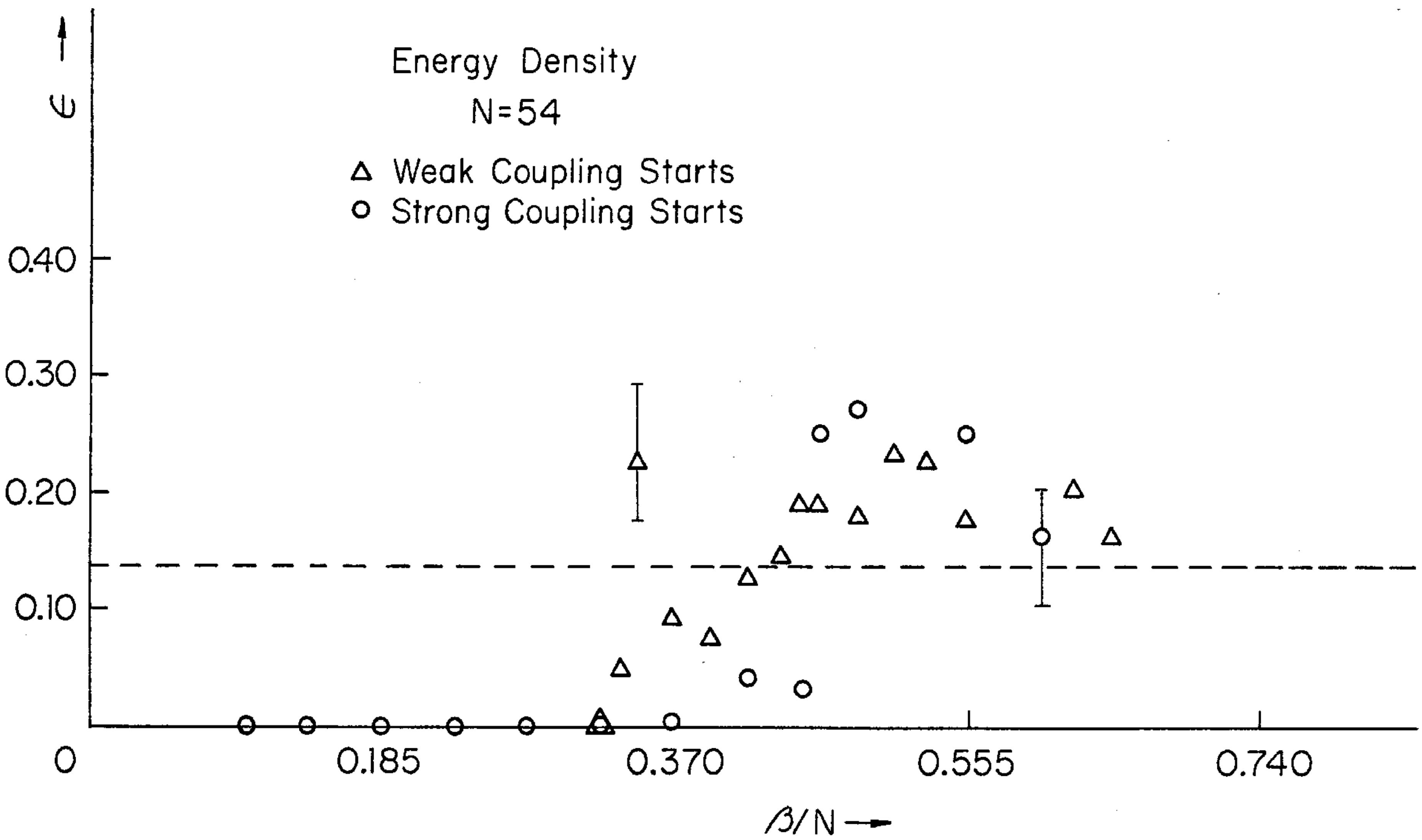

⟨WL⟩
N = 54
Wilson Line
Strong Coupling Starts
Weak Coupling Starts
1.0
0.8
0.6
0.4
0.2
0
0.185
0.370
0.555
0.740
0.925
β/N

Fig. 3



Energy Density
N=54
Weak Coupling Starts
Strong Coupling Starts
ε
0.40
0.30
0.20
0.10
0
0.185
0.370
0.555
0.740
β/N

Fig. 4