MIT-CTP-2508, TIFR-TH/96-02

Excitations of D-strings, Entropy and Duality

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We examine the BPS and low energy non-BPS excitations of the D-string, in terms of open strings that travel on the D-string. We use this to study the energy thresholds for exciting a long D-string, for arbitrary winding number. We also compute the leading correction to the entropy from non-BPS states for a long D-string, and observe the relation of all these quantities with the corresponding quantities for the elementary string.

January, 1996

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1. Introduction

Recently the idea that massive elementary particles behave as black holes at strong coupling [1] has proved to be very fruitful in string theory [2] [3]. In particular, it has been argued that this could lead to an understanding of black hole entropy [2]. In fact in [4] BPS black holes in heterotic string theory compactified on T^6 were shown to have the same entropy (defined as the area of the stretched horizon) as that expected from the degeneracy of BPS states in the elementary string theory. The importance of BPS states lies in the fact that their masses are not changed by quantum corrections and that a certain class of these are absolutely stable. In fact the correspondence has been understood [5] [6] in certain other BPS black holes in this theory having nonzero horizon area [7].

Evidence in favor of the identification of elementary BPS states of heterotic strings with extremal black holes has been also obtained in scattering processes of these states in [8] and [9] (in [9], as well as in [10] such black holes are obtained by compactifying macroscopic strings [11]), and in scattering of massless scalar states from such BPS states in [12] (where the inelastic thresholds are also examined).

If the above reasoning is correct other BPS states in string theory like those which carry RR charges should behave in a similar manner. In fact Type IIB string theory is conjectured to be self dual in ten dimensions. In this theory there are macroscopic string solutions [11] which are to be identified with the BPS states of the elementary string carrying NSNS charge. An $SL(2, Z)$ symmetry which mixes the NSNS and RR rank three gauge fields can be used to generate other solutions which carry both NSNS and RR charges [13] - in particular strings which carry only RR charges. In a recent work, Polchinski [14] has shown how to describe objects in string theory which carry RR charges - through open strings with Dirichelt boundary conditions in some of the directions, giving 'D-branes' [15] [16]. These are exact descriptions of RR p-branes (p is odd for Type IIB and even for Type IIA). The D-brane descrption has been used to provide strong evidence for the existence of these $SL(2, Z)$ multiplets [17],[18].

In this paper we consider D-strings in the Type-IIB theory. We examine the set of open strings that can live on this D-string, and thereby identify and count the BPS excitations of the D-string. We also examine low energy non-BPS excitations in this manner and compute their entropy - such excitations should be long lived if the D-string is sufficiently long and the level of excitation small. We also consider the scattering of massless probes from a long D-string using the results of [19] and show that at low energies the expected Coulomb form is obtained. Furthermore the threshold for inelastic scattering from elementary string states [12] is shown to match exactly with that obtained with our identification of the non-BPS excited states. In all of the above it is interesting to note the distinction between the D-string wound n_w times and the case of n_w singly wound D-strings placed next to each other.

Backgrounds of such excitations have been analysed in [20]. Such backgrounds, however, describe coherent states of excitations of the D-string, while in the discussion below we wish to count all excitations.

2. The macroscopic NSNS and RR Strings

We will use the Einstein metric $G^{(E)}$ throughout this paper.

The low energy effective action of the type IIB theory is

$$S = \frac{1}{2\kappa^2}\int d^Dx\,\sqrt{-G^{(E)}}\,[R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{12}e^{-\phi}(H^{(1)})^2 - \frac{1}{12}e^{\phi}(H^{(2)})^2] \qquad (2.1)$$

$H^{(1,2)}$ are the rank three NSNS and RR field strengths respectively related to the corresponding gauge fields $B^{(i)}$ by $H^{(i)} = dB^{(i)}$. We have omitted the other fields which are irrelevant for our purposes. This action has a $SL(2,Z)$ symmetry which mixes the two rank two gauge potentials [13]. A particular case of this duality is the transformation

$$\phi \rightarrow -\phi \quad B^{(i)} \rightarrow \epsilon^{ij}B^{(j)} \qquad (2.2)$$

Suppose we add to the action (2.1) the source term from the worldsheet of the elementary string

$$S_{source} = \frac{T}{2}\int d^2\xi[(e^{\phi/2}G^{(E)}_{AB}(X)\delta^{ab} + B^{(1)}_{AB}(X)\epsilon^{ab})\partial_a X^A\partial_b X^B] \qquad (2.3)$$

We take 10-dimensional spacetime with X^9 compactified on a circle of circumference L. Then we have the following solution for the low energy fields if the elementary string is wound n_w times around X^9 [9], [10]

$$\begin{aligned} &ds^2 = A^{-3/4}[-dt^2 + dz^2] + A^{1/4}d\vec{x}.d\vec{x} \\ &A = 1 + \sigma\Lambda, \qquad e^{-2\phi} = e^{-2\phi_0}A, \qquad B^{(1)}_{90} = e^{\frac{\phi_0}{2}}A^{-1} \\ &\Lambda = \frac{\kappa^2}{3\omega_7}\frac{1}{r^6} \qquad \sigma = n_w T e^{\phi_0/2} \end{aligned} \qquad (2.4)$$

Here $\vec{x} = (x^1, \cdots x^8)$ and $r^2 = \vec{x} \cdot \vec{x}$ and ω_7 is the volume of the unit seven sphere. The ADM mass of the macrosopic string may be obtained from the asymptotic form of g_{00} and is given by

$$M = \sigma L \tag{2.5}$$

We can add transverse oscillations to the string which also carry a corresponding amount of momentum along the string; such solutions are also described in [9][10]. We will however be later interested in oscillations of the string that are not necessarily coherent classical deformations, and for these the classical fields must be approximated by quantum and phase averages.

A string solution in lower dimensions may be obtained easily by compactifying $10 - d$ of the transverse dimensions $\vec{x}$ on a compact manifold of volume V_c [9], [10]. For distances much larger than the size of the compact manifold one has a solution which is identical to (2.4), with the sole change being in the form of Λ which now becomes Λ_d

$$\Lambda_d = \frac{2\kappa^2}{V_c(d-4)\omega_{d-3}} \frac{1}{\rho^{(d-4)}} \tag{2.6}$$

where ρ is the radial distance in the theory reduced to d dimensions.

A string solution with RR charges may be now obtained using the 'duality transformation' [13]. For a string carrying purely RR charge the solution is obtained from (2.4) by reversing the sign of the ϕ and ϕ_0 and replacing $B^{(1)}$ by $B^{(2)}$ [3].

3. String states and macroscopic strings

Our goal is to compare states of the elementary string carrying NSNS charge with states of the dual string carrying RR charge. As in section 2, we will consider the elementary string to be wound n_w times around the compact direction X^9. The mass of such states (in Einstein metric) is given to lowest order in the coupling by

$$\begin{aligned} m^2 &= (n_w L T^{(S)} + \frac{2\pi n_p}{L})^2 + 8\pi T^{(S)}(N_R - \delta_R) \\ &= (n_w L T^{(S)} - \frac{2\pi n_p}{L})^2 + 8\pi T^{(S)}(N_L - \delta_L) \end{aligned} \tag{3.1}$$

[3] Note that the relation (17a) in [13] describing the antisymmetric tensor field for general string solutions with both NSNS and RR charges is not quite correct : the vector $B^{(i)}$ in that equation has to be replaced by $\mathcal{M}_0^{-1}\ B$ where $\mathcal{M}_0$ denotes the value of the matrix $\mathcal{M}$ at infinity.

where $T^{(S)} = Te^{\frac{\phi_0}{2}}$ and n_w, n_p are integers giving the winding and momentum in the X^9 direction, and $\delta_{L,R} = 0, 1/2$ for the Ramond and Neveu-Schwarz sectors respectively. In the following we will consider very long strings, so that $L\sqrt{T^{(S)}} >> 1$.

The D-string, on the other hand, appears as a solitonic string with tension $T^{(D)} = Te^{-\phi_0/2}$. Open strings have Dirichlet boundary conditions on the D-string:

$$\begin{aligned}(\alpha_n^\mu \ + \ \tilde{\alpha}_{-n}^\mu)|B> &= 0, \quad (\psi_n^\mu - i\tilde{\psi}_{-n}^\mu)|B> = \ 0, \quad \mu = 0, 9 \\ (\alpha_n^i \ - \ \tilde{\alpha}_{-n}^i)|B> &= 0, \quad (\psi_n^i + i\tilde{\psi}_{-n}^i)|B> = \ 0, \quad i = 1 \ldots 8\end{aligned} \tag{3.2}$$

We will examine the excitations of the D-string by studying configurations of open string states that can live on the D-string. Due to Dirichlet boundary conditions in the eight transverse dimensions the open string states can have nonzero momenta only along the longitudinal directions X^0 and X^9. The lowest mass states of the open string are the massless excitations

$$\begin{aligned}\Psi_b \ &= \psi_{-1/2}^i|p>, \quad p_0 = |p_9|, \quad p_i = 0, \quad (i = 1 \ldots 8) \\ \Psi_f \ &= |p>_\alpha, \quad p_0 = |p_9|, \quad p_i = 0, \quad (i = 1 \ldots 8, \quad \alpha = 1 \ldots 8)\end{aligned} \tag{3.3}$$

Here the two classes of states come from the NS and R sector of the open string respectively. α is the spacetime spinor index; we are in the light cone gauge for the open string where both vector and spinor indices run over 8 possibilities.

Apart from the states in (3.3) there are open string states which involve higher oscillator content. We will not consider them since in general these states will decay at strong coupling.[4]

For the discussion below we use the term 'ground state' of the elementary string for the state having some nonzero winding number n_w but $n_p = 0$, $N_R = \delta_R$, $N_L = \delta_L$. Correspondingly, we take the 'ground state' of the D-string to be one with no transverse excitations and no $B^{(1)}$ charge.

3.1. Ground States

Consider the elementary string with $n_w = 1$, in the 'ground state' as described above. This state has a degeneracy equal to $16 \times 16 = 256$, where the 16 states in each of the left and right sectors come from the 8 bosonic ground states in the NS sector and the 8

[4] We thank M. Douglas for pointing this out to us.

fermionic ground states in the R sector. Thus there are 128 bosonic and 128 fermionic 'ground states' of the elementary string.

For the D-string, we consider the description of [17] where the low energy field theory coming from the open strings is the dimensionsal reduction to the 2-dimensional D-string world sheet of the 10 dimensionsal supersymmetric U(1) gauge theory. This theory has 8 scalars corresponding to the transverse deformations of the D-string, and 8 majorana spinors on the 2-dimensional space. The latter give 8 zero modes, which generate $2^8 = 256$ degenerate ground states of the supersymmetric field theory.

3.2. BPS states with $n_w = 1$

There are an infinite set of BPS saturated states in the fundamental Type II string. These are states with either (i) $N_R = \delta_R$ with arbitrary N_L or (ii) $N_L = \delta_L$ with arbitrary N_R and with arbitrary values of n_p and n_w in either case [2] [3] [4]. For such states the mass formula (3.1) is exact. We will choose $N_R = \delta_R$ and also restrict for the moment to $n_w = 1$. One therefore has, from (3.1), $N_L = \delta_L + n_p$ ($n_p > 0$). The mass is

$$m = \bar{m}_{NSNS} + \frac{2\pi n_p}{L}, \qquad \bar{m}_{NSNS} = T^{(S)} L \tag{3.4}$$

The degeneracy of such states is given by the number of ways one can decompose n_p into levels of the 8 bosonic and 8 fermionic oscillators. As is well known, for large n_p the degeneracy behaves as an exponential of $\sqrt{n_p}$. This leads to an entropy [2] [4]

$$S \ \sim \ 2\pi\sqrt{cN_L/6} \ = \ 2\pi\sqrt{2n_p} \tag{3.5}$$

where we have used the fact that the central charge c comes from 8 transverse bosons and 8 transverse fermions and thus totals 12. This should be identified with the "black string" entropy. One expects that the black string entropy is proportional to the length of the string. This is indeed so if we rewrite the answer in terms of the parameters which appear in the corresponding classical solution.

We identify the corresponding states of the D-string as those that have open strings in the states of the form (3.3), with

$$p_9 \ = \ \frac{2\pi m}{L}, \quad m > 0, \quad \sum m = n_p \tag{3.6}$$

The mass is

$$m = \bar{m}_{RR} + \frac{2\pi n_p}{L}, \qquad \bar{m}_{RR} = T^{(D)} L \tag{3.7}$$

In the low energy limit such a state has the metric corresponding to the string tension $\frac{\bar{m}_{RR}}{L}$ and momentum $P = 2\pi n_p/L$.

The degeneracy of such D-string states is given by the number of ways one can decompose the integer n_p into the positive integers m's of the individual open string states. Note that there are 8 bosonic and 8 fermionic single string states of the type (3.3) for a given momentum. It is immediate that we get the same combinatorics as for the elementary string, where the *oscillator levels* on the left sector were positive integers that had to total upto n_p to generate all the BPS states. Thus we get the same entropy (3.5) at the same total momentum $P = 2\pi n_p/L$ for the D-string as for the elementary string, in accordance with duality.

To describe the BPS states corresponding to with $N_L = 0$, $N_R \neq 0$ we take the momentum p_9 of each open string to be negative instead of positive.

We expect these excited states of the D-string to be stable because if for instance a pair of open strings were to decay to a closed string, by conservation of p_0 and p_9 the closed string would have no transverse momentum p_i. ($p_0^2 \geq p_9^2 + p_i^2$; but $p_0 = p_9$ from the open strings.) If there are noncompact directions among the X^i then the overlap of the open strings (which are confined to the neighbourhood of the D-string) and the zero p_i state of the closed string vanishes, so there is no phase space for decay.

3.3. Non-BPS states with $n_w = 1$

We now consider the non-BPS states specified by some momentum $P = 2\pi n_p/L$ ($n_p > 0$) and $n_w = 1$. For elementary string states it follows from (3.1) that these are described by

$$N_R = \delta_R + n, \qquad N_L = \delta_L + n_p + n, \qquad (n = \text{integer}) \tag{3.8}$$

The mass (at lowest order in coupling) for such a state m_n is given by

$$m_n = \sqrt{[\bar{m}_{NSNS} + (2\pi|n_p|/L)]^2 + 8\pi T^{(S)} n} \tag{3.9}$$

where $\bar{m}_{NSNS}$ is given in (3.4). For large $L\sqrt{T^{(S)}}$ one has

$$m_n = \bar{m}_{NSNS} + \frac{2\pi|n_p|}{L} + \frac{4\pi n}{L} + O(\frac{1}{T^{(S)}L^3}) \tag{3.10}$$

In particular the lowest mass state of this kind has $n_p = 0$ and $n = 1$. The degeneracy of such states comes from the many possibilities for the oscillator excitations, in both the left and the right sectors independently. We thus have, for large n the total degeneracy behaving as

$$D(n) \sim e^{2\pi\sqrt{2(n_p+n)}}\ e^{2\pi\sqrt{2n}} \tag{3.11}$$

This gives the entropy for the non-extremal string. For example for $n_p = 0$ one has an entropy

$$S \sim 4\pi\sqrt{2n} = \sqrt{[4\pi(m_n^2 - \bar{m}_{NSNS}^2]/T^{(S)}]} \tag{3.12}$$

To construct the corresponding non-BPS states for the D-string we let there be open strings on the D-string with both signs of $p_9 = 2\pi m/L$, with $\sum m = n_p$ as before but with $\sum |m| = |n_p| + 2n$. The mass of such a state is given by

$$m_n \;\approx\; \bar{m}_{RR} + \frac{2\pi|n_p|}{L} + \frac{4\pi n}{L} \tag{3.13}$$

Let us analyse the approximation in the above equation. A pair of oppositely moving open strings scatter through a disc diagram to another pair of open strings. This diagram can generate shifts in the energy of a state of the excited D-string. Let all the four open strings have momenta $p_9 = \pm 2\pi n/L,\ p_0 \equiv \omega = |p_9|$. Then the amplitude for scattering per unit time is $\sim e^{\phi_0}T^{(S)}(L\omega)^{-2}L(\omega^2/T^{(S)})^2 \sim (T^{(D)}n^2L^3)^{-1}$. We expect energy shifts for the states of this order, which is the correct order to be dual to the term dropped in (3.10). (In this amplitude calculation one should note that the leading terms in ω in the 4 point disc amplitude cancel after summing over all the cyclically inequivalent orderings of vertex opertors.)

We now wish to count non-BPS excitations of the D-string (at large coupling e^{ϕ_0}) and see if they agree with the non-BPS excitations of the elementary string (at small coupling). As mentioned above, the masses of non-BPS states on the D-string will get corrections due to open string scattering to open strings. Further, there is a process where two open strings travelling in opposite directions interact and decay into a closed string. We take our limits in the following way. First we take the coupling sufficiently strong so that only the states (3.3) are the stable open string states; it is reasonable to expect that the higher oscillator mode open strings decay rapidly and can be ignored. We fix a certain upper limit for the integers $n,\ n_p$ in (3.13); we will count states upto these excitation numbers. Then we take the limit L large. In this limit it becomes hard for the open strings to 'find' each other and interact, so that both the energy corrections to the non-BPS states and the rate of decay of these states (to closed strings) go to zero. Then we can reliably count these non-BPS states.

But now we immediately see that this count is again identical to that in the elementary string case since there is a one-to-one correspondence between open strings with positive and negative m on the one hand and the oscillator mode numbers in the right and left sectors of the Type II elementary string on the other. We therefore get the same expression (3.11) for the degeneracy and thus for the entropy of 'slightly nonextremal' states of the D-string.

3.4. States with $n_w > 1$

Now consider the case $n_w > 1$. This generalization is trivial for the elementary string states since everything follows from the mass formula (3.1). For BPS states the expressions (3.4) are modified, giving $m = n_w L T^{(S)} + \frac{2\pi n_p}{L}$, $N_L = \delta_L + n_w n_p$, leading to an entropy $S \sim 2\pi\sqrt{2 n_w n_p}$.

In the case of the D-string we need an ansatz for the behavior of the open string momenta for the case $n_w > 1$. In particular this ansatz must distinguish between the case (a) where we have n_w different D-strings, each with winding number unity, next to each other, and the case (b) where we have a single D-string that has winding number n_w. To see that a distinction is required note that for the analogous cases in the elementary string, the excitation energies and degeneracies are different for these two cases.

For the case (a) where we have n_w different singly wound D-strings it is sufficient to let the open strings have momenta $2\pi m/L$ as in the discussion of sections 3.2, 3.3, but with a Chan-Paton factor at each end of the open string which takes n_w values corresponding to which of the D-strings the end is on; this gives an n_w^2 fold degeneracy to the excitations for a single D-string [16] [17].

For the case (b) of the single D-string with winding number n_w the following ansatz appears to be adequate. We postulate that each single open string state can carry momentum $p = \frac{2\pi m}{n_w L}$ (m an integer) but the total momentum carried by all the open strings has to equal $\frac{2\pi n_p}{L}$, n_p an integer. Thus the translation of the entire system in the compact direction through L returns the wavefunction of the system to itself with no phase.

It is easy to see that with this ansatz we reproduce the number of BPS and non-BPS states that we find from the elementary string, by carrying out the steps similar to those in the above subsections. In particular for BPS states we have to partition the integer $n_w n_p$ among positive integers, just as for the elementary string. To see why this ansatz for the momenta is reasonable, note that the open strings describe oscillations of the D-string which now has length $n_w L$; thus they must have correspondingly long wavelengths (i.e. low energies).

In particular let us check the lowest energy excitation having zero total momentum. The elementary string has for such a state the mass $\frac{4\pi}{n_w L}$ (coming from the obvious modification of (3.10)). This agrees with the energy of for the lowest energy excitation of the D-string: a pair of open string states with momenta $\pm\frac{2\pi}{n_w L}$ and hence carrying total energy $\frac{4\pi}{n_w L}$.

In the description of [17] we have a supersymmetric $U(n_w)$ Yang Mills theory on the D-brane. On the basis of the above discussion we would expect that for the supersymmetric vacuum corresponding to a single long D-string, the lowest excitations should have energies $E = \frac{4\pi}{n_w L}$, momentum $P = 0$. This energy is lower (for $n_w > 2$) than the energy of Goldstone modes in a periodic box of length L.

4. Scattering from string states and excitation thresholds

In a recent paper [12] the threshold for exciting a BPS state of the elementary string to the lowest non-BPS state with the same charges was computed. In view of the conjectured duality the scattering of probes from the D-string should agree with that from elementary string states.

First we note that the elastic scattering cross section for neutral scalars at low energies must be the coulomb scattering obtained from the classical gravitational field of the string. [5]

We work in a situation where five of the transverse directions x^i , $i = 4, \cdots 8$ are compactified on an internal manifold of volume V_c. We consider scattering of massless gravitons of the closed string whose polarizations ϵ_{ij} are nonzero only for $i = 4, \ldots 8$ and whose momenta are nonzero only along the four noncompact directions, i.e. $p^i \neq 0$ for $i = 0, \cdots 3$. The target (D-string or elementary string state) is taken to be in the lowest state for the given winding.

The scattering cross section from a NSNS state of mass M may be in fact read off from the scattering in the heterotic string theory computed in [12], by specialising to the case where the charges come entirely from winding and momentum in the string direction.

[5] A discussion of low energy elastic scattering has also been presented recently in [21], where scattering of RR scalars and polarisation dependences have also been studied.

The differential cross section in the rest frame of the target has the expected Coulomb behavior at low energies :

$$\frac{d\sigma}{d\Omega} \sim (G_N M)^2 \ \mathrm{cosec}^4 \frac{\theta}{2} \tag{4.1}$$

where G_N is the Newton's constant in 4 dimensions and M is the mass. In (4.1) θ is the angle between the initial (spatial) momentum $\vec{p}$ and the final momentum $\vec{k}$. As shown in [12] this cross section agrees with that obtained from classical scattering of a massless wave from a black hole of mass M. The threshold for inelastic scattering corresponds to excitation of a non-BPS state of the type (3.9) with $n = 1$, $n_p = 0$. Simple kinematics leads to the threshold energy [12]

$$p_0^{th} \approx \frac{4\pi}{n_w L} \tag{4.2}$$

The scattering from a fixed D-string can be computed by evaluating the two point function of our "graviton" operators on a disc with boundary conditions appropriate for D-strings. This calculation has been already performed in [19] and we will simply use their result. The amplitude for this process is

$$\mathcal{T} \sim n_w \kappa^2 \epsilon_{ij}^{(1)} \epsilon_{ij}^{(2)} s \ \frac{\Gamma(1 - \frac{s}{2\pi T})\Gamma(-\frac{t}{8\pi T})}{\Gamma(1 - \frac{s}{2\pi T} - \frac{t}{8\pi T})} \tag{4.3}$$

where for our process we have

$$s = k_0^2 - k_9^2 = p_0^2 - p_9^2, \qquad t = -(p+k)^2 = 2(k_0^2 + \vec{p}\cdot\vec{k}) \tag{4.4}$$

and we have used the momentum conservation for the longitudinal directions $p_0 = -k_0$, $p_9 = -k_9$ and the mass shell conditions $k_0 = |\vec{k}|, \ \ p_0 = |\vec{p}|$. The factor of n_w as compared to the formulae in [19] follows from the fact that a D-string with winding number n_w has n_w units of RR charge. In the low energy limit $k_0 \to 0$ the matrix element approaches the standard Coulomb behaviour and one gets a cross section

$$\frac{d\sigma}{d\Omega} \ \sim \ (\frac{e^{-\phi_0} T^{(S)} n_w \kappa^2}{V_c})^2 \mathrm{cosec}^4 \ \frac{\theta}{2} = \ (T^{(D)} n_w L \kappa_4^2)^2 \mathrm{cosec}^4 \ \frac{\theta}{2} \tag{4.5}$$

which agrees with the dependence on parameters with (4.1) since $T^{(D)} n_w L = M$ and $G_N = 8\pi\kappa_4^2$. (κ_4 is the effective four dimensional coupling.)

The threshold for inelastic scattering may now come from any of the following sources.

(a) First, the D-string can be excited to the lowest non-BPS state described in the previous section, i.e. a pair of open string modes in their lowest allowed oscillator state and with equal and opposite longitudinal momenta $\pm\frac{2\pi}{n_w L}$. The threshold energy, $\frac{4\pi}{n_w L}$ matches with the result (4.2) for the elementary string. This is the lowest threshold for large L.

(b) One can excite states of the open string which have higher number of oscillators. The threshold here is independent of L and equal to $\sqrt{8\pi T^{(S)}} = \sqrt{8\pi} T^{1/2} e^{\frac{\phi_0}{4}}$. This is the relevant threshold for small L but will be a mild resonance at strong coupling since these do not describe stable states. This kind of excitation is considered in [21] for 0-branes representing black holes.

(c) One can excite a pair of open string states which wind in opposite senses around the compact directions x^i , $i = 4, \cdots 8$. The corresponding threshold energy is $2T^{(S)}a = 2Tae^{\frac{\phi_0}{2}}$, where a is the smallest circumference of compactification. When $L >> \frac{1}{T^{(S)}a}$ this threshold is higher than the first one discussed above.

None of these thresholds appears to represent the classical threshold that would folllow from extending the calculation of [12] to the long black string. In fact the details of that calculation depend only on the metric in the directions perpendicular to the string, and the result is simply that the threshold is at wavelengths

$$\lambda \sim \frac{\kappa_5^2 M}{L} = \frac{\kappa^2}{V_c} T e^{-\phi_0/2} n_w \ \sim\ \frac{1}{V_c} e^{3\phi_0/2} T^{-3} n_w \tag{4.6}$$

where we have used the mass per unit length of the D-string. Since this thresold does not depend on L it cannot be of type (a) above. The threshold (b) above differs in its T, ϕ_0 dependence from (4.6). The threshold of type (c) above depends on the smallest compact dimension; while (4.6) depends only on the total volume of all the compact dimensions V_c.

5. Discussion

We have observed the correspondence between excitations of the elementary string and configurations of open strings travelling on the D-string. The latter were described in a manner reminiscent of the Green-Schwarz language for the elementary string. In particular we estimated the entropy from non-BPS states for a long D-string. Even for a large (but fixed) value of the coupling, we could take the length L sufficiently large so that the density of non-BPS excitations is very low and thus the decay rate and mass corrections for the non-BPS states can be ignored. These long lived excitations can be counted in a manner similar to the count of BPS states. The above approach to estimating non-BPS entropy should have wider validity. We found that as we go away from the extremal limit the leading correction to the entropy goes as $\sim \sqrt{m - m_{\rm ex}}$ where $m - m_{\rm ex}$ is the mass in excess of the BPS mass. In a recent work the entropy of five dimensional extremal holes with both electric and magnetic charge was shown to equal the area of the horizon as given in [22]. If we take a black string in six dimensional space-time then we can apply the above estimates. The correction to the horizon area in [22] for slightly nonextremal holes indeed goes like $\sim \sqrt{m - m_{\rm ex}}$, in accordance with the above expectation.

It would also be interesting to compute the amplitude for a D-string excited in the above non-BPS states to fall into a black hole made from other D-strings, and to see if the non-BPS excitations escape as 'Hawking radiation' in this process.

6. Acknowledgements

We have benefitted from discussions with A. Belopolsky, M. Douglas, M. Li, W. Taylor, S.P Trivedi, S. R. Wadia and B. Zwiebach. S.R.D. would like to thank the Theory Group of KEK for hospitality during the completion of this paper. S.D.M. is partially supported under DOE cooperative agreement number DE-FC02-94ER40818.

References

[1] S. Hawking, *Month. Not. Roy. Astro. Soc.* **152** (1971) 75; A. Salam, in *Quantum Gravity : an Oxford Symposium* (eds. C. Isham, R.Penrose and D. Sciama, Oxford University Press, 1975); G. 't Hooft, *Nucl. Phys.* **B335** (1990) 138; J. Preskill, *Physica Scripta* **T36** (1991) 258; C. Holzhey and F. Wilczek, *Nucl. Phys.* **B380** (1992) 447

[2] L. Susskind, hep-th/9309145; J. Russo and L. Susskind, *Nucl. Phys.* **B437** (1995) 611

[3] M. Duff, M. Khuri, R. Minasian and J. Rhamfeld, *Nucl. Phys.* **B418** (1994) 195; M. Duff and J. Rahmfeld, *Phys. Lett.* **345B** (1995) 441

[4] A. Sen, *Nucl. Phys.* **B440** (1995) 421; *Mod. Phys. Lett.* **A10** (1995) 2081; for an earlier suggestion see C. Vafa as quoted in [2]

[5] F. Larsen and F. Wilczek, hepth/9511064.

[6] A. Strominger and C. Vafa, hepth/9601029

[7] M. Cvetic and D. Youm, hepth/9512127 and references therein.

[8] J. Gauntlett, J. Harvey, M. Robinson and D. Waldram, *Nucl. Phys.* **B 411** (1994) 461; R. Khuri and R. Myers, hep-th/9508045

[9] C.G. Callan, J.M. Maldacena and A.W. Peet, hep-th/9510134

[10] A. Dabholkar, J. Gauntlett, J. Harvey and D. Waldram, hepth/9511053.

[11] A. Dabholkar and J. Harvey, *Phys. Rev. Lett.* **63** (1989) 2073; A. Dabholkar, G. Gibbons, J. Harvey and F. Ruiz-Ruiz, *Nucl. Phys.* **B340** (1990) 33.

[12] G. Mandal and S.R. Wadia, hepth/9511218

[13] J. Schwarz, hepth/9508143

[14] J. Polchinski, hepth/9510017

[15] J. Dai, R. Leigh and J. Polchinski, *Mod. Phys. Lett.* **A4** (1989) 2073; P. Horava, *Phys. Lett.* **B231** (1989) 251; R. Leigh, *Mod. Phys. Lett.* **A4** (1989) 2767; M. Green, *Phys. Lett.* **B266** (1991) 325 and *Phys. Lett.* **B329** (1994) 435

[16] J. Polchinski, *Phys. Rev.* **D50** (1994) R 6041

[17] E. Witten, hep-th/9510135

[18] A. Sen, hep-th/9510229 and 9511026; C. Vafa, hep-th/9511088

[19] I. Klebanov and L. Thorlacius, hepth/9510200

[20] C.G. Callan and I. Klebanov, hepth/9511173

[21] S.S. Gubser, A. Hashimoto, I.R. Klebanov and J.M. Maldacena, hepth/ 9601057

[22] G. Gibbons and K. Maeda, *Nucl. Phys.* **B298** (1988) 741