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Interactions involving D-branes

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We investigate some aspects of the spectrum of D-branes and their interactions with closed strings. As argued earlier, a collection of many D-strings behaves, at large dilaton values, as a single multiply wound string. We use this result and T-duality transformations to show that a similar phenomenon occurs for effective strings produced by wrapping p-branes on a small (p-1)-dimensional torus, for suitable coupling. To understand the decay of an excited D-string at large dilaton values, we study the decay of an elementary string at small dilaton values. A long string, multiply wound on a circle, with a small excitation energy is found to predominantly decay into another string with the same winding number and an unwound closed string (rather than two wound strings). This decay amplitude agrees, under duality, with the decay amplitude computed using the Born-Infeld action for the D-string. We compute the absorption cross section for the D-brane model studied by Callan and Maldacena. The absorption cross section for the dilaton equals that for the scalars obtained by reduction of the graviton, and both agree with the cross section expected from a classical hole with the same charges.

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1. Introduction

We have learnt over time that there exists a large class of solitonic objects in string theory which are essential to give the theory its dual nature. A subset of these known as D-branes [1], carrying Ramond-Ramond charges, can be studied through open strings that have Dirichlet boundary conditions on hypersurfaces in spacetime which represent the locations of these extended solitons. Such open strings represent the possible excitations of the soliton, in particular the collective modes of its low energy deformations. The low energy field theory of these open strings is the gauge field theory described by open strings, dimensionally reduced to the worldvolume of the D-brane.

Thus each D-brane carries in particular a $U(1)$ gauge field on its surface. Witten [2] conjectured that when two D-branes approach each other, there is an enhancement of symmetry from $U(1) \times U(1)$ to $SU(2)$, due to open strings that can stretch from one brane to the other. More generally, with N D-branes close to each other, the symmetry is enhanced to $U(N)$.

This is a valid picture at weak elementary string coupling. Let the coupling constant of closed string theory be $g = e^{\phi}$. The tension of D-branes is proportianal to $1/g$, so at small g the D-branes are heavy, and can be well localised in space, with the velocities from quantum fluctuations being small. In this case one can use the approximation that the D-branes are at fixed locations in space, and as close to each other as we wish. The ends of the open string can lie on any one of the N branes, and thus we associate a Chan-Paton factor taking N values with each end of the open string. This gives rise to open strings describing a $U(N)$ symmetry group. In such calculations one usually assumes that the D-branes are infinitley long along the directions which lie within the brane surface.

Some of the important physical applications of D-branes require, however, an understanding of these objects at strong coupling and wound around compact directions. One such application involves regarding configurations of D-branes as black branes or black holes. So long as the branes are extremal - corresponding to BPS states - it is possible to make exact statements of e.g. the degeneracy of states leading to an understanding of the Hawking-Beckenstein entropy [3][4]. However if we are interested in the absorption of matter leading to the formation of nonextremal states or in the decay of a non-extremal state leading to Hawking radiation one is forced to consider the physics at non-infinitesimal coupling. This is because at weak coupling the 'thickness' of the soliton will be larger than the Schwarzschild radius of the soliton, and one will be studying excitations and de-excitations of an ordinary soliton (rather than a black hole) when one studies interactions of closed strings with open strings that are attached to the soliton.

However, it was shown in [5] that at sufficiently strong coupling the nature of the excitation spectrum for D-strings is not that suggested by the above description of Chan-Paton factors. Consider the Type IIB theory on $M^9 \times S^1$, with the compactified direction having length L. Take a collection of D-strings wound around this compact direction having a total RR charge equal to n_w. By S-duality of the Type IIB theory such D-strings at strong coupling should behave exactly like a macroscopic elementary string at weak coupling. The normalisable state of the latter is however known to be a string wound n_w times so that in particular its excitations live on a circle of size $n_w L$. Thus the D-string with RR charge n_w is represented at strong coupling by a single string wound n_w times rather than n_w singly wound D-strings. If we bring n_w D-strings close to each other to form a bound state they would join to form a single multiply wound string such that the low energy excitations are collective modes of a long string of length $n_w L$.

This fact becomes especially important for excited D-brane configurations which represent nonextremal black branes with nonzero horizon area in the extremal limit, e.g. D-strings bound to 5 D-branes, as considered in [3] and [6]. This configuration corresponds to a five dimensional black hole. The degeneracy of states with given charges is in exact agreement with the Hawking-Beckenstein formula, both for extremal [3] and slightly nonextremal holes [7], thus realizing the program of [8] and [9].

It was argued in [10] that the thermodynamics of open string states used in [6] to obtain the extremal and nonextremal entropies is correct for the "fat" black hole limit only when one considers the branes as single branes which are multiply wrapped around the compact direction. A related phenomenon was found by [11] where a D-string lies within a collection of Q_5 parallel close by 5-D-branes. It was argued that the excitation spectrum of the D-string equals that of a single long string with tension $1/Q_5$ and total length $n_w Q_5$, where n_w is the RR charge of the D-string.

In this paper we do the following:

(a) We investigate whether higher dimensional branes also share this property of D-strings that the bound state of a collection of them behaves as if it were one 'long' brane instead of a collection of closely spaced parallel branes. By using T-dulaities we relate the D-string spectrum to the spectrum of $(n_c + 1)$-D-branes wrapped on an n_c dimensional torus. The size of this torus is small, so we obtain an effective string in the remaining directions, with this effective string turning out to behave as one long string for a suitable range of parameters like coupling and size of wrapping torus. In particular for a bound state of n_w 5-D-branes, even at $g << 1$, the excitations are not those suggested by naive Chan- Paton factors at the ends of open strings, if the dimensions of the wrapping torus are less than $g\sqrt{\alpha'}$ – the effective string behaves as a long string of length n_w times the length of the direction on which the effective string is wrapped.

(b) We wish to investigate the amplitudes for multiply wound D-strings to absorb and emit massless quanta. If we had only one D-string (RR charge n_w equal to unity) then we can study its interactions using open strings (attached to the D-string) and closed strings (travelling througout spacetime) by an examination of elementary string diagrams. This is less clear when there are many D-strings close to each other and the elementary string interaction g is strong enough to invalidate a naive description where we only add Chan-Paton factors to the ends of the open string. But in the large g limit we can make use of the duality with the elementary string, whose interaction cross sections we do know how to compute.

We take an initial state of the closed elementary string to be have winding number n_w around the compact direction, together with a simple choice of oscillator excitation. Such an excited state can decay in two possible ways. In the first the closed string splits into two closed strings with winding numbers $n_w^{(1)} > 0$ and $n_w^{(2)} > 0$ ($n_w^{(1)} + n_w^{(2)} = n_w$) and no oscillator excitations; the relative momentum of the decay products carries the initial energy of excitation. In the second mode we just get a string with winding number n_w with no oscillator excitation, and the energy is carried away by a massless graviton (winding number zero). The second process is found to have a much larger amplitude if $n_w L >> \sqrt{\alpha'}$, and the excitation energy is small compared to the rest mass of initial wound string.

(c) We use the Born-Infeld action for the D-string to compute the amplitude for a long wavlength vibration on a D-string to decay by emitting a graviton or a dilaton. This amplitude agrees under duality with that found in the corresponding process for the elementary string in (b) above.

(d) The results of (b) and (c) above provide some justification for an assumption used in [12] for computing the emission from a 5-brane and a collection of D-strings [6], which was compared with black hole emission. The assumption used was that the D-string excitations behave as the excitations of one long string, and that the decay amplitude for low energy emission is provided by the BI action for the string. We calculate here the cross section for absorption of low energy scalars into the 5-brane-1-brane model, and obtain (as expected) the same result as found through computation of emission in [12] . The dilaton is found to have the same absorption cross section as the scalars coming from the Kaluza-klein reduction of the graviton.

2. Excitation spectrum of a multiply wound string.

Let us review the argument of [5]. Consider a type IIB elementary string, for the sake of definiteness. We compactify the direction X^9 on a circle of length L. To lowest order in coupling, the mass of an elementary string state is given by

$$\begin{aligned} m^2 &= (n_w L T^{(S)} + \frac{2\pi n_p}{L})^2 + 8\pi T^{(S)}(N_R - \delta_R) \\ &= (n_w L T^{(S)} - \frac{2\pi n_p}{L})^2 + 8\pi T^{(S)}(N_L - \delta_L) \end{aligned} \tag{2.1}$$

Here $T^{(S)} = Te^{\frac{\phi_0}{2}}$ is the tension of the elementary string. n_w, n_p are integers giving the winding and momentum in the X^9 direction. $\delta_{L,R} = 0, 1/2$ for the Ramond and Neveu-Schwarz sectors respectively.

Let us consider very long strings, so that $L\sqrt{T^{(S)}} >> 1$. Consider the lowest excitation that has no net momentum: $n_p = 0$. Thus $(N_R - \delta_R) = (N_L - \delta_L) = 1$. An "unexcited" string has winding number n_w, no momentum (thus in particular $n_p = 0$), and $(N_R - \delta_R) = (N_L - \delta_L) = 0$. The energy of the excitation over the energy of this unexcited state is

$$\delta m = \sqrt{(n_w T^{(S)} L)^2 + 8\pi T^{(S)}} - (n_w T^{(S)} L) \approx \frac{4\pi}{n_w L} \tag{2.2}$$

This result corresponds to the transverse vibrations of a string of length $n_w L$. But this result, valid for $g \to 0$, must hold, by duality, also for the D-string in the limit $g \to \infty$. Thus if we have a threshold bound state of n_w D-strings, at large elementary string coupling ($g \to \infty$), then the excitation spectrum of this state must correspond to that of one long string of length $n_w L$. Indeed it was shown in [5] that the ensemble of such open strings with fractional momenta, but with a total momentum which is integer (in units of $2\pi/L$) leads to an entropy which agrees with that of the elementary string spectrum.

This excitation spectrum is to be contrasted with the spectrum obtained by attaching Chan-Paton factors to the ends of the open strings that give excitations of the D-string. This would represent closely spaced but separately wound D-strings. The lowest energy of such an open string is $\frac{2\pi}{L}$, and the lowest excitation that has vanishing total momentum is $\frac{4\pi}{L}$ (one open string travelling in each direction on the D-string). The Chan-Paton factors give this energy level a multiplicity of n_w^2. It is possible that this is the correct representation of the D-string spectrum at $g \to 0$; in that case as g increases we must have a splitting of degenerate levels to a set of levels that correspond to one long string.

3. Other branes and duality

As observed above the multiply wound D-string, at large g, has the excitation spectrum of one long string rather than many singly wound strings. It would be good to see an analogue of this for higher branes, and in fact for various combinations of branes. Here we provide a modest result in this direction.

We will start with a D-string wound around a compact direction, and further assume that a certain number n_c of additional directions are compactified on circles. We choose the coupling and lengths of compact directions such that we know that the spectrum of the D-string is that of one long string. Now we T-dualise in the n_c compact directions, thus creating a (n_c+1)-D-brane from each D-string. The energy spectrum of excitations will of course remain the same. Note that the coupling would change under the T-duality, and in fact the (n_c+1)-D-branes obtained will have all the n_c compact directions very small, so we would have obtained an effective string, made from wrapping (n_c+1)-D-branes on n_c small compact directions. It is for these effective strings that we would have established that the excitation spectrum is that of a single long string.

Note that

$$T^{(D)} = T^{(S)} g^{-1}$$

We define also the length scale associated with these tensions

$$L^{(D)} = L^{(S)} g^{1/2}$$

where $L^{(S)}$ is such that under T-duality a circle of length $\lambda L^{(S)}$ goes to a circle with length $\lambda^{-1} L^{(S)}$.

We start with the following fact. Take the elementary string, at $g << 1$, wrapped on a circle of length L (direction X^9) that is order $L^{(S)}$ or larger, and with any other compact dimensions also having length L_c of order $L^{(S)}$ or larger:

$$L = AL^{(S)}, \qquad A > 1$$

$$L_c = BL^{(S)}, \qquad B > 1$$

Such a string with winding number n_w has an excitation spectrum of one long string. Here the restrictions on the lengths of compact directions are imposed because if some direction becomes sufficiently small, the one loop corrections to the mass of the elementary string can become large, even for small g. This is because the light states of the string wrapping around that compact direction propagate in higher loop string diagrams. (Such higher loop corrections would of course be exactly zero if $g = 0$, but for the dualities that we are about to use we need to start with nonzero g.)

The S-dual of this configuration is a D-string multiply wrapped in the X^9 direction, with X^9 compactified on a circle of length

$$L = AL^{(D)} = AL^{(S)}g^{1/2}$$

(with $A > 1$ as above). The other n_c compact directions are $X^{9-n_c}, \ldots, X^8$. These are on circles of length

$$L_c = BL^{(D)} = BL^{(S)}g^{1/2}$$

(with $B > 1$ as above). The dual coupling is $g_D = g^{-1}$. We take $g^D << 1$, which means $g >> 1$. Then the spectrum is that of a single long D-string, with energy threshold

$$E_T = \frac{4\pi}{An_w L^{(D)}}$$

Under a T-duality, the coupling changes to

$$g' = [L'/L]^{1/2} g$$

If we T-dualise all the n_c directions above, then length of these compact directions and the value of the coupling change

$$L'_c = B^{-1} L^{(S)} g^{-1/2}$$

$$g' = gB^{-n_c} g^{-n_c/2} = B^{-n_c} g^{1-n_c/2}$$

Since $g >> 1$, and $B > 1$, the new lengths of the n_c compact directions are much smaller than the string length $L^{(S)}$. The resulting branes describe an effective string extending in the X^9 direction, with a tension (mass per unit length) given by

$$T^{(M)} = T^{(S)} g'^{-1} (\frac{L'_c}{L^{(S)}})^{n_c} = T^{(S)} g^{-1} = T^{(D)'} B^{-n_c} g^{-n_c/2}$$

The length of this effective string is

$$L = AL^{(D)} = AL^{(D)'}(g'/g)^{-1/2} = AL^{(D)'}B^{n_c/2}g^{n_c/4} = AL^{(M)}$$

where $L^{(M)} = L^{(S)}g^{1/2}$ is the length scale associated with the tension $T^{(M)}$. Thus the length of this effective string is $A > 1$ times the length scale set by its own effective tension, just like the D-string that we started with. Note that

$$\frac{L'_c}{L^{(M)}} \;=\; B^{-1}g^{-1} \;<<\; 1 \tag{3.1}$$

so that the compactification torus for the branes is indeed much smaller than the length scale defined by the tension of the effective string, thus justifying the statement that we have obtained an effective string rather than a higher dimensional brane.

As a paricular example take $n_c = 4$, so that we get a 5-D-brane wrapped on a small T^4, giving an effective string in the remaining directions that is magnetically charged under the RR gauge field. (The D-string was electrically charged under this gauge field.) The new coupling is

$$g' = B^{-4}g^{-1}$$

Since $B > 1$ and $g >> 1$, we have $g' << 1$. Thus at weak elementary string coupling g', suppose we take n_w 5-D-branes, and wrap 4 directions on circles of length $L'_c = B^{-1}L^{(S)}g^{-1/2}$. Then (even though the elementary string coupling is small) we will find that the excitations are not given by open strings moving in an X^9-direction box of length L; instead they are given by open strings moving in an X^9-direction box of length n_wL.

Note that by these duality arguments we have not been able to say anything about the excitation spectrum of 5-D-branes with all dimensions of brane large, and $g >> 1$. In this latter case we do not expect the spectrum of a 1-dimensional object, so duality arguments starting from the elementary string are unlikely to access this domain.

4. Decay amplitudes for the elementary string

Let the spacetime be $M^9 \times S^1$ with X^9 compactified on a circle of length L. Consider an elementary string state with winding number $n_w^{(1)}$ around X^9, zero momentum along X^9 as well as all other directions, and an excited state of oscillators. There could be two possible channels for the decay of this excited state:

(a) The initial excited state may decay into the 'ground state' with the same winding number $n_w^{(1)}$, while emitting a closed string state with zero winding.

(b) The initial state can decay into two closed strings each with nonzero winding number (say $n_w^{(2)}, n_w^{(3)}$, with $n_w^{(2)} + n_w^{(3)} = n_w^{(1)}$). If the initial excitation had the lowest allowed energy, then these final states will have no oscillator excitations; the initial oscillator energy will be manifested as the energy of relative motion of the two final strings.

We would like to know which of these decay modes dominates. By duality this will tell us the dominant decay mode of a multiply wound D-string, at $g = e^{\phi} >> 1$.

We use the NSR formalism, and investigate the decay of a particular class of initial excitations. The results should indicate the physics for an arbitrary excitation.

The fields on the closed string world sheet have the mode expansions [13]

$$X_L^\mu = \tfrac{1}{2}[x^\mu + \frac{1}{\pi T^{(S)}} p_L^\mu(\tau - \sigma) + \frac{i}{\sqrt{\pi T^{(S)}}} \sum_{n \neq 0} \frac{\alpha_n^\mu}{n} e^{-2in(\tau-\sigma)}] \tag{4.1}$$

and similarly for X_R^μ (with $\tau - \sigma \to \tau + \sigma$ and $\alpha \to \tilde{\alpha}$). For the fermion fields we have (for the NS sector)

$$\psi_-^\mu = \frac{1}{\sqrt{\pi T^{(S)}}} \sum_{r = Z + \frac{1}{2}} \psi_r^\mu e^{-2ir(\tau-\sigma)} \tag{4.2}$$

and similarly for ψ_+^μ (with $\tau - \sigma \to \tau + \sigma$ and $b \to \tilde{b}$).

We take the initial excited state to be

$$|I >= \eta_r \tilde{\eta}_{r'} \epsilon^{(1)}_{aa'} \frac{\alpha^r_{-N}}{\sqrt{N}} \psi^a_{-1/2} \frac{\tilde{\alpha}^{r'}_{-N}}{\sqrt{N}} \tilde{\psi}^{a'}_{-1/2} |k_{1L}, k_{1R} > \tag{4.3}$$

This state has $N_L = N_R = N$. The final state is taken to be

$$|F >= \epsilon^{(3)}_{cc'} \psi^c_{-1/2} \tilde{\psi}^{c'}_{-1/2} |k_{3L}, k_{3R} > \tag{4.4}$$

The vertex operator for the state which is emitted is

$$V(u,v) = \epsilon^{(2)}_{bb'} e^{ik_{2L}X_L} \; e^{ik_{2R}X_R} \; [\partial_u X^b + \tfrac{1}{2}(k_{2L} \cdot \psi)\psi^b] \; [\partial_v X^{b'} + \tfrac{1}{2}(k_{2R} \cdot \tilde{\psi})\tilde{\psi}^{b'}] \tag{4.5}$$

where (u, v) are the related to the coordinates on the cylinder by $u = \tau + \sigma$ and $v = \tau - \sigma$.

In the above the polarisations are normalised by

$$\epsilon^{(i)}_{ab} \epsilon^{(i)ab} = 1, \quad \eta_r \eta^r = 1, \quad \tilde{\eta}_r \tilde{\eta}^r = 1 \tag{4.6}$$

To lowest order in the elementary string coupling g the decay amplitude is then given by

$$\mathcal{A} = 8(\pi T^{(S)})^2 \kappa < F|V(0,0)|I > \tag{4.7}$$

and may be easily evaluated to be

$$\begin{aligned}\mathcal{A} =& 8(\pi T^{(S)})^2 \kappa \epsilon^{(1)}_{aa'} \epsilon^{(2)}_{bb'} \epsilon^{(3)}_{cc'} \eta_r \eta_{r'} \\ & [\frac{\sqrt{N}}{\sqrt{\pi T^{(S)}}} \delta_{ac}\delta_{br} - \frac{1}{2\pi T^{(S)}} \frac{k^r_{2L}}{2\sqrt{\pi T^{(S)}}\sqrt{N}} (k^b_{1L}\delta_{ac} + k^c_{2L}\delta_{ab} - k^a_{2L}\delta_{bc})] \\ & [\frac{\sqrt{N}}{\sqrt{\pi T^{(S)}}} \delta_{a'c'}\delta_{b'r'} - \frac{1}{2\pi T^{(S)}} \frac{k^{r'}_{2R}}{2\sqrt{\pi T^{(S)}}\sqrt{N}} (k^{b'}_{1R}\delta_{a'c'} + k^{c'}_{2R}\delta_{a'b'} - k^{a'}_{2R}\delta_{b'c'})]\end{aligned} \tag{4.8}$$

The overall coefficient in (4.7) has been fixed by comparing with the three graviton vertex which follows from the Einstein action in the following way. The Einstein action for the traceless components of the metric in the harmonic gauge becomes, upto terms with three gravitons ($g_{ab} = \eta_{ab} + h_{ab}$)

$$S = \frac{1}{8\kappa^2}[(h_{ab,c}h^{ab,c}) - (h_{ab,c}h^{ab}_{,d}h^{cd} + 2h_{ab,c}h^{c,b}_{d}h^{ad})] \tag{4.9}$$

Consider the amplitude for a process where a graviton h_{12} with momentum k_1 goes into a graviton h_{12} with momentum k_3 and a graviton h_{34} with momentum k_2. The tree level answer may be easily computed from (4.9). To do this we have to remember that we have to use fields which are properly normalized (i.e. have the standard kinetic energy term). In particular the off diagonal metric components have the standard kinetic term after the rescaling $h_{12} \to \sqrt{2}\kappa h_{12}$ etc. The result for this process is

$$A = \frac{4\kappa}{2\sqrt{2}}(k^3_1 k^4_3 + k^4_1 k^3_3) \tag{4.10}$$

This has to be compared with the string theory answer for the three graviton vertex with polarizations $\epsilon^{ab}_i = \epsilon^{ba}_i$ with $i = 1, \cdots 3$ and the only nonzero components being $\epsilon^{12}_1 = \epsilon^{21}_1 = \epsilon^{34}_2 = \epsilon^{43}_2 = \epsilon^{12}_3 = \epsilon^{21}_3 = \frac{1}{\sqrt{2}}$. The string theory answer (see e.g. [13]) can then be seen to agree with the Einstein gravity answer with the normalization given in (4.7).

In the following we concentrate on the case where the emitted state has zero momentum in the string direction X^9. Then in the rest frame of the initial elementary string the various momenta are

$$\begin{aligned} k_{1L} &= (k^0_1, \vec{0}, n^{(1)}_w L T^{(S)}) & k_{1R} &= (k^0_1, \vec{0}, -n^{(1)}_w L T^{(S)}) \\ k_{2L} &= (k^0_2, \vec{k}_2, n^{(2)}_w L T^{(S)}) & k_{2R} &= (k^0_2, \vec{k}_2, -n^{(2)}_w L T^{(S)}) \\ k_{3L} &= (k^0_3, \vec{k}_3, n^{(3)}_w L T^{(S)}) & k_{3R} &= (k^0_3, \vec{k}_3, -n^{(3)}_w L T^{(S)}) \end{aligned} \tag{4.11}$$

with $\vec{k}_2 + \vec{k}_3 = 0$. If the emitted state is a closed string with no winding one has to set $n_w^{(2)} = 0$. In (4.11) k_i^0 stands for the on-shell values

$$\begin{aligned} k_1^0 &= \sqrt{(T^{(S)} L n_w^{(1)})^2 + 8\pi T^{(S)} N} \\ k_2^{(0)} &= \sqrt{(T^{(S)} L n_w^{(2)})^2 + \vec{k}_2^2} \\ k_3^{(0)} &= \sqrt{(T^{(S)} L n_w^{(3)})^2 + \vec{k}_3^2} \end{aligned} \tag{4.12}$$

An interesting feature of the amplitude is that in the special case where all the polarizations are transverse to the string direction X^9, $\mathcal{A}$ is independent of the values of $n_w^{(i)}$. In particular it does not depend on whether the emitted state is a wound string or a graviton like state. In the decay rate all such differences would arise from the normalizations of the states (which involve k_i^0 and hence the $n_w^{(i)}$) and phase space factors.

It is now easy to see why for $L >> \sqrt{\alpha'}$ the decay into two wound strings is suppressed relative to decay into a wound string and a massless state. The factor in the decay rate coming from the normalizations of the states is

$$\mathcal{N} = [(2k_1^0 V)(2k_2^0 V)(2k_3^0 V)]^{-1} \tag{4.13}$$

Consider the case where the transverse momenta $\vec{k}_2$ are small compared to $T^{(S)}L$. Then for emission of a wound state one has $k_i^0 \sim n_w^{(i)} L T^{(S)}$ for all i so that the factor (4.13) is

$$\mathcal{N} \approx [8 n_w^{(1)} n_w^{(2)} n_w^{(3)} (L T^{(S)})^3 \; V^3]^{-1} \tag{4.14}$$

On the other hand for the emission of a massless state one has $n_w^{(2)} = 0$ and $k_2^0 = |\vec{k}_2|$ so that the factor from normalization is

$$\mathcal{N} \approx [8 n_w^{(1)} n_w^{(3)} |\vec{k}_2| (L T^{(S)})^2 \; V^3]^{-1} \tag{4.15}$$

Thus emission of a state with nonzero winding is suppressed by a factor of order $\sim |k_2|/(LT^{(S)})$ compared to the emission of a graviton with no winding.

At very low energies, only the first terms in (4.8) contribute. These terms have the feature that the polarizations of the initial and final states, $\epsilon^{(1)}, \epsilon^{(3)}$ do not affect the polarization of the emitted state. In particular when the polarization of the macroscopic string state is unchanged by the emission we have the particularly simple answer at low energies

$$8(\pi T^{(S)})^2 \frac{\kappa N}{\pi T^{(S)}} \; \epsilon^{(2)}_{rr'} \eta^r \tilde{\eta}^{r'} \tag{4.16}$$

Thus the amplitude for an excitation with left and right polarisations i, j, to emit the graviton h_{12}, is

$$8(\pi T^{(S)})^2 \frac{\kappa N}{\pi T^{(S)}} \frac{1}{\sqrt{2}} \tag{4.17}$$

where we have used that $\epsilon_{12} = \epsilon_{21} = 1/\sqrt{2}$.

It may be also checked that for small k_2 the leading contribution at a given oscillator level comes from the type of state considered above, rather than states of the same oscillator level obtained by applying multiple creation operators like $\prod_i \alpha_{-m_i}|0>$ with $\sum_i m_i = N$. This is because for such states with $i > 1$ there cannot be any term in the amplitude which is independent of k_2.

We conclude that for $g = e^{\phi} >> 1$ a multiply wound D-string would preferentially decay by emitting a massless closed string states like a graviton rather than split into pairs of strings each with nonzero winding. Thus it makes sense to study the decay of a nonextremal D-string into gravitons and examine to what extent this resembles Hawking radiation.

We can also compute the amplitude for the excited string to emit a dilaton. The polarisation tensor is[14]

$$\epsilon_{\mu\nu} \;=\; \frac{1}{\sqrt{8}}[\eta_{\mu\nu} - l_\mu k_\nu - l_\nu k_\mu] \tag{4.18}$$

where k_μ is the momentum of the dilaton, and l_μ is any null vector satisfying $k_\mu l^\mu = -1$.

Then we see that if we have an excitation of the initial string with polarisations i on each of the left and right sides, then the amplitude to emit a dilaton is

$$8(\pi T^{(S)})^2 \frac{\kappa N}{\pi T^{(S)}} \frac{1}{\sqrt{8}} \tag{4.19}$$

Note that this amplitude is 1/2 times the amplitude found for the graviton emission above.

We close this section by noting that the emission of quanta of the axion field $B_{\mu\nu}$ is as likely as the emission of gravitons. In fact the emission is to coherent superpositions of the graviton and the axion; the emitted quanta from the excited string state (4.3) are of the form $\frac{1}{\sqrt{2}}[h_{12} + B_{12}]$. The vertex operator for emission of such a quantum is given by with $\epsilon_{12} = 1, \epsilon_{21} = 0$. (Thus $\epsilon_{ab}\epsilon^{ab} = 1$ as before.) The amplitude for decay to this mode is thus $\sqrt{2}$ higher than the decay to the graviton itself, and thus the probablility of decay per unit time is 2 times the decay rate to the graviton. Since the decay rate to axions equals that to gravitons, we find that the total decay rate obtained is the same whether we use the graviton and axion as our fields or if we use $\frac{1}{\sqrt{2}}[h_{\mu\nu} \pm B_{\mu\nu}]$ as our fields.

5. D-String amplitudes

We now compare the elementary string amplitude derived in the previous section with amplitude for emission of a massless closed string state from an excited D-string at low energies. The amplitude for such a process has been computed in [15]. For our purposes it is most efficient to use the Born-Infeld action to derive the result.

The DBI action which describe the low energy dynamics of a D-string may be written in terms of the coordinates of the D-string $X^\mu(\xi^m)$ (where μ runs over all the 10 indices whereas ξ^m are parameters on the D-string worldsheet) and the gauge fields on the D-string worldsheet $A^m(\xi^m)$ as follows [16]

$$S_{BI} = T \int d^2\xi \; e^{-\phi(X)} \sqrt{\det[G^{(S)}_{mn}(X) + B_{mn}(X) + 2\pi\alpha' F_{mn}]} \tag{5.1}$$

where F_{mn} denotes the gauge field strength on the D-string worldsheet and $G^{(S)}_{mn}$ and B_{mn} are given by

$$G^{(S)}_{mn} = G^{(S)}_{\mu\nu}(X)\partial_m X^\mu \partial_n X^\nu \qquad\qquad B_{mn} = B_{\mu\nu}(X)\partial_m X^\mu \partial_n X^\nu \tag{5.2}$$

Here $G^{(S)}_{\mu\nu}$ is the target space metric in the string frame, $B_{\mu\nu}$ is the antisymmetric tensor field, and T is a tension related to the tension $T^{(S)}$ of the fundamental string through $T = T^{(S)} e^{-\phi/2}$.

We will concentrate on the coupling of the D-string to gravitons and the dilaton, and so ignore the B field and the field strength F for the following calculation. As mentioned at the end of the previous section, the B field couples to the D-string as efficiently as the gravitons, but we may separate gravity from the B field for convenience, which is what we will do below. We also shift to the Einstein metric in the following, given by $G_{\mu\nu} = e^{-\phi/2} G^{(S)}_{\mu\nu}$. Then the DBI action may be written as

$$S_{BI} = T \int d^2\xi \; e^{-\phi(X)/2} \sqrt{\det[G_{mn}(X)]} \tag{5.3}$$

The bulk action is

$$S_{\rm bulk} \;=\; \frac{1}{2\kappa^2} \int d^{10}x \sqrt{G} [R \;-\; \frac{1}{2}\partial\phi\partial\phi \;+\; \ldots..] \tag{5.4}$$

where we write only the terms that we shall need.

We will work in the static gauge which means

$$X^0 = \xi^0 \qquad\qquad X^9 = \xi^1 \tag{5.5}$$

The D-string worldsheet is then the X^0, X^9 plane. In this gauge the massless open string fields which denote the low energy excitations of the brane are the transverse coordinates $X^i(\xi^0, \xi^1), \quad i = 1, \cdots 8$.

In the following we will set the gauge field and the RR field to be zero. The lowest order interaction between the metric fluctuations around flat space and the open string modes is obtained by expanding the metric as $G_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(X)$, expanding the transverse coordinates $X^i(\xi)$ around the brane position $X^i = 0$ and treating $h_{\mu\nu}$ and X^i to be small. One then has

$$\begin{aligned} g_{mn} &= [\eta_{mn}] \ + \ [h_{mn}] \ + \ [X^i_{,m}X^j_{,n}\eta_{ij} + X^i_{,m}h_{in} + X^j_{,n}h_{mj}] \ + \ [X^i_{,m}X^j_{,n}h_{ij}] \\ &\equiv [\eta_{mn}] \ + \ g^{(1)}_{mn} \ + \ g^{(2)}_{mn} \ + \ g^{(3)}_{mn} \end{aligned} \tag{5.6}$$

The above relation is exact, but we have grouped terms according to the order of smallness, assuming that in later use we will treat X^i, h_{ij} as being small. We have

$$\det(G) \ = \ -\det[1 + C] \tag{5.7}$$

where

$$C = \eta^{-1}[g^{(1)}_{mn} \ + \ g^{(2)}_{mn} \ + \ g^{(3)}_{mn}] \tag{5.8}$$

Note that for a 2×2 matrix C,

$$\det[1 + C] = 1 + trC + \det C \tag{5.9}$$

Then upto terms involving two open string fields we have

$$\begin{aligned} &\frac{1}{2}[\delta_{ij} + h_{ij} - \frac{1}{2}h^\alpha_\alpha\delta_{ij}]\partial_\alpha X^i\partial^\alpha X^j \\ &+ \frac{1}{2}(\partial_k h^\alpha_\alpha)X^k + \frac{1}{4}(\partial_k\partial_i h^\alpha_\alpha)X^iX^k + h_{i\alpha}\partial^\alpha X^i + (\partial_j h_{i\alpha})X^j\partial^\alpha X^i \\ &- \frac{1}{2}\delta_{ij}(h_{00}\partial_1 X^i\partial_1 X^j + h_{11}\partial_0 X^i\partial_0 X^j - 2h_{10}\partial_0 X^i\partial_1 X^j) \end{aligned} \tag{5.10}$$

For purely transverse gravitons, i.e. only $h_{ij} \neq 0$ this simplifies to

$$\frac{1}{2}(\delta_{ij} + h_{ij})\partial_\alpha X^i\partial^\alpha X^j \tag{5.11}$$

Consider a D-string which is excited above the BPS state by addition of a pair of open string states with momenta (on the worldsheet) (p_0, p_1) and (q_0, q_1) respectively. These open strings are to be identified with quanta of the variable X^i in the above BI action. Note that the field with a standard kinetic term is not X^i but

$$\tilde{X}^i \;=\; \sqrt{T} X^i \tag{5.12}$$

The polarizations of the open string states are chosen to be transverse. The decay of this state into the extremal state is given by the process of annihilation of this pair into a closed string state, like a graviton. For a graviton which is also transversely polarized to the D-string with momentum $(k_0, k_1, \vec{k})$ (where $\vec{k}$ denotes the momentum in the transverse direction), the leading term for this amplitude for low graviton energies can be read off from (5.11) as

$$\mathcal{A}_D = \lambda^i_{(1)} \lambda^j_{(2)} \epsilon_{ij} p \cdot q \tag{5.13}$$

where the polarisations are normalised as

$$\epsilon_{ij}\epsilon^{ij} = 1, \qquad \lambda_{(1)i}\lambda^i_{(1)} = 1, \qquad \lambda_{(2)i}\lambda^i_{(2)} = 1 \tag{5.14}$$

When the outgoing graviton does not have any momentum along the string direction one has

$$p \cdot q = p^0 q^0 - p^1 q^1 = 2|p_1|^2 \tag{5.15}$$

where we have used momentum conservation in the string direction and the masslessness of the modes.

Writing this another way, the amplitude per unit time for a pair of open strings with equal and opposite momenta to collide and emit a graviton is

$$R_h \;= \sqrt{2}\kappa(2|p_1|^2)\frac{1}{\sqrt{2|p_1|}}\frac{1}{\sqrt{L}}\frac{1}{\sqrt{2|p_1|}}\frac{1}{\sqrt{L}}L\frac{1}{\sqrt{2\omega_h}}\frac{1}{\sqrt{V}} \;=\; \sqrt{2}\kappa|p_1|\frac{1}{\sqrt{2\omega_h}}\frac{1}{\sqrt{V}} \tag{5.16}$$

Here the first two factors come from the fact that the field with standard kinetic term corresponding to say the graviton h_{12} is $(2\kappa)\frac{1}{\sqrt{2}}h_{12}$. The last factors are from the normalisations present in the fields and the volume of the interaction region L. (L is the total length of the D-string.)

Let us also compute the amplitude for the open strings to collide and emit a dilaton quantum. From the action (5.3), the relevant contribution is

$$S_{BI} \;\rightarrow\; -\frac{\phi}{2}\; T\; \frac{1}{2}\; \delta_{ij}\partial_\alpha X^i \partial^\alpha X^j \tag{5.17}$$

¿From the bulk action (5.4) we note that the field corresponding to the dilaton with correctly normalised kinetic term is

$$\tilde{\phi} \;=\; \frac{\phi}{\sqrt{2}\kappa} \tag{5.18}$$

Then we find that the amplitude for two open strings (both with the same polarisation but with equal and oposite momenta) to collide and emit a dilaton is

$$\begin{aligned} R_d \;&=\; (\sqrt{2}\kappa)\frac{1}{4}2(2|p_1|^2)\frac{1}{\sqrt{2|p_1|}}\frac{1}{\sqrt{L}}\frac{1}{\sqrt{2|p_1|}}\frac{1}{\sqrt{L}}L\frac{1}{\sqrt{2\omega_d}}\frac{1}{\sqrt{V}} \\ &=\; \frac{1}{\sqrt{2}}\kappa|p_1|\frac{1}{\sqrt{2\omega_d}}\frac{1}{\sqrt{V}} \end{aligned} \tag{5.19}$$

In (5.19) the factor $\sqrt{2}\kappa$ comes from (5.18) and the factor 1/4 comes from the coefficient in the BI action in (5.17). There is an additional factor of 2 since we are considering the emission of diagonal elements of h_{ij} which arise from two open string states of the same polarization. Then each open string annihilation operator in the BI action term $h_{ii}\partial X^i \partial X^i$ can kill either of the initial open strings states.

Note that $R_d = R_h/2$.

6. Comparison of elementary and D-string S-matrices

We now compare the S-matrix for the decay of an excited elementary string into a massless graviton with the S-matrix of the decay of an excited D-string into the same polarization state. For definiteness we will consider the polarization state h_{12}.

First consider the elementary string. From the results of section 4 (equations (4.7), (4.8)) we get that for $n_w L >> \sqrt{\alpha'}$ and small excitation number N (which together imply that $k_2^0 << {T^{(S)}}^{1/2}$) the amplitude per unit time to decay to the graviton is

$$\begin{aligned} A'_E &\approx (\frac{2}{2\sqrt{2}})(8(\pi T^{(S)})^2)\ \frac{\kappa}{\pi T^{(S)}}\ N\ V[(2T^{(S)}Ln_wV)^2(2k_2^0V)]^{-1/2} \\ &= \frac{2\pi N}{n_w L}\frac{\sqrt{2}\kappa}{\sqrt{2k_2^0 V}} \end{aligned} \tag{6.1}$$

The origin of the various factors has been explained in section 4. The states are normalized in the nine dimensional spatial volume V. The overall V comes from the momentum conserving delta function after setting the initial and final momenta to be equal.

The D-string decay amplitude per unit time is similarly obtained from (5.15) as

$$A_D = \sqrt{2}\kappa(2|p_1|^2)L[(2|p_1|L)(2|p_1|L)(2k_2^0V)]^{-1/2} = \frac{\sqrt{2}\kappa|p_1|}{\sqrt{(2k_2^0)(V)}} \tag{6.2}$$

where the origin of the various terms is exactly the same as in (6.1) with the difference that the open string states are normalized on the D-string rather than in the entire space. The factor of $\sqrt{2}$ comes from the fact that the field with standard kinetic term is $\sqrt{2}\kappa h_{12}$.

The D-string is believed to be dual to the elementary string. If this is true these two amplitudes (6.1) and (6.2) must be equal. Under the duality transformation the oscillator states of the elementary string become the momentum states of the open strings on the D-string, with the oscillator number being identified with the quantized open string momenta. In fact the D-string answer (6.2) is in exact agreement with the elementary string answer (6.1) with the identification

$$|p_1| = \frac{2\pi N}{n_w L} \tag{6.3}$$

In a similar way we verify that the amplitude for an excited elementary string to emit a dilaton equals the amplitude for a D-string to emit a dilaton, using equations (4.19) and (5.19).

These results show again that the emission predicted by the BI action for a D-string with a RR charge n_w equals that for an elementary string at $g << 1$ that is multiply wound n_w times.

7. The Absorption cross-section

In [3] a model was given using D-branes which, at strong coupling, would correspond to an extremal black hole in 5 dimensions. This black hole carries three nonzero charges, and so has nonzero horizon area. The entropy of the D-brane system agreed with the Bekenstein entropy given by this area. In [6] it was shown that a slightly nonextremal configuration of these branes radiates with the temperature expected from black hole thermodynamics, and moreover the emission rate is proportional to the horizon area implied by the charges of the near-extremal hole. In [12] it was shown that the emission of low energy scalar quanta (obtained by dimensional reduction of the 10-d graviton) from the slightly non-extremal configuration of branes agreed exactly with the radiation expected from the corresponding black hole. (For some other results on nonextremal branes see [17].)

Below we compute the cross section for this collection of branes to absorb (a) scalars derived from a Kaluza-Klein reduction of the graviton and (b) the dilaton. These two results (a) and (b) will be found to agree. This agreement is essential if they are to be compared to absorption by a black hole, since a black hole absorbs all uncharged scalars with the same cross-section. As expected from general arguments of detailed balance, this cross section agrees with that computed from an analysis of emission in [12].

The absorbing system is the D-string, with winding number n_w around X^9 which is compactified on a circle of length L. We assume, following [18], that the D-string is constrained to move in only four out of its eight transverse directions by a 5-D-brane, to which it is bound. There is a thermal distribution of momentum modes on the D-string (say left moving), with total momentum $2\pi N/L$.

A D-string of length L may be considered as a system with some discrete energy levels with spacing ΔE which is independent of E. Consider an inital state at $t = 0$ where the D-brane system is in its BPS ground state and a closed string state of energy k_0 is incident on it. Let the amplitude to excite the D-string to any one of the excited levels per unit time be R. (For t large, only the levels in a narrow band will contribute, and in this band we can use the same R for each level.) Then the amplitude that the system in an excited state with energy E_n at a given time t is given by

$$A(t) = Re^{-iE_n t} \int_0^t dt' e^{i(E_n - k_0)t'} = Re^{-\frac{i}{2}(E_n + k_0)t} [\frac{2\sin[(E_n - k_0)t/2]}{(E_n - k_0)}] \tag{7.1}$$

The total number of quanta absorbed in time t is thus given by

$$P(t) = \sum_n |R|^2 [\frac{2\sin[(E_n - k_0)t/2]}{(E_n - k_0)}]^2 \rho(k_0) \tag{7.2}$$

where $\rho(k_0)$ denotes the occupation probablity of the graviton in the initial state with energy k_0. For large length of the D-string L we can replace the sum by an integral

$$\sum_n \to \int \frac{dE}{\Delta E} \tag{7.3}$$

in which case the rate of absorption $\mathcal{R}_{\mathcal{A}} = P(t)/t$ evaluates to

$$\mathcal{R}_{\mathcal{A}}(t) = \frac{2\pi |R|^2}{\Delta E} \rho(k_0) \tag{7.4}$$

For our case of the D-string on the 5-brane,

$$\Delta E \ = \ \frac{4\pi}{n_w L} \tag{7.5}$$

Note that because we have the spectrum of one long string of length $n_w L$ rather than n_w strings of length L, we have closely spaced levels for n_w large, and thus the approximation (7.3) is improved. It is possible that interactions further smooth out the discrete level separation (7.5) towards a continuum, but we shall not investigate this issue here.

Consider the absorption of a quantum of the graviton h_{12}, with no momentum or winding along the compact directions. There are two open string states that can be created on the D-string in absorbing this graviton. We can have the string with poilarisation 1 travelling left on the D-string and the open string with polarisation 2 travelling right, or we can have the polarisations the other way round. This means that there are two series of closely spaced levels that will do the absorption, and so the final rate of absorption computed from (7.4) will have to be doubled.

¿From (5.13) we find for the amplitude per unit time for the graviton to create any one of these two possible open sring configurations to be

$$R = \sqrt{2}\kappa |p_1| \frac{1}{\sqrt{2k_2^0}} \frac{1}{\sqrt{L}} \frac{1}{\sqrt{V_c}} \frac{1}{\sqrt{V_T}} \rho_L^{(1/2)}(|p_1|) \tag{7.6}$$

where we have separated the term $\frac{1}{\sqrt{V}}$ into contributions from the string direction X^9, the remaining four compact directions (denoted by the subscript c) and the transverse noncompact patial directions (denoted by the subscript T). We have also included the term

$$[\rho_L(|p_1|)]^{1/2} = [\frac{T_L}{|p_1|}]^{(1/2)}, \quad T_L = \frac{S_L}{\pi n_w L} \tag{7.7}$$

which gives the Bose enhancement factor due to the population of left moving open string states on the D-string [6]. Here S_L is the entropy of the extremal configuration, given by the count of the possible ways to distribute the N quanta of momentum among different left moving vibrations of the D-string:

$$S_L \;=\; 2\pi\sqrt{n_w N} \tag{7.8}$$

and equals the Bekenstein entropy of the black hole with the same charges as the D-brane configuration.[3]

The absorption cross section is given by

$$\sigma \;=\; 2\mathcal{R}_{\mathcal{A}}/\mathcal{F} \tag{7.9}$$

[3] For a derivation of the results of [6] in the notation used here, see [12].

where $\mathcal{F} = \rho(k_0)V_T^{-1}$ is the flux, and the factor of 2 was explained before eq. (7.6).

Note that

$$\frac{\kappa^2}{LV_c} = 8\pi G_N^5 \tag{7.10}$$

and that for the given choice of momenta

$$k_0 = 2|p_1| \tag{7.11}$$

Then we find

$$\sigma = A \tag{7.12}$$

where $A = 8\pi G_N^5 \sqrt{n_w N}$ is the area of the extremal black hole with one 5-D-brane, n_w windings of the 1-D-brane, and momentum charge N. This result agrees, as expected, with the calculation of [12] where the cross section was computed from the emission of quanta from the slightly nonextremal configuration of branes, and the result (7.12) was shown to agree with the classical cross section for absorption of scalar quanta.

Now consider the absorption of the dilaton. As shown in sections 4,5, the amplitude per unit time for two open strings of the same polarisation to collide and emit a dilaton is $1/2$ times the amplitude for open strings of polarisation 1 and 2 to collide and emit the 5-dimensional scalar given by h_{12}. The rate of emission for the dilaton is thus $n/4$ times the rate for emission of h_{12} quanta, where n is the number of polarisations allowed for the open strings. Since we have $n = 4$, the emission rate for the dilaton equals that for the other scalars, and repeating the above calculation shows that the absorption cross section will be the same for the two cases as well.

8. Discussion

We were interested in examining the absorption cross section for 5-dimensional scalars in the D-brane configuration discussed in [3][6]. This configuration would give an extremal black hole with nonzero horizon area, if the charges and the coupling were appropriately large.

It is not clear how to access the region of parameter space that corresponds to the black hole through simple D-brane calculations. The classical cross section for absorption of waves into the black hole is of course computable, in particular for low energy waves one can follow the methods of [19] or [20]. Such a calculation was done for the extremal black hole under consideration in [12], and for a nonextremal version in [21]. As in the 3+1 dimensional Schwarzschild case, the cross section equals the area A of the horizon.

With D-branes, what we have computed is the absorption for the case when we have one 5-D-brane, a given number of 1-D-branes, and momentum on the 1-D-branes. For $g = e^{\phi} << 1$, and long wavelength oscillations, one believes that the BI action for the D-string should be a good description. We have shown that for $g >> 1$, the results given by the BI action are still obtained. This was done using that fact that the D-string at $g >> 1$ behaves like an elementary string at $g << 1$, and in the latter case we know how to compute the deacy rates again. In particular in the elementary string case we know how to handle the issue of the decay of a multiply wound string, and the result agrees with using the BI action for a single long string rather than a collection of closely spaced individually wrapped strings. This is useful because from the description of the D-brane excitations as open strings with ends on the D-brane, we do not quite know how to handle the oscillations of bound states of several parallel branes. A naive placing of Chan-Paton factors at the ends of the open string is in fact not correct at least for large g, where duality with the elementary string gives the excitation threshold of a single long string.

Note that just because we know how to handle the D-string at both large and small g does not mean that we can access the black hole limit. The latter may imply values for the charges n_w, N, and the coupling g that do not permit using the lowest order perturbation theory that we have done here to study the interactions of scalars with the D-branes. (In particular, a D-string at large g is just like an elementary string at small g, so we do not access the black hole limit by just taking a D-string and going to large g.) What is interesting is that the low energy absorption cross section nevertheless agrees between the black hole and the D-brane cases. In particular the fact that only four directions of oscillations are allowed in the D-brane model was essential in getting the dilaton absorption cross section to agree with the cross section of the other scalars, this agreement being a basic requirement of black hole theormodynamics. (In the D-brane case this might be a consequence of the supermultiplet structure in the D-brane model.) Another interesting feature is that in the 3+1 dimensional hole, the absorption cross section for spin-0 and spin-1/2 quanta are proportional to the area A for low energy quanta, while the cross sections for higher spins vanish at low energy, being multiplied by higher powers of $A\omega^2$ [19]. In the D-brane model we get emission of scalars from the collision of two bosonic open string states on the D-string, and the emission of spin-1/2 quanta from the collision of a left moving bosonic string and a right moving fermionic string. It is important to have the left moving string to be bosonic, so that we get the bose enhancement factor (7.7) without which the cross section vanishes in the classical limit. We find again that only spin-0 and spin-1/2 quanta have nonvanishing cross sections at low energy.

It is interesting to note that the computation of absorption from the classical black hole geometry is a calculation involving only the five noncompact directions, with the wavefunction having no nontrivial dependence in the compactified directions. The D-brane calculation, by contrast, involves converting the energy of the incoming graviton (which again has a wavefunction with nontrivial dependence only in the five noncompact directions) into a pair of open strings that have momentum in the *compact* directions. Thus these quite different mechanisms have lead, in the present low energy calculation, to the same cross section. It is possible that the agreement between the black hole and the D-brane cases might be a consequence of some combination of the following: the supersymmetry of the extremal configuration, the near extremality of the absorption/emission process, and the low energy of the quanta considered. Investigation of these issues in in progress.

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