arXiv:hep-th/9804149v2 15 May 1998

Three Brane Action and The Correspondence Between N=4 Yang Mills Theory and Anti De Sitter Space

Sumit R. Das[a] and **Sandip P. Trivedi**[b]

[a] *Tata Institute For Fundamental Research*
Homi Bhabha Road, Mumbai 400 005, India

[b] *Fermi National Accelerator Laboratory*
P.O.Box 500
Batavia, IL 60510, USA

Abstract

Recently, a relation between $N = 4$ Super Yang Mills in 3+1 dimensions and supergravity in an AdS_5 background has been proposed. In this paper we explore the idea that the correspondence between operators in the Yang Mills theory and modes of the supergravity theory can be obtained by using the D3 brane action. Specifically, we consider two form gauge fields for this purpose. The supergravity analysis predicts that the operator which corresponds to this mode has dimension six. We show that this is indeed the leading operator in the three brane Dirac-Born-Infeld and Wess-Zumino action which couples to this mode. It is important in the analysis that the brane action is expanded around the anti de-Sitter background. Also, the Wess-Zumino term plays a crucial role in cancelling a lower dimension operator which appears in the the Dirac-Born-Infeld action.

1. Introduction and Summary

One of the interesting outcomes of recent progress in string theory has been the relationship between gauge theories and gravity, particularly in the context of black holes. In particular, classical scattering of various fields from non-dilatonic black holes like extremal three branes are reproduced by correlators of the gauge theories living on the brane worldvolume [1]. Noting the fact that the near-horizon geometry of such black holes is in fact a five dimensional anti-de Sitter (AdS) space, Maldacena has conjectured that the large N limit of a conformally invariant d- dimensional Yang Mills theory in fact *contains* supergravity (and IIB superstring theory) in $(d+1)$ dimensional AdS space [2]. This has led to some progress in understanding the strong coupling behavior of these gauge theories in the large-N limit.

The conjecture was further investigated in [3] and [4] where a concrete prescription was given for relating observables in the supergravity and the Yang Mills theories. The idea in [3], [4] was to consider AdS space together with a boundary. The dependence of the supergravity action on the boundary values of fields then yields the required generating functional from which Greens functions can be calculated.

In this paper we will be concerned with $3+1$ dimensional $SU(N)$ Yang Mills theories with $N = 4$ supersymmetry. The proposal of [2] relates this theory to ten- dimensional Type IIB supergravity compactified on $AdS_5 \times S_5$. Using the recipe outlined above, several two point correlation functions were calculated in [3] and [4] and recently some three point functions have been computed in [5]. For some special operators, which are either chiral or protected from renormalization effects for other reasons agreement was found between the anomalous dimensions as calculated in the Yang mills and the supergravity theories. For other operators this led to a determination of the anomalous dimensions in the large N limit.

A large class of such operators are marginal or relevant in the Yang-Mills theory. It is important to examine the relation between higher dimensional operators and supergravity modes as well. Some higher dimensional operators have been studied in [6]. These may have two different roles in the supergravity context. For some supergravity modes, like the fixed scalar, these are responsible for the leading order absorption by the black 3-brane and related to the correlator in the AdS space itself. For other modes, like the dilaton, they are responsible for corrections to the leading result and probe the 3-brane metric beyond the AdS throat [6].

In this note we continue the study of this set of ideas by focussing on another supergravity mode: the two index NS-NS antisymmetric tensor field with a polarization parallel to some directions of the brane [1]. The propagation of this mode in AdS space was investigated in [7], where, after incorporating the mixing with the R-R two form field it's mass was

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[1] Another mode is obtained by interchanging the roles of the NS-NS and R-R two form fields, our results apply to this case as well.

determined. More recently in [8] the cros-section for scattering this field off a extremal black hole was calculated. These studies show that the scattering of the two form field (with a polarization parallel to the brane) should be suppressed for small energies. Correspondingly the operator in the Yang Mills theory to which it couples should have a total dimension of six. Determination of this operator will be the goal of this paper. We should mention that this operator has been identified in [9], [10], from considerations of superconformal symmetry [2].

So far, in the literature, the principle behind a precise correspondence between various modes in the supergravity theory and operators in the Yang Mills theory has not been spelt out. In this note we explore the idea that the operators of the Yang Mills theory can be obtained by expanding an action consisting of the (non-abelian) Dirac-Born-Infeld (DBI) and Wess-Zumino (WZ) terms. We will see that, for the antisymmetric tensor, the correct Yang Mills operator can be identified in this way, but *only if* the action is expanded about AdS spacetime and the accompanying five form field strength background. In many ways this is the natural expectation. The supergravity mode being considered is a perturbation about AdS space. Thus one expects that to consistently couple it, the DBI action should also be expanded about the AdS background. Related points have been recently made in [11]. The conformal symmetry of the three brane action has been studied in [12].

It should be emphasized that here the DBI plus WZ action will be used to identify the correct operators in the AdS-Yang Mills correspondence. Whether higher order corrections to the correlator in the full 3-brane geometry require the gauge field dynamics to be governed by the DBI-WZ action remains to be seen. Some evidence in favor of this has been presented in [6].

One noteworthy feature about our analysis is that the WZ term plays an important role in it. The leading operator obtained from the DBI action has engineering dimension of four. But the coupling to this operator is cancelled by a contribution coming from the Wess-Zumino term. Thus the leading operator obtained from the whole action, which couples to this supergravity mode, has dimension six at tree level. As was mentioned above, the supergravity analysis shows that this must in fact be its total dimension. We learn in this way that the operator studied here does not acquire any anomalous dimension in the large N limit. This is also true of the dimension eight operator studied in [6].

One more point about our discussion below needs to be mentioned. In expanding the action we will need to decide where the DBI-WZ action lives in AdS space. As was mentioned above, the basic idea in the discussion of [3] and [4] is that the boundary values for the supergravity modes act as sources for the Yang Mills field operators. This suggests the DBI-WZ action should be expanded around the boundary of AdS space. In fact, as we will see

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[2]We thank the authors of [9],[10] for correspondence in this regard.

here, this yields a consistent answer. We do not, however, take this to mean that there are D3-branes physically located at the boundary.

Clearly, this analysis needs to be extended for other modes as well. For one class of Yang Mills operators the coupling to the supergravity modes has been written down from superconformal symmetry considerations in [9]. It will be interesting to see if these and the couplings to all the other supergravity modes can be obtained by expanding the DBI-WZ action. We hope to report more fully on these questions in the future. Let us mention one final point. Our discussion in this note suggests that in the conformally non-invariant cases, the supergravity theory should correspond to a Yang Mills theory not in flat space-time but in the supergravity background instead.

2. The Supergravity Analysis

A system of N parallel D3- branes is described by an extremal black hole geometry with a metric:

$$ds^2 = H^{-1/2}(dx^i)^2 + H^{1/2}(dx^a)^2, \tag{2.1}$$

with

$$H = 1 + \frac{R^4}{r^4}. \tag{2.2}$$

Here $x^i, i = 0, \cdots 3$, refer to the four coordinates parallel to the brane world volume, x^a to the coordinates transverse to the brane, $r^2 = (x^a)^2$ is the transverse coordinate distance away from the branes, and $R^4 = 4\pi g_s N(\alpha')^2$, where g_s is the string coupling, related to the Yang-Mills coupling g_{YM} by $g_s = g_{YM}^2$. In the near horizon region, $r \ll R$, this metric reduces to that of $AdS_5 \times S^5$:

$$ds^2 = \frac{r^2}{R^2}(dx^i)^2 + \frac{R^2}{r^2}(dx^a)^2. \tag{2.3}$$

we see that the S_5 has a radius R and the geometry is smoothly varying if $g_{YM}^2 N$ is large. The dilaton is constant in this background, while the self dual five form field strength in the near horizon region is given by:

$$F_{0123r} = \frac{r^3}{R^4}. \tag{2.4}$$

Following [7] and [8] we now consider the propagation of the NS-NS and R-R two form fields in the AdS background. The NS-NS and R-R gauge potentials will be denoted by $B_{\mu\nu}$ and $A_{\mu\nu}$ and the corresponding fields strengths by $H_{\mu\nu\rho}$ and $F_{\mu\nu\rho}$ respectively. The equations governing the propagation of these modes are [13]:

$$
\begin{aligned}
\nabla^\mu H_{\mu\nu\rho} &= \left(\frac{2}{3}\right) F_{\nu\rho\kappa\tau\sigma} F^{\kappa\tau\sigma} \\
\nabla^\mu F_{\mu\nu\rho} &= -\left(\frac{2}{3}\right) F_{\nu\rho\kappa\tau\sigma} H^{\kappa\tau\sigma}.
\end{aligned}
\tag{2.5}
$$

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We see that in the presence of a non-zero five-form field strength the $B_{\mu\nu}$ and $A_{\mu\nu}$ fields mix with each other.

Here we only consider an s-wave mode with the NS-NS two form being polarised along the brane directions. For simplicity, the only component of $B_{\mu\nu}$ that is non-zero will be chosen to be B_{12}. Eq. (2.5) can now be used to solve for $F_{\mu\nu\rho}$ in terms of B_{12} and gives [3] :

$$F_{0r3} = -4 \ F_{r3012} \ g^{11} g^{22} \ B_{12}, \tag{2.6}$$

with all the other components of the three form RR field strength being zero. In the subsequent discussion we consider a perturbation with energy ω, with a resulting time dependence $e^{-i\omega t}$. ¿From eq. (2.5), eq. (2.6), we then get an equation for B_{12}:

$$\frac{1}{\sqrt{-g}} \frac{1}{g^{11}} \frac{1}{g^{22}} \partial_r (\sqrt{-g} g^{rr} g^{11} g^{22} \partial_r B_{12}) - w^2 g^{00} B_{12} = -16 F^2_{0123r} g^{00} g^{rr} g^{33} g^{11} g^{22} B_{12}. \tag{2.7}$$

Substituting for F_{0123r}, from eq. (2.4) now gives:

$$\frac{1}{\sqrt{-g}} \frac{1}{g^{11}} \frac{1}{g^{22}} \partial_r (\sqrt{-g} g^{rr} g^{11} g^{22} \partial_r B_{12}) - w^2 g^{00} B_{12} = \frac{16}{R^2} B_{12}. \tag{2.8}$$

Thus we see that the mode corresponds to a field with mass $m = \frac{4}{R}$. In the correspondence between Supergravity and Yang Mills theory the region, $r \gg R$ is particularly relevant. In this region the second term on the left hand side of eq. (2.8) can be neglected, giving rise to two solutions with, $B_{12} \sim r^{\pm 4}$. Of these the case,

$$B_{12} \sim c \ r^4, \tag{2.9}$$

can be extended to a non-singular solution in the small r region, it will be the relevant one for the subsequent discussion.

We can also solve for the R-R two form A, corresponding to eq. (2.9). From eq. (2.6), in the $r \gg R$ region it is given by:

$$A_{03} = -B_{12} \tag{2.10}$$

with all other components being zero.

Now that our analysis of the supergravity mode is complete we can use the prescription of [3], [4] to calculate the anomalous dimension of the Yang Mills operator that corresponds to it. The general formula relating the anomalous dimension, Δ, to the mass for a p form is:

$$(\Delta - p)(\Delta + p - 4) = m^2, \tag{2.11}$$

where the mass is measured in units of R. In this case, $m^2 = 16$, eq. (2.8), setting in addition $p = 2$, gives:

$$\Delta = 6. \tag{2.12}$$

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[3]In our conventions $F_{\mu\nu\rho} = \partial_\mu A_{\nu\rho} + \partial_\nu A_{\rho\mu} + \partial_\rho A_{\mu\nu}$.

Thus the operator in the Yang Mills theory corresponding to the mode discussed here has a total dimension of 6 in the large N limit. In the next section we turn to determining this operator.

Before doing so though, let us pause to make contact with the general analysis in [7]. Here we have focussed on an S-wave mode of the two index gauge field. In [7] the propogation of all the higher Kaluza Klein harmonics arising from the two form field (and in fact all the other supergravity modes) were analyzed as well. The propogation equation for the two form, eq. (2.56) in [7], was elegantly factorised giving rise to two families, eq. (2.62), and eq. (2.64) of [7] (also shown in FIG 3 as the two $a_{\mu\nu}$ modes). The S-wave mode discussed in this paper is the $k = 0$ member of the second family, eq. (2.64). The first family, eq. (2.63), only involves $l = 1$ and higher angular momentum modes. In fact the $l = 1$ mode of this family (which transforms like a 6 of $SU(4)$ and is the mode shown in Fig 3 with a circle) is discussed in [9] where it was identified with a dimension three operators in the Yang Mills theory.

3. Identifying the Operator in the Yang Mills Theory

So far in the literature on this subject, a precise procedure for identifying operators in the Yang Mills theory, that correspond to a particular perturbation mode in the supergravity theory, has not been given. The supergravity analysis determines the dimension of the operator. In some cases supersymmetry determines the operator, e.g. the conserved currents in [9]. However, even in these cases the overall normalisation is not determined *a priori*. For example, the relevant power of the string coupling in the normalisation cannot be determined in this manner. This point becomes clear when one tries to extract absorption cross-sections from the Yang-Mills theory. The leading power of energy in the Yang-Mills calculation is determined by the total dimension of the operator, but the power of string coupling depends on further details of the operator, as is clear from the analysis of [14].

Here, we will explore the idea that these operators can be obtained by expanding an action containing a DBI and WZ term [4]. This method for identifying operators has been used earlier in [16] for five dimensional black holes and in [6] for the 3-brane, where the action was expanded about flat space-time and shown to give consistent results. The present discussion will have two important new features. First, as we will discuss, it will be crucial to expand the action about AdS space, rather than flat space, to identify the operator correctly. In many ways, this is the natural thing to do. The supergravity modes we are interested in are perturbations about AdS space. Thus to couple them consistently one also expects

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[4]Strictly speaking we are proposing to expand the action which governs the dynamics of D3-branes. If additional terms besides the DBI and WZ terms are present in such an action, as has been suggested in [15], one would expect to keep them as well. We note here that the additional terms discussed in [15] are of dimension 12 and higher; such high dimension operators do not alter our conclusions.

to expand the brane action about the AdS background. Secondly, our analysis will involve the WZ term in an important way. In particular a cancellation, between two contributions , which arise from the DBI and WZ terms respectively, to the coupling of a dimension four operator, will be important in identifying the leading operator. We will see that this procedure yields a consistent result. In fact, the leading operator which couples to the mode of section 2 has dimension six. The supergravity analysis showed that it's total dimension is six as well. Thus, we find that the operator does not acquire any anomalous dimensions in the large N limit.

It is worth drawing attention to one other aspect of the analysis at the outset. The action we use can be identified with the world volume theory for a set of D3- brane probes. In the discussion below we will take these branes to be placed at the boundary of AdS space, at $r \gg R$. This is in line with the discussion of [3], [4]. In these references, the idea was to compute correlation functions by, roughly speaking, regarding boundary values of the supergravity fields as sources for the Yang Mills theory . This implies that the Yang Mills operators should also be identified by working at the boundary of AdS space.

The action for a D3-brane has been studied in [17], [18], and is given by [5]:

$$S = - \int d^4\xi \sqrt{-det(G_{ij} + \mathcal{F}_{ij})} + \int (\hat{C}_{(4)} + \mathcal{F} \wedge \hat{A} + \hat{C}_{(0)} \mathcal{F} \wedge \mathcal{F}). \tag{3.1}$$

The two terms above correspond to the DBI action and the WZ term respectively. It is worth defining the various terms above carefully. G_{ij} refers to the induced world-volume metric, obtained as the pull-back of the spacetime metric. Similarly,

$$\mathcal{F}_{ij} = F_{ij} - \hat{B}_{ij}, \tag{3.2}$$

where F_{ij} stands for the gauge field on the D3-brane and $\hat{B}_{ij}$ is the pullback of the NS-NS two form potential. In the W-Z term $\hat{C}_{(4)}$, $\hat{A}$ and $\hat{C}_{(0)}$ refer to the pullback of the R-R four form, two form and zero form fields respectively. Strictly speaking we are interested here in the action for N D3 branes. It has been suggested in [19] that this can be obtained by appropriately symmetrising the terms obtained from the single brane action. To begin with we will work with the abelian single brane action. The color factors will be introduced by appropriate symmetrisation towards the end. Some of our conclusions do not depend on the details of this symmetrisation procedure.

In the following discussion it is useful to distinguish between three kinds of indices: $\xi^i, i = 0, \cdots 3$, refer to the world-volume coordinates and are purely bosonic. Z^M, refers to ten dimensional superspace coordinates; M can stand for bosonic coordinates denoted by $m = 1, \cdots 10$, or for fermionic coordinates denoted by μ. Finally, we denote frame indices by

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[5]We have set the dilaton $e^\phi = 1$ and chosen units for the string tension so that the coefficient in front of the DBI term is unity.

A; A can stand for bosonic tangent vectors, denoted by $a = 1, \cdots 10$, or for spinor tangent vectors denoted by α. In this note we will be interested in expanding this action about AdS space to linear order in the perturbation, eq. (2.10), eq. (2.9). Following, [18] we will choose a static gauge, where $X^m = \xi^m, m = 0, \cdots 3$. The remaining $X^m, m = 4 \cdots 9$ will be denoted by ϕ^m. In addition the kappa symmetry of eq. (3.1) will be used to set half the fermionic coordinates to zero [6] [18]. The remaining 16 component Majorana-Weyl fermion will then be denoted by λ.

With this notation in hand the induced metric is given by

$$G_{ij} = \frac{\partial Z^M}{\partial \xi^i} \frac{\partial Z^N}{\partial \xi^j} \ E_M^A \ E_N^B \eta_{AB}. \tag{3.3}$$

with E_M^A being given by:

$$\begin{aligned}
E_m^a &= \frac{r}{R}\eta_m^a, m = 0, \cdots 3 \\
&= \frac{R}{r}\eta_m^a, m = 4, \cdots 9 \\
E_\mu^a &= (\bar{\lambda}\Gamma^a)_\mu \\
E_m^\alpha &= 0 \\
E_\mu^\alpha &= \delta_\mu^\alpha.
\end{aligned}$$

¿From eq. (3.3), (3.4) it then follows that the induced metric is given by [7]:

$$\begin{aligned}
G_{ij} &= \frac{r^2}{R^2}\eta_{ij} + \frac{R^2}{r^2}\frac{\partial \phi^m}{\partial \xi^i}\frac{\partial \phi^n}{\partial \xi^j}\eta_{mn} \\
&\quad - \left(\bar{\lambda}\ (\Gamma_i \frac{r}{R} + \Gamma_m \frac{\partial \phi^m}{\partial \xi^i}\frac{R}{r})\ \frac{\partial \lambda}{\partial \xi^j} + i \leftrightarrow j\right) + \sum_{a=0}^{9} \bar{\lambda}\Gamma^a \frac{\partial \lambda}{\partial \xi^i}\ \bar{\lambda}\Gamma_a \frac{\partial \lambda}{\partial \xi^j}. \tag{3.4}
\end{aligned}$$

Similarly, the pullback of the rank-2 tensor is defined as

$$B_{ij} = B_{AB}\partial_i Z^M \partial_j Z^N E_M^A E_N^B \tag{3.5}$$

and the superspace components of B_{AB} are given in [17]. Using these we get

$$\begin{aligned}
\mathcal{F}_{ij} &= F_{ij} - \left(\bar{\lambda}\ (\Gamma_i \frac{r}{R} + \Gamma_m \frac{\partial \phi^m}{\partial \xi^i}\frac{R}{r})\ \frac{\partial \lambda}{\partial \xi^j} - i \leftrightarrow j\right) - B_{ij} \\
&\quad + \frac{R}{r}\left(B_{ik}\ \bar{\lambda}\Gamma^k\partial_j \lambda - i \leftrightarrow j\right) - \frac{R^2}{r^2}(\bar{\lambda}\Gamma^k\partial_i\lambda\ \bar{\lambda}\Gamma^l\partial_j\lambda\ B_{kl}) \tag{3.6}
\end{aligned}$$

Adding eq. (3.4) and (3.6) we get that the DBI action is given by :

$$S_{DBI} = -\int d^4\xi\ (\frac{r}{R})^4\ \sqrt{-det(\eta_{ij} + M_{ij})} \tag{3.7}$$

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[6] Our conventions for spinors and Dirac matrices are the same as those in [18]. Namely, the Γ matrices are 32×32 real matrices satisfying, $\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$, with $\eta = (-1, 1, \cdots 1)$.

[7] To avoid confusion let us note that all indices on Gamma matrices here are frame indices.

where

$$
\begin{aligned}
M_{ij} &= \frac{R^4}{r^4}\partial_i\phi^m\partial_j\phi^n\eta_{nm} + \frac{R^2}{r^2}F_{ij} - 2\bar{\lambda}[\frac{R}{r}\Gamma_i + \frac{R^3}{r^3}\Gamma_m\partial_i\phi^m]\partial_j\lambda \\
&\quad + \frac{R^2}{r^2}\sum_{a=0}^{9}(\bar{\lambda}\Gamma^a\partial_i\lambda)(\bar{\lambda}\Gamma_a\partial_j\lambda) - \frac{R^2}{r^2}B_{ij} \\
&\quad + \frac{R^3}{r^3}[B_{ik}\bar{\lambda}\Gamma^k\partial_j\lambda - B_{jk}\bar{\lambda}\Gamma^k\partial_i\lambda] - \frac{R^4}{r^4}B_{kl}\,(\bar{\lambda}\Gamma^k\partial_i\lambda)\,(\bar{\lambda}\Gamma^l\partial_j\lambda).
\end{aligned}
\tag{3.8}
$$

This gives rise to the following terms in the DBI action linearly dependent on the NS-NS two form :

$$
\begin{aligned}
\int d^4\,\xi L_{BI} &= \int d^4\xi\left[-\frac{1}{2}B_{ij}F^{ji} + \frac{r}{R}\,\bar{\lambda}\,\Gamma_i\partial_j\lambda B^{ji}\right] - \frac{R}{r}\left[\bar{\lambda}\Gamma^m F_{mi}\partial_j\lambda B^{ij}\right] \\
&\quad - \left[\frac{1}{2}\frac{R^4}{r^4}F_{im}F^{mj}F_{jk}B^{ki}\right] + \frac{1}{2}\frac{R}{r}B_{ij}F^{ji}\bar{\lambda}\,\Gamma^l\partial_l\lambda \\
&\quad - \frac{1}{8}\frac{R^4}{r^4}F_{lm}F^{ml}F_{ij}B^{ij} + \cdots.
\end{aligned}
\tag{3.9}
$$

The ellipses above refer to operators of dimension eight and higher that occur in the expansion. Also, for the sake of brevity we have regrouped terms so that the indices above take values in ten dimensions and are raised and lowered by the flat space metric. Thus for example, F_{im} refers to the ten dimensional field strength [8].

The action described above is for a single D3-brane. While a definitive action is not known for N branes, we may adopt the symmetrized trace prescription of [19], so that the various quantities above have to be replaced by matrices and traced over. The first term in (3.9) then receives a contribution only from the $U(1)$ piece of $U(N)$ and is subdominant in the large N limit. Thus the lowest dimension operator which couples to B_{ij} in eq. (3.9) has dimension 4 and is given by the first term in eq. (3.9). The subsequent operators listed in eq. (3.9) all have dimension 6.

The contribution from the WZ term arises from the wedge product of $\mathcal{F}$ and $\hat{A}$ in eq. (3.1), which can be expanded as:

$$
L_{WZ} = \frac{1}{4}\mathcal{F}_{ij}\hat{A}_{kl}\epsilon^{ijkl}.
\tag{3.10}
$$

We remind the reader that in eq. (3.10) $\hat{A}_{kl}$ stands for the pullback of the R-R two form. This is given by:

$$
\hat{A}_{ij} = A_{ij} - \frac{R}{r}\left[\bar{\lambda}\Gamma^k\partial_j\lambda A_{ik} - i \leftrightarrow j\right] + \cdots
\tag{3.11}
$$

where the ellipses again denote higher dimensional operators. ¿From eq. (2.10) it follows that in the general case, for a perturbation B_{ij} polarised along the brane directions, the R-R

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[8]The last two terms in (3.9) were not included in the first version of this paper, we regret this error

two form is given by [9]:

$$A_{kl} = \frac{1}{2} \, \epsilon_{klij} \, B^{ij}. \tag{3.12}$$

Substituting eq. (3.11) and eq. (3.12) in eq. (3.10) then leads to :

$$L_{WZ} = -\frac{r}{R}\bar{\lambda}\Gamma^i\partial_j\lambda B^{ji} - \frac{R}{r}\bar{\lambda} \, \Gamma^m\partial_j\lambda \, F_{im} \, B^{ij} + \frac{1}{2}\frac{R}{r}F_{ij}B^{ij}\bar{\lambda}\Gamma^l\partial_l\lambda. \tag{3.13}$$

In obtaining eq. (3.13) we have used eq. (3.6) for $\mathcal{F}$ and expanded keeping only terms linear in B_{ij}. As in eq. (3.9), we have regrouped terms and expressed them in a notation where all indices take values in ten dimensions (the indices are raised and lowered by the flat space metric). Finally, for reasons mentioned above, we have omitted a term in eq. (3.13) which goes like $B_{ij}F^{ij}$.

On adding eq. (3.9) and (3.13) we now see that the dimension four operators that couple to B_{ij} do indeed cancel between the two equations [10]. Furthermore the two types of dimension six operators which involves the fermionic fields also cancel between (3.9) and (3.13). Thus the leading operators are of dimension six and of the form:

$$\mathcal{O}_6 = -(\frac{R}{r})^4 \, \left[\frac{1}{2} \, F_{jm}F^{mk}F_{ki}B^{ij} + \frac{1}{8}F_{lm}F^{ml}F_{ij}B^{ij}\right]. \tag{3.14}$$

While we have suppressed the color indices here, the operators in eq. (3.14) should be understood as being symmetrised in color space. Eq. (3.14) is the main result of this paper.

A few comments are in order at this stage.

Our starting point was the assumption that the coupling between various supergravity modes and operators in the Yang Mills theory can be obtained by expanding the action consisting of the DBI and W-Z terms about AdS space. The consistent answer obtained above provides evidence in support of this assumption. Note in particular that the analysis above probed terms in the action of dimension six and involved the W-Z term in a non-trivial way.

It was crucial in the above analysis to expand the action about AdS space. What would have happened if we had expanded about flat space? In this case, without any five -form field strength, the supergravity equations, eq. (2.5), would not have coupled the NS-NS and R-R two form gauge potentials together and would have lead to two massless modes. Correspondingly, on expanding the brane action for the coupling to the NS-NS two form, one would have only got a contribution of the form of eq. (3.8) from the DBI action with no

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[9]Here all indices are being summed using the flat space metric. Our conventions for the ϵ symbol are chosen so that , $\epsilon^{0123} = +1$.

[10] As was mentioned above, we have so far been suppressing color indices. For the dimension four operator involved here there is a unique color singlet that can be formed ; the cancellation then follows in the full Non- Abelian theory as well.

contribution from the WZ term. Thus the leading operator in the Yang Mills theory would have had dimension four which is different from what we got above.

It will be interesting to study this set of ideas further for the other supergravity modes as well. We have looked at the S-wave mode for the two form fields here. As was mentioned above, the propagation of higher partial waves was studied in [7]. One would like to determine the corresponding Yang Mills operators as well [11]. For one class of supergravity modes (including an $l = 1$ mode of the two form field) the coupling to the Yang Mills theory has been written down in [9] from superconformal considerations. It is worth examining if these couplings can be obtained by expanding the DBI plus WZ action.

4. Note Added in revised version :

While this revised version was being written [20] appeared, where a large class of gauge theory operators corresponding to supergravity modes, including the operator derived in this paper, have been shown to follow from the proposal of this work. This paper has also noted the omission of the second term in $\mathcal{O}_6$ in our earlier version.

5. Acknowledgement

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