

Emission from parallel p -brane Black Holes

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The entropy of a near-extremal black hole made of parallel D-branes has been shown to agree, upto a numerical factor, with that of the gas of massless open string states on the brane worldvolume when the string coupling is chosen suitably. We investigate the process of emission or absorption of massless S-wave neutral scalars by these black holes. We show that with rather mild assumptions about the nature of the interactions between the scalar and open string states, the D-brane cross-section generally fails to reproduce the universal low energy black hole cross-section except for 1-branes and 3-branes.

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1. Introduction

Recently the success of the idea that string states behave as black holes at strong coupling has revolutionized our understanding of black hole thermodynamics [1,2,3,4,5,6,7,8,9,10,11,12,13].

When the product of the string coupling and the charge is large enough D-brane configurations are expected to behave as black holes. For D-brane bound states which correspond to five dimensional extremal black holes with large horizons, the black hole entropy is indeed reproduced exactly [4], a result which is understood from the non-renormalization of BPS states. The entropy continues to agree for slight departures from extremality [5,6] and the very low energy decay rate was found to be proportional to the horizon area [5] which is consistent with the semiclassical Hawking radiation from these holes [7]. Remarkably the two emission (or absorption) rates were shown to be *exactly* equal [8,9] and even the greybody factors agree exactly [10] in the dilute gas regime ². Recent results for such black holes seem to suggest, however, that there could be enough low energy non-renormalization properties which justify extrapolation of weak coupling results to the strong coupling regime [16], [17]. Most of these results have been extended to four dimensional holes and for charged scalar emission ³ [18] and agreements upto numerical factors have been obtained for emission at higher angular momentum [19,20,21].

For parallel D-branes, the extremal state corresponds to black holes of zero size and one does not expect to reproduce the black hole entropy exactly upto numerical coefficients, though one expects the D-brane entropy to be proportional to the area of the stretched horizon like NS-NS holes or corresponding holes in heterotic string theory [3]. With a slight departure from extremality the horizon area becomes non-zero and along the lines of the proposal of [1] one should be able to understand the entropy after a proper treatment of mass renormalization of these states. Indeed, as a part of their general analysis of non-extremal holes in string theory, Horowitz and Polchinski [22] have shown that if the string coupling g is chosen such that the curvature (in string metric) at the horizon is of the order of the string scale, the entropy of a slightly non-extremal black hole made of

² Such detailed agreements are not present beyond the dilute gas regime [14] [15]

³ There seem to be some disagreements for emission of charged scalars when individual energies are large [15].

N parallel D-branes is exactly reproduced by that of the gas of the N^2 massless modes of the open strings ⁴. When gN is much smaller than this matching value the horizon curvature is strong, the semiclassical black hole description is bad while the description in terms of perturbative D-branes is good. When gN is large the horizon curvature is small, rendering a black hole description reliable, while the D-branes become strongly coupled and difficult to describe. The above result shows that at the matching point there is no sharp discontinuity of the mass.

This “correspondence principle” also predicts the correct entropy in the regime far from extremality where for small gN the entropy is described by that of a single long string [22]. In this paper we will, however, restrict our attention to the near-extremal limit.

It is natural to ask whether the D-brane configuration also absorbs and emits like a black hole at this matching point. As mentioned above it is not completely clear why absorption should work even for D-brane bound states with a well-understood extremal entropy. Nevertheless it is important to get as much “phenomenology” as possible. This is particularly relevant in view of the suspicion that non-renormalization theorems are at work - these non-renormalization properties should hold for all slightly non-extremal parallel D-branes [17]. In fact the S-wave and P-wave absorption by *extremal* 3-branes have been shown to agree *exactly* with the classical answers, [24] presumably because of the non-singular nature of these black holes.

In the following we calculate the emission cross-section of massless neutral scalars by such parallel D-branes with zero net momentum by making rather mild assumptions about the nature of interactions of the bulk scalars with the open string states. In particular, we allow processes in which an arbitrary number of open string states can go into such a scalar via some local interaction at any order of the open string perturbation theory. We find that at the lowest energies the D-brane cross-section does not in general reproduce the universal black hole answer which is the horizon area [25], except for 1-branes and 3-branes. For 1-D branes the leading process is that of two open strings going into a closed string while for the 3 D-brane the leading process involves four open strings going into a closed string. Our results should also apply to branes with some net momentum.

⁴ Note that this is in a different regime than the work of [23] where the nonextremal entropy was shown to arise from the excitations of a single long string whose degrees of freedom are essentially the N diagonal components of the adjoint fields.

2. Non-extremal entropy

A p -brane with RR charge N has a classical solution with the ten dimensional string metric [26]

$$ds^2 = f^{-1/2}[-(1 - \frac{r_0^n}{r^n})dt^2 + dy^i dy_i] + f^{1/2}[(1 - \frac{r_0^n}{r^n})dr^2 + r^2 d\Omega_{n+1}] \quad (2.1)$$

where $n \equiv 7-p$, y^i are the coordinates along the brane which is assumed to be compactified on a torus of volume V_p and r denotes the radial coordinate in the $9-p$ dimensional noncompact space. The harmonic function $f(r)$ is given by

$$f(r) = 1 + \frac{r_0^n \sinh^2 \alpha}{r^n} \quad (2.2)$$

Other properties of the classical solution may be found in [26] and [22]. The charge N is given by

$$N \sim \frac{r_0^n}{g} \sinh 2\alpha \quad (2.3)$$

In (2.3) we have used string units and will continue to do so in the rest of the paper. g is the string coupling. The horizon is at $r = r_0$ and the classical Beckenstein-Hawking entropy is given by

$$S_{BH} \sim \frac{r_0^{n+1} V_p}{g^2} \cosh \alpha \quad (2.4)$$

The extremal limit is $r_0 \rightarrow 0$ and $\alpha \rightarrow \infty$ with N held fixed. In this limit S_{BH} is zero. The total energy is given by

$$E \sim \frac{r_0^n V_p}{g^2} [\frac{n+2}{n} + \cosh 2\alpha] \quad (2.5)$$

The curvature at the horizon becomes of the order of the string scale when [22]

$$r_0 \sim (\cosh \alpha)^{-1/2} \quad (2.6)$$

in string units, and at this point the effective open string coupling gN becomes

$$gN \sim (\sinh 2\alpha)(\cosh \alpha)^{-n/2} \quad (2.7)$$

For the near-extremal situation (large α), $r_0 \sim e^{-\alpha/2}$ and

$$gN \sim r_0^{n-4} \quad S_{BH} \sim \frac{r_0^{n-1} V_p}{g^2} \quad (2.8)$$

so that the area of the horizon A_H is

$$A_H \sim r_0^{n-1} \tag{2.9}$$

at the matching point.

It was shown in [22] that at this value of the string coupling the entropy S_{BH} for a near extremal hole with large σ agrees (upto a numerical factor) with the entropy of the gas of N^2 massless open string modes

$$S_g \sim N^2 V_p T^p \tag{2.10}$$

where T is the temperature, when the energy of this gas

$$\Delta E \sim N^2 V_p T^{p+1} \tag{2.11}$$

is set equal to the *excess* energy of the corresponding black hole above extremality. Equation (2.10) shows that at this matching point the temperature at this matching point is

$$T \sim r_0 \tag{2.12}$$

As explained in [22] it is immaterial whether we use the asymptotic or horizon values of T and V_p in (2.10) since the redshift factor cancels in this equation.

For near-extremal 3-branes the entropy of the gas agrees with the black hole entropy (upto a constant) for *any value of the string coupling constant* [22], as first noticed in [27]. This may be easily seen by equating (2.11) with the excess energy obtained from (2.5) to determine T in terms of r_0 and α (after using (2.7) to relate gN to r_0 and α) and substituting the result in (2.10). The answer agrees with S_{BH} precisely when $p = 3$.

3. Absorption cross-section

From the point of view of the theory of massless open string modes on the p -brane worldvolume, this slightly nonextremal state decays by some number of open string modes colliding to go into a massless closed string state which we take to be a scalar (from the $10 - p$ dimensional point of view). Consider n_o such open string modes with momenta $p_i, i = 1, \dots, n_o$ going into a closed string mode with momenta (q, k) where q denotes the momentum components along the brane worldvolume and k denotes the components in the noncompact $d = 9 - p$ dimensional space with volume V_d .

We will consider a completely general interaction with the form factor $f(p_i, \omega)$. We allow some of the initial open string modes to be fermionic: let there be n_B bosonic and n_F fermionic open string modes, $n_o = n_B + n_F$. n_F is even since the final state is bosonic. If the process occurs at the tree level the matrix element is given by

$$M \sim g^{n_o/2} \delta^{p+1} (\sum p_i - q) \left[\prod_{i=1}^{n_B} \frac{1}{(|p_i| V_p)^{\frac{1}{2}}} \right] \left[\prod_{i=1}^{n_F} \frac{1}{V_p^{\frac{1}{2}}} \right] \left(\frac{1}{\omega V_p V_d} \right)^{\frac{1}{2}} f(p_i, k) \quad (3.1)$$

To obtain the decay rate we have to average over all initial states which are taken from a thermal bath at temperature T ⁵ and sum over all the final states⁶. Thus there is a factor of the Bose-Einstein distribution function $\rho_B(|p_i|, T)$ for each of the bosonic open string modes and a fermi distribution function $\rho_F(|p_i|, T)$ for each fermionic mode. This yields a decay rate

$$\Gamma \sim N^n \int \left[\prod_{i=1}^{n_B} V_p d^p p_i \rho_B(|p_i|, T) \right] \left[\prod_{i=1}^{n_F} V_p d^p p_i \rho_F(|p_i|, T) \right] |M|^2 [V_d d^d k] \quad (3.2)$$

The factor of N^n comes from summing over the ‘‘colors’’ of the worldvolume fields which are in the adjoint representation of $U(N)$.

We will consider wavelengths larger than all other length scales in the problem. In particular the energy of the outgoing mode ω is taken to be much smaller than the temperature T . In this regime we can approximate the distribution functions by

$$\rho_B(p_i, T) \sim \frac{T}{|p_i|} \quad \rho_F(p_i, T) \sim \frac{1}{2} \quad (3.3)$$

The dependence of Γ on ω may be then read off by simply counting the powers of momenta. The form factor will contribute some power of ω which we take to be

$$f \sim \omega^M \quad (3.4)$$

M is then basically the number of derivatives in the position space interaction term. A local interaction means that M is nonnegative and integer. Thus the low energy decay rate becomes

$$\Gamma \sim (gN)^{n_o} \omega^{\beta-1} T^{n_B} [d^d k] \quad (3.5)$$

⁵ Once again the redshift factor cancels since only ratio of the energy to the temperature appear in the distribution functions

⁶ The net momentum is zero so that there is no chemical potential for the momentum

where

$$\beta = (n_B + n_F)(p - 2) + 2n_F + 2M - (p + 1) \quad (3.6)$$

Since the outgoing particle is a boson we have to divide by one factor of the Bose distribution function and the phase space volume to obtain the absorption cross-section

$$\sigma \sim (gN)^{n_o} \omega^\beta T^{(n_B-1)} \quad (3.7)$$

In the above we assumed that the process occurs at the tree level of the open string theory. It is straightforward to extend the result to the situation where the process appears at the loop level. If the number of holes in the worldsheet diagram is $h + 1$ one has an additional overall factor of $(gN)^{2h}$ so that (3.7) becomes

$$\sigma_h \sim (gN)^{n_o+2h} \omega^\beta T^{(n_B-1)} \quad (3.8)$$

We now compare this D-brane answer for the absorption cross-section to the classical answer which at low energies is given by [25]

$$\sigma_{cl} = A_H \quad (3.9)$$

regardless of the type of black hole we are considering. By the correspondence principle of [22] we expect that these two cross-sections can possibly match only at the value of the coupling where the entropies match. At this point (2.9) and (2.12) imply

$$\sigma \sim \omega^\beta A_H^\gamma \quad (3.10)$$

where β is given by (3.6) and

$$\gamma = \frac{(n_B - 1) + (n_B + n_F + 2h)(3 - p)}{6 - p} \quad (3.11)$$

The case $p = 6$ is special and will be dealt with later.

The D-brane cross-section matches the classical answer when

$$\beta = 0 \quad \gamma = 1 \quad (3.12)$$

which has the following solutions for n_B and n_F

$$\begin{aligned} n_B &= \frac{6M - 3 + p(5 - 2M) + 2hp(p - 3)}{6 - p} \\ n_F &= \frac{18 - 8M + 2p(M - 3) - 2h(p - 3)(p - 2)}{6 - p} \end{aligned} \quad (3.13)$$

We have to find solutions for these equations for given p with positive integer values for n_B, n_F, h and M . Furthermore since the minimal process has to involve at least two open strings $n_o = n_B + n_F \geq 2$ and since the final state is a boson n_F must be even.

Remarkably these conditions are rather restrictive and by a case by case study it is easily seen that there are precisely two solutions

$$\begin{aligned} p = 1 & \quad n_B = 2 \quad n_F = 0 \quad M = 2 \quad h = 0 \\ p = 3 & \quad n_B = 4 \quad n_F = 0 \quad M = 0 \quad h = 0 \end{aligned} \tag{3.14}$$

For the 6-brane the D-brane absorption cross-section becomes

$$\sigma_{p=6} \sim r_0^{-(3n_F+2n_B+6h+1)} \omega^{4n_B+6n_F+2M-7} \tag{3.15}$$

In this case one has $n = 1$ and the horizon area or the classical entropy does not depend on r_0 (or equivalently α). Thus for (3.15) to agree with the classical answer the power of r_0 has to be zero. This, however, cannot happen since n_B is strictly positive and n_F is non-negative.

The two cases where the D-brane answer does agree with the classical answer upto a numerical factor correspond to familiar terms in the tree level worldbrane action of a $U(N)$ gauge theory. For the 1-brane, the relevant interaction is the familiar term of the form $\phi \partial X \partial X$ where ϕ is the dilaton and X stands for a transverse coordinate. For the 3-brane the interaction term involves the commutator term in the gauge field on the brane, e.g.

$$\text{Tr } \phi [A_\alpha, A_\beta][A^\alpha, A^\beta] \tag{3.16}$$

or coupling of the ten dimensional graviton longitudinal to the brane to the energy momentum tensor.

4. Discussion

The spirit of the calculation of absorption cross-section for D-brane bound states with large horizons in the extremal limit or for extremal 3-branes is rather different from the considerations of this paper. In the former situations one compares the tree level D-brane result with the first term in the expansion of the classical cross-section in powers of string coupling [24], [17]. Such an expansion is possible because of low energies. Then the non-trivial point to check is that there are no open string loop corrections to the tree

level D-brane answer at the same order of the energy [17]. In this paper we tried to compare D-brane and black holes at a specified value of the coupling, at which the horizon curvature becomes of the order of the string scale. In this sense it is not surprising that the cross-sections do not agree, even though the entropy does.

It is nevertheless interesting to note that the two cases which lead to agreement (apart from numerical factors) are also essentially cases which lead to exact agreement of cross-sections obtained so far. The low energy degrees of freedom of the five dimensional black hole are in fact those of a long string. Absorption of S-wave scalars by *extremal* 3-branes have been shown to agree exactly with the classical answer [24]. A puzzling point is that for absorption by extremal 3-branes (which have of course zero horizon area) the leading contribution in the D-brane cross-section comes from the interaction of two open strings with a closed string and leads to the correct answer proportional to $\omega^3 R^8$ where R is the gravitational radius. For a non-extremal brane it is the four point coupling which gives the leading contribution proportional to the area, while the term involving two open strings behaves as $\omega^2 T$ for $\omega \ll T$.

In Type IIB theory 3-branes are special and provide a good laboratory to understand the D-brane- black hole connection, since the extremal geometry is nonsingular and the dilaton is a constant. In fact for this case the entropy of near-extremal D-branes agree with the black hole entropy upto numerical factors *regardless of the value of the coupling* [27]. On the other hand we found that the absorption agrees only at the matching point required by the correspondence principle which happens when $gN \sim 1$ and independent of r_0 . The significance of this is not clear.

After completion of this work we received a preprint [28] where it is shown that the grey body factors for NS-NS holes are not correctly predicted from the corresponding string calculation, though the leading area dependence is recovered. The fact that the 1D-brane cross-section also reproduced the leading result is consistent with this since the effective action for emission of quanta from excited fundamental strings is rather similar to the D-string effective action [9].

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