

Comparing decay rates for black holes and D-branes

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We compute the leading order (in coupling) rate of emission of low energy quanta from a slightly nonextremal system of 1 and 5 D-branes. We also compute the classical cross-section, and hence the Hawking emission rate, for low energy scalar quanta for the black hole geometry that corresponds to these branes (at sufficiently strong coupling). These rates are found to agree with each other.

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1. Introduction

Recently D-branes [1] have provided significant insights into the thermodynamics of black holes in string theory [2]. In particular, a configuration of 1-D branes and 5-D branes compactified on a suitable five dimensional compact space (which can be $K3 \times S^1$ or T^5) represents a six dimensional black string, or a five dimensional black hole for distance scales larger than the size of the string. For such a configuration which is BPS saturated, it was shown that the degeneracy of states for a given mass and charges leads to an entropy which is *exactly* equal to the Beckenstein-Hawking entropy of the corresponding black hole [3]. This is a realization of the idea that the degeneracy of string states is responsible for Hawking-Beckenstein entropy [4], which was pursued for holes with NS-NS charges and zero extremal area in [5] and through quantisation of metric fluctuations for nonzero area holes in [6]. For other work on extremal holes, see [7].

The nature of non-BPS states of D-branes was discussed for the D-string in [8]. Remarkably, it was found in [9] and [10] that the degeneracy of slightly excited non-BPS states also agrees with the Beckenstein-Hawking entropy for the corresponding nonextremal hole. The nature of excitations responsible for the nonextremal entropy is in fact quite similar to that of the D-string - these are now open strings moving along the 1 brane, but with polarizations which lie entirely in the brane directions³. In [12] it was found that the non-BPS excitations of some other black branes have thermodynamical properties that agree with predictions from black hole thermodynamics, though there are also many exceptions that arise at least from a naive application of this correspondence. See [13] for some recent work on nonextremal holes.

Even more remarkably, [9] showed that the rate of decay of such an excited state via annihilation of oppositely moving open strings into a closed string which then escapes from the brane qualitatively (i.e. apart from numerical factors) agrees with the expectations from hawking radiation and the effective temperature of the outgoing state agrees exactly with the hawking temperature of the nonextremal hole. (The analogue of this calculation for absorption has been recently analysed in [14] where it is also shown that the classical cross-section for absorption of low energy scalars by the black hole is proportional to the area like the D-brane calculation in [9].)

³ The nature of these open strings depend on whether the 5 brane charge is equal to or greater than 1. See [11]

These results for nonextremal holes could appear surprising, since the regime in which the branes act as black holes is when the open string coupling is large, whereas the above calculations were performed at weak coupling. (It was argued in [8], however, that when the length of a D-string is large, the weak coupling results remain accurate for long wavelength modes even at somewhat larger coupling.) The issues in this regard are not clear, and there is no clear reason to expect that non-BPS processes agree between D-brane calculations at low orders in coupling and semiclassical results based on the geometry produced by the branes at stronger coupling.

In this paper we carry out two calculations:

- (a) In the first calculation we take the configuration discussed in [3] and [9]. We take the case with a single 5-brane wound on a T^5 , and a collection of D-strings (wound along one of the directions of the T^5), with net momentum along the D-string direction. We compute the rate of emission of low energy quanta from a slightly nonextremal state of this configuration. To compute the decay amplitude we use the Born-Infeld action for describing the long wavelength fluctuations of the D-string within the 5-D-brane. (Note that as discussed in [8] and [15] the D-strings should behave as one long string rather than a collection of singly wound strings, and we adopt this model here.) From this action we derive the coupling of the oscillations of the string to the gravitons in the 10-dimensional spacetime. Because of the compactification on T^5 , we further decompose these gravitons into scalars, vectors and gravitons of the 5-dimensional non-compact spacetime. Because the D-string can only vibrate within the 5-brane, of these fields only the 5-dimensional scalars will be emitted.
- (b) In the second calculation we compute the probability that a low energy scalar is absorbed by an extremal black hole with geometry corresponding to the charges carried by the D-branes in calculation (a) above. Here we follow the method of [16], which has also been used in the context of a slightly nonextremal 5-dimensional hole recently in [14]. It is found that the effective cross-section is the area of the horizon, just as was the case in the 3+1 dimensional case of [16]. At least in 3+1 dimensions the absorption cross section (and therefore the emission rate) of low energy particles of spin one and spin two vanish for low energies [17]. It is plausible that such is the case here as well, but we do not address that calculation in this paper.

We find that the absorption rate for scalars agree between the calculations (a) and (b). Photons and gravitons are not emitted at this order in the calculation (a); if the 3+1 dimensional calculation is an accurate guide to the 4+1 dimensional extremal case then there is agreement here as well.

2. General features of D-string amplitudes

In a companion paper [18] we have considered various aspects of amplitudes for the decay of an excited D-string into a massless closed string state of low energy. Since the relevant massless excitations of the 1D brane and 5D brane configuration are similar to those of a D-string, we recall some of the basic features. For such low energy processes we can use the Dirac-Born-Infeld action to compute the amplitudes. This may be written in terms of the coordinates of the D-string $X^\mu(\xi^m)$ (where μ runs over all the 10 indices whereas ξ^m are parameters on the D-string worldsheet) and the gauge fields on the D-string worldsheet $A^m(\xi^m)$ as follows [19]

$$S_{BI} = T \int d^2\xi e^{-\phi(X)} \sqrt{\det [G_{mn}(X) + B_{mn}(X) + F_{mn}]} \quad (2.1)$$

where F_{mn} denotes the gauge field strength on the D-string worldsheet and G_{mn}, B_{mn} are the background (string) metric and NS-NS 2-form fields induced on the worldsheet

$$G_{mn} = G_{\mu\nu}^{(S)}(X) \partial_m X^\mu \partial_n X^\nu \quad B_{mn} = B_{\mu\nu}(X) \partial_m X^\mu \partial_n X^\nu \quad (2.2)$$

T is a tension related to the D-string tension by $T^D = e^{-\phi/2} T$. We will work in the static gauge which means

$$X^0 = \xi^0 \quad X^1 = \xi^1 \quad (2.3)$$

In this gauge the massless open string fields which denote the low energy excitations of the brane are the transverse coordinates $X^i(X^0, X^1)$, $i = 2, \dots, 9$.

In the following we will set the gauge field and the RR field to be zero. The lowest order interaction between the metric fluctuations around flat space and the open string modes is obtained by expanding the metric as $G_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(X)$, expanding the transverse coordinates $X^i(\xi)$ around the brane position $X^i = 0$ and treating $h_{\mu\nu}$ and X^i to be small. Here κ is the ten dimensional gravitational coupling present in the bulk action (in terms of the einstein metric)

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{g} [R - \frac{1}{2}(\nabla\phi)^2 + \dots] \quad (2.4)$$

For purely transverse gravitons, i.e. only $h_{ij} \neq 0$, the terms upto two open string fields is (after rescaling $X \rightarrow \sqrt{T^D} X$)

$$\frac{1}{2}(\delta_{ij} + 2\kappa h_{ij}) \partial_\alpha X^i \partial^\alpha X^j \quad (2.5)$$

Note that to this order the lagrangian is independent of T^D . We will be interested in the decay amplitude for a closed string state in a specific polarization state, say h_{67} . Note that the quadratic action for the h_{ij} which follows from the bulk action (2.4) is (in the harmonic gauge)

$$\int d^{10}x \frac{1}{2} (\partial h_{ij})(\partial h_{ij}) \quad (2.6)$$

Since h_{ij} are symmetric the field h_{67} does not have a properly normalized kinetic term. The properly normalized field is then $\bar{h}_{67} = \sqrt{2}h_{67}$. This means that the interaction term with a \bar{h}_{67} with two open string fields is, from (2.5),

$$\sqrt{2}\kappa \bar{h}_{67} \partial X^6 \partial X^7 \quad (2.7)$$

Consider a D-string which is excited above the BPS state by addition of a pair of open string states with momenta (on the worldsheet) (p_0, p_1) and (q_0, q_1) respectively. The decay of this state into the extremal state is given by the process of annihilation of this pair into a closed string state, like a graviton. For a graviton represented by \bar{h}_{ij} with momentum (k_0, k_1, \vec{k}) (where \vec{k} denotes the momentum in the transverse direction), the leading term for this amplitude for low graviton energies can be read off from (2.7) as

$$\mathcal{A}_D = \sqrt{2}\kappa A_D = \sqrt{2}\kappa p \cdot q \quad (2.8)$$

When the outgoing graviton does not have any momentum along the string direction one has

$$p \cdot q = p^0 q^0 - p^1 q^1 = 2|p_1|^2 \quad (2.9)$$

where we have used momentum conservation in the string direction and the masslessness of the modes. As shown in [20] a direct conformal field theory calculation of the decay rate agrees with the above answer for low energies and transverse polarizations.

The pair of colliding open strings is part of a one dimensional gas of open strings. To obtain the decay rate of the nonextremal state one has to compute the decay rate for this specific initial state and then average over initial states. The latter averaging is responsible for the thermal nature of the outgoing closed string.

3. D-brane thermodynamics

In this section we discuss the thermodynamics for massless open string states moving on the D-brane. We essentially have the statistical mechanics of massless particles on the brane. Typically we have a net momentum in one of the directions so actually we have an ensemble with given total energy E and total momentum P along say the X^1 direction. For large size of the D-brane we can approximate this by a canonical type ensemble characterized by an inverse momentum β and a chemical potential α as follows. Let there be n_r particles with energy e_r and momentum in the X^1 direction p_r . Define a partition function \mathcal{Z} by

$$\mathcal{Z} = e^h = \sum_{states} \exp \left[-\beta \sum_r n_r e_r - \alpha \sum_r n_r p_r \right] \quad (3.1)$$

Then α, β are determined by requiring

$$E = -\frac{\partial h}{\partial \beta} \quad P = -\frac{\partial h}{\partial \alpha} \quad (3.2)$$

The average number of particles n_r in state (e_r, p_r) is then given by

$$\rho(e_r, p_r) = \frac{1}{e^{\beta e_r + \alpha p_r} \pm 1} \quad (3.3)$$

where as usual the plus sign is for fermions and the minus sign is for bosons. Finally the entropy S is given by the standard thermodynamic relation

$$S = h + \alpha P + \beta E \quad (3.4)$$

For the case of a D-string with f species of bosons and f species of fermions the above quantities may be easily evaluated

$$\begin{aligned} P &= \frac{fL\pi}{8} \left[\frac{1}{(\beta + \alpha)^2} - \frac{1}{(\beta - \alpha)^2} \right] & E &= \frac{fL\pi}{8} \left[\frac{1}{(\beta + \alpha)^2} + \frac{1}{(\beta - \alpha)^2} \right] \\ S &= \frac{fL\pi}{4} \left[\frac{1}{\beta + \alpha} + \frac{1}{\beta - \alpha} \right] \end{aligned} \quad (3.5)$$

Since we have massless particles in one spatial dimension, they can be either right moving, with $e_r = p_r$ or left moving $e_r = -p_r$. The distribution functions then become

$$\begin{aligned} \rho_R &= \frac{1}{e^{(\beta + \alpha)e_r} \pm 1} & \text{R} \\ \rho_L &= \frac{1}{e^{(\beta - \alpha)e_r} \pm 1} & \text{L} \end{aligned} \quad (3.6)$$

Thus the combinations $T_R = 1/(\beta + \alpha)$ and $T_L = 1/(\beta - \alpha)$ act as *effective* temperatures for the right and left moving modes respectively. In fact all the thermodynamic quantities can be split into a left and a right moving piece : $E = E_R + E_L$ $P = P_R + P_L$ $S = S_R + S_L$ in an obvious notation. The various quantities $E_L, E_R, P_L, P_R, S_L, S_R$ may be read off from (3.5). From (3.5) we get

$$T_R = \sqrt{\frac{8E_R}{L\pi f}} \quad T_L = \sqrt{\frac{8E_L}{L\pi f}} \quad (3.7)$$

The extremal state corresponds to $P_L = E_L = 0$ so that $E = P$. Finally we note that

$$T_L = \frac{4S_L}{\pi f L} \quad T_R = \frac{4S_R}{\pi f L} \quad (3.8)$$

This relation will be crucial in physical properties of the decay rate which we now calculate.

Finally we note that the left right splitting is a feature of statistical mechanics of massless particles in one (spatial) dimension, which is relevant to the D-string as well as other examples of D-strings bound to 5D branes to be considered below. For higher dimensional branes, like the three brane, such simplifications may not apply, except for certain limiting values of the parameters. However the method of introducing a chemical potential for the total momentum can be used in this case as well, as we will discuss in a later communication.

4. The decay rate for D-string

The S-matrix element for the decay is given by

$$S_{fi} = \sqrt{2}\kappa(2\pi)^2 \delta(p_0 + q_0 - k_0) \delta(p_1 + q_1 - k_1) \frac{(-iA_D)}{\sqrt{(2p_0L)(2q_0L)(2k_0V_9)}} \quad (4.1)$$

where V_9 denotes the nine dimensional spatial volume and L is the length of the D-string. If R is the radius of compactification of the string direction, then for a multiply wound string one has $L = 2\pi n_w R$. The total spatial volume $V_9 = V_8(2\pi R)$ where V_8 is the volume of the noncompact space. The decay rate for this pair to produce a graviton of zero momentum in the string direction and with a *given* polarization is then

$$\Gamma(p, q, k) = \frac{\kappa_9^2(2\pi)^2}{4L} \delta(p_0 + q_0 - k_0) \delta(p_1 + q_1 - k_1) \frac{|A_D|^2}{p_0 q_0 k_0 V_8} \frac{V_8 [d^8 k]}{(2\pi)^8} \quad (4.2)$$

where we have introduced the nine dimensional gravitational coupling $\kappa_9^2 = \frac{\kappa^2}{2\pi R}$.

To obtain the total rate to produce a graviton with the given momentum we have to average over all initial states. This includes a sum over the momenta and polarizations of the open string states. When the open string momentum quantum numbers are large (but still much smaller than the net momentum quantum number) this means that we have to multiply by the relevant thermal distribution functions obtained in the previous section. Denoting the distribution functions by $\rho(p_0, p_1)$ and $\rho(q_0, q_1)$ the total decay rate is

$$\Gamma(k) = \int_{-\infty}^{\infty} \frac{Ldp_1}{2\pi} \int_{-\infty}^{\infty} \frac{Ldq_1}{2\pi} \Gamma(p, q, k) \rho(q_0, q_1) \rho(p_0, p_1) \quad (4.3)$$

We will consider the emission of a graviton of a specified polarization, say $\epsilon_{67} = 1$ and the others zero. Let us first specialize to the case where the outgoing particle has $k_1 = 0$. Then this graviton may be produced only by the annihilation of two open strings with exactly equal and opposite momenta. There are two possible polarizations of the initial state. The first would be when the left moving open string has a polarization in the 6 direction and the right moving in the 7 direction and the second would be the other way round. However once the momentum integral in (4.3) is over the entire range $[-\infty, \infty]$ we have automatically summed over these two polarization states.

We will evaluate the rate for low energies where we can use the expression for \mathcal{A}_D in the previous section. The integral over q_1 sets $q_1 = -p_1$ and one is left with

$$\Gamma(k) = \frac{\kappa_9^2 L [d^8 k]}{k_0 (2\pi)^8} \int_{-\infty}^{\infty} dp_1 \delta(2|p_1| - k_0) |p_1|^2 \rho(|p_1|, p_1) \rho(|p_1|, -p_1) \quad (4.4)$$

The integration can be done

$$\begin{aligned} & \int_{-\infty}^{\infty} \delta(2|p_1| - k_0) |p_1|^2 \rho(|p_1|, p_1) \rho(|p_1|, -p_1) \\ &= 2 \cdot \frac{1}{2} \left(\frac{k_0}{2}\right)^2 \rho\left(\frac{k_0}{2}, \frac{k_0}{2}\right) \rho\left(\frac{k_0}{2}, -\frac{k_0}{2}\right) \end{aligned} \quad (4.5)$$

where the factor 2 comes from the two values of p_1 where the delta function clicks and the factor of $\frac{1}{2}$ comes from the Jacobian involved in performing the integration. Note that the two distributions which appear are the left and right distributions defined in (3.6).

Since the initial state is only slightly nonextremal, we have $E_L = E_{BPS} + \Delta E$ and $E_R = \Delta E$ and $\Delta E \ll E_{BPS}$ Then (3.7) implies that $\frac{p_0}{T_L}$ is small.

First consider the case where the open string states are bosonic. Then we can approximate $\rho_L(p_0)$ in (4.4) as

$$\rho_L(p_0) \sim \frac{T_L}{p_0} = \frac{4S_L}{p_0 L \pi f} \quad (4.6)$$

where we have used (3.8). In this case of a multiply wound D-string we have $f = 8$. The final result is

$$\Gamma(k) = \frac{\kappa_9^2 S_{ext}}{2^{10} \pi^9} [d^8 k] \frac{1}{e^{\frac{k_0}{T_{eff}}} - 1} \quad (4.7)$$

where we have defined an effective temperature of the graviton spectrum to be

$$T_{eff} = 2T_R \quad (4.8)$$

and have used the fact that $S_L \sim S_{ext}$ where S_{ext} is the extremal entropy. It is important to note that the size of the compact direction has disappeared from this answer.

When the initial open string states are fermionic $\rho_L(p_0)$ becomes order unity in the near extremal case rather than the large quantity (4.6) for the bosonic case. Thus when the extremal entropy is large, these states do not contribute to $\Gamma(k)$ in the leading order.

The crucial fact about (4.7) is that the answer comes out to be proportional to the extremal entropy. This is a consequence of the fact that we have essentially one dimensional thermodynamics where the temperature is proportional to the entropy. In the next section we will deal with brane configurations which correspond to nonzero extremal horizon area and where the extremal entropy agrees with the Hawking-Beckenstein formula and hence proportional to the horizon area. We then get a result proportional to the horizon area.

5. Decay Rate for 1-Brane 5-brane configurations

The results of the previous section may be used to calculate the decay process for situations where the configuration of branes produce a spacetime which has a large horizon in the extremal limit. In the following we will do so for the model similar to that considered in [9]. We will consider a configuration of one 5D-brane wrapped around a T^5 in the $(X^5 - X^6 - X^7 - X^8 - X^9)$ direction and single D-string wound Q_1 times around the X^5 direction. The radius of the X^5 direction is R which is taken to be large. When the T^4 in the $(6 - 7 - 8 - 9)$ direction is small this represents a black string in six dimensions one of which is compact but large.

In this case, as argued in [11] the low energy excitations of the system are described by massless modes of open strings which begin and end on the D-string and whose polarization

vectors λ^i lie in the $6 - 7 - 8 - 9$ plane. These modes thus live on an effective length $L = 2\pi Q_1 R$. There are 4 such bosonic and 4 fermionic modes. The extremal state corresponds to the case when all these open strings are moving in the same direction and a nonextremal situation corresponds to strings moving in either directions.

We thus have a situation similar to the case of a single D-string discussed in the previous sections with the number of flavors f in Section 5 being 4. As pointed out in [15] the thermodynamic formulae are valid when extensivity holds. This is indeed the case here in the “fat hole” limit. For example in the extremal limit $E_R = 0$ and $E_L = \frac{N}{R}$ and from (3.7) one has

$$T_L L = 2\sqrt{NQ_1} \quad (5.1)$$

which is large when Q_1 and N are of the same order and large. The extremal entropy is

$$S_{ext} = 2\pi\sqrt{NQ_1} = \frac{A_H}{4G_5} \quad (5.2)$$

where A_H is the horizon area and G_5 is the five dimensional Newton constant.

The amplitude for the decay of a nonextremal hole is identical to that in section 4. However since the low energy open string modes have polarizations λ^i where $i = 6 \dots 9$ it follows from (2.8) that at lowest order the only gravitons which are produced have polarization in these directions. From the five dimensional point of view these are in fact scalars. In the following we will examine the decay rate for the production of a given polarization state, say the situation in which only ϵ_{67} is nonzero.

The S-matrix element is identical to (4.1). However the decay rate $\Gamma(p, q, k)$ for an outgoing closed string mode which has no momentum along the X^5 direction is a slight modification of (4.2)

$$\Gamma(p, q, k) = \frac{\kappa_5^2 (2\pi)^2}{4L} \delta(p_0 + q_0 - k_0) \delta(p_1 + q_1 - k_1) \frac{|A_D|^2}{p_0 q_0 k_0 V_4} \frac{V_4 [d^4 k]}{(2\pi)^4} \quad (5.3)$$

V_4 denotes the volume of the spatial noncompact four dimensions, while $\kappa_5^2 = \frac{\kappa^2}{2\pi R \tilde{V}_4}$ where \tilde{V}_4 denotes the volume of the compact directions $X^6 \dots X^9$. The total decay rate is given by (4.3), the $\Gamma(p, q, k)$ in this equation being given by (5.3).

We now use the low energy result for A_D given in (2.8) and (2.9) and integrate over q_1 and p_1 . These integrations are identical to those in the previous section and one has

$$\Gamma(k) = \frac{\kappa_5^2 L [d^4 k]}{(2\pi)^4 k_0} \left(\frac{k_0}{2}\right)^2 \rho_L(k_0/2) \rho_R(k_0/2) \quad (5.4)$$

For low energies we use (4.6) with $f = 4$ to get

$$\rho_L(p_0) \sim \frac{T_L}{p_0} = \frac{A_H}{4\pi G_5 L p_0} \quad (5.5)$$

Plugging (5.5) into (5.4), and performing the p_1 integral as in the previous section and using $\kappa_5^2 = 8\pi G_5$ we finally get

$$\Gamma(k) = \frac{A_H}{16\pi^4} [d^4 k] \rho_R(k_0/2) \quad (5.6)$$

Then the total energy emitted in an energy range $k_0, k_0 + dk_0$ per unit time is obtained by multiplying (5.6) with k_0 and writing out the phase space factor $[d^4 k] = 2\pi^2 k_0^3 dk_0$. We finally obtain

$$\frac{dE(k)}{dt} = \frac{A_H}{8\pi^2} \frac{k_0^4 dk_0}{e^{\beta_H k_0} - 1} \quad (5.7)$$

The Hawking temperature is $T_H = 1/\beta_H = 2T_R$ and as noted in [9] agrees with the temperature defined by the surface gravity at the horizon.

6. The classical cross section.

In this section we find the classical cross section for absorption of low energy scalar quanta into the extremal black hole having the charges of the D-brane model discussed in the previous section. The method of computing such low energy cross-sections for four dimensional holes was given in [16], and a calculation for a slightly nonextremal hole in five dimensions has been recently carried out in [14]. We will carry out the calculation for the extremal hole, and observe that it agrees with the extremal limit of [14], though the actual details of the calculation differ in the extremal case and in the slightly nonextremal case.

The extremal metric for the five dimensional black hole is given by

$$ds^2 = -[f(r)]^{-\frac{2}{3}} dt^2 + [f(r)]^{\frac{1}{3}} dr^2 + [f(r)]^{\frac{1}{3}} r^2 d\Omega_3^2 \quad (6.1)$$

where

$$f(r) = \left(1 + \frac{Q_1}{r^2}\right) \left(1 + \frac{Q_2}{r^2}\right) \left(1 + \frac{Q_3}{r^2}\right) \quad (6.2)$$

The massless minimally coupled scalar wavefunction is

$$\phi(r, t) = R(r) e^{-i\omega t} \quad (6.3)$$

where we restrict to spherically symmetric wavefunctions, since as shown in [16], the higher angular momentum components are not absorbed in the limit of low frequencies. The wave equation reduces to

$$\left[\frac{d^2}{dr^2} + \omega^2 f(r) - \frac{3}{4r^2}\right]\psi(r) = 0 \quad (6.4)$$

where

$$\psi(r) = r^{3/2}R(r) \quad (6.5)$$

The idea of [16] is to solve this equation approximately in three regions, and match the solutions across the boundaries of the regions. It is assumed that it is adequate to keep only the lowest order terms in ω in each region; we will adopt this assumption here as well.

6.1. Outer region:

The outermost region is $r \gg Q_i^{1/2}$. In this region we get

$$f = 1 + \frac{Q}{r^2} \quad (6.6)$$

where

$$Q = Q_1 + Q_2 + Q_3 \quad (6.7)$$

Defining $\rho = \omega r$, the equation is

$$\left[\frac{d^2}{d\rho^2} + \left(1 + \frac{Q\omega^2 - 3/4}{\rho^2}\right)\right]\psi = 0 \quad (6.8)$$

The solution may be written as

$$\psi(\rho) = \alpha F(\rho) + \beta G(\rho) \quad (6.9)$$

where the two independent solutions may be written in terms of Bessel functions

$$\begin{aligned} F &= \sqrt{\frac{\pi}{2}}\rho^{1/2}J_{(1-Q\omega^2)^{1/2}}(\rho) \\ G &= \sqrt{\frac{\pi}{2}}\rho^{1/2}J_{-(1-Q\omega^2)^{1/2}}(\rho) \end{aligned} \quad (6.10)$$

In the region $\rho \gg 1$ we have

$$\begin{aligned} F &= \cos(\rho - \pi/2(1 - Q\omega^2)^{1/2} - \pi/4) \\ G &= \cos(\rho + \pi/2(1 - Q\omega^2)^{1/2} - \pi/4) \end{aligned} \quad (6.11)$$

In terms of a shifted coordinate $\rho' = \rho - \frac{\pi}{4}$ we get

$$\psi = e^{i\rho'} \left[\frac{i}{2} (-\alpha e^{i\pi/4 Q\omega^2} + \beta e^{-i\pi/4 Q\omega^2}) \right] + e^{-i\rho'} \left[\frac{i}{2} (\alpha e^{-i\pi/4 Q\omega^2} - \beta e^{i\pi/4 Q\omega^2}) \right] \quad (6.12)$$

implying a reflection coefficient

$$\mathcal{R} = -e^{i\frac{\pi}{2} Q\omega^2} \frac{1 - \frac{\beta}{\alpha} e^{-i\frac{\pi}{2} Q\omega^2}}{1 - \frac{\beta}{\alpha} e^{i\frac{\pi}{2} Q\omega^2}} \quad (6.13)$$

This will give for the absorption probability

$$|\mathcal{A}|^2 = 1 - |\mathcal{R}|^2 \quad (6.14)$$

We thus need to find α , β , from the requirement that only an inward moving wave exists at the horizon; we will compute these parameters below.

From the behavior of the Bessel functions at small argument we find that for $\rho \ll 1$, the solution to the wave equation is

$$R(r) \sim \sqrt{\frac{\pi}{2}} \omega^{\frac{3}{2}} \left[\frac{1}{2} \alpha + \frac{\beta Q}{r^2} \right] \quad (6.15)$$

6.2. Intermediate region

This region has $r \sim Q_i^{1/2}$. Then the equation (for $Q\omega^2 \ll 1$) is

$$\frac{1}{r^3} \frac{d}{dr} r^3 \frac{dR}{dr} = 0 \quad (6.16)$$

which gives

$$R = C + \frac{D}{r^2} \quad (6.17)$$

6.3. Near horizon region:

This region is $r \ll Q_i^{1/2}$. Here the wave equation is

$$\frac{1}{r^3} \frac{d}{dr} r^3 \frac{dR}{dr} + \frac{\omega^2 P}{r^6} (1 + \mu r^2) R = 0 \quad (6.18)$$

where $P = Q_1 Q_2 Q_3$ and $\mu = 1/Q_1 + 1/Q_2 + 1/Q_3$. Define

$$u = -\frac{1}{2r^2}, \quad \rho = u\omega\sqrt{P}, \quad \eta = \frac{1}{4}\mu\omega\sqrt{P} = \frac{1}{4}\omega\sqrt{Q_1 Q_2 Q_3} (1/Q_1 + 1/Q_2 + 1/Q_3) \quad (6.19)$$

Then the wave equation is

$$\frac{d^2 R}{d\rho^2} + \left(1 - \frac{2\eta}{\rho}\right)R = 0 \quad (6.20)$$

The two independent solutions are given by Coulomb functions [21]. Very close to the horizon, $\rho \rightarrow -\infty$ the two independent solutions are

$$\begin{aligned} F_0 &= \sin\left[\pi/4 + \rho + \eta \log\left(\frac{\eta}{2\rho}\right)\right] \\ G_0 &= \cos\left[\pi/4 + \rho + \eta \log\left(\frac{\eta}{2\rho}\right)\right] \end{aligned} \quad (6.21)$$

The particular linear combination we have to pick is

$$R(r) = G_0(r) - iF_0(r) \quad (6.22)$$

to get an ingoing wave at large negative ρ . At small $|\rho|$ we get

$$R \sim \frac{1}{\sqrt{2}}[(1-i) - (1+i)\rho] = \frac{1}{\sqrt{2}}[(1-i) + (1+i)\frac{\omega\sqrt{P}}{2r^2}] \quad (6.23)$$

6.4. Matching the solutions

Comparing (6.23) with (6.17) one gets

$$C = \frac{1}{\sqrt{2}}(1-i) \quad D = \frac{1}{2\sqrt{2}}(1+i)\omega\sqrt{P} \quad (6.24)$$

Matching the solution (6.15) with (6.17) and using (6.24) we get

$$\alpha = \frac{2(1-i)}{\omega^{3/2}\sqrt{\pi}} \quad \beta = \frac{(1+i)\sqrt{P}}{2\sqrt{\pi}Q\omega^{1/2}} \quad (6.25)$$

We can now easily compute the absorption probability $|A|^2$ from (6.14),

$$|A|^2 = \frac{1}{2}\pi\omega^3\sqrt{P} \quad (6.26)$$

This may be expressed in terms of the area of horizon $A_H = 2\pi^2\sqrt{P}$ as

$$|A|^2 = \frac{1}{4\pi}\omega^3 A_H \quad (6.27)$$

6.5. The absorption cross section

We now calculate the absorption cross section of a plane wave incident on the hole. Let us expand a plane wave as follows

$$e^{i\omega z} = K \frac{e^{-i\omega r}}{r^{3/2}} Z_{000} + \text{other terms} \quad (6.28)$$

where we have kept only the radially inward momentum component and ignored modes that are not spherically symmetric. (As mentioned above, we expect that the higher angular momentum waves have an absorption coefficient that is suppressed at low energies.) The quantity Z_{000} is normalized over a three sphere $Z_{000} = \frac{1}{\sqrt{2\pi^2}}$

Once K is known the absorption cross-section of the S-wave is given by

$$\sigma = |K|^2 |A|^2 \quad (6.29)$$

An efficient way of determining K is to integrate both sides of (6.28) over all angles with the standard measure of a three sphere. This projects out only the spherically symmetric part, thus the first term of the right hand side of (6.28). One gets

$$4 \frac{2\pi^{\frac{3}{2}}}{\omega r} \Gamma\left(\frac{3}{2}\right) J_1(\omega r) = \sqrt{2\pi^2} K \frac{e^{-i\omega r}}{r^{\frac{3}{2}}} \quad (6.30)$$

Using the asymptotic form of the Bessel function $J_1(kr)$ and isolating the outgoing component one easily gets

$$|K|^2 = \frac{4\pi}{\omega^3} \quad (6.31)$$

Thus the absorption crosssection is

$$\sigma = |K|^2 |A|^2 = A_H \quad (6.32)$$

where we have used (6.27).

Thus we get a result similar to that in [16], where it was found that for the 3+1 dimensional Schwarzschild hole the low energy cross section for scalars is the area of the horizon. Thus this cross sections differs by a factor of order unity from the geometrical cross section, which is the effective cross section expected for quanta with wavelength much smaller than the horizon size.

The ‘inner region’ that we used above had the form of an infinite throat with constant diameter, and the solution of the wave equation in this region was taken as (6.21). But this solution assumes that the length of the throat L_h is much larger than the wavelength ω^{-1} of the wave. While this condition is indeed satisfied by a throat of infinite length, one sees that if the hole is slightly nonextremal, then the horizon is encountered after a finite length of the throat, and the calculation must stop at this horizon. In fact in the calculation of [14] the hole was close to extremality, but the limit of $\omega \rightarrow 0$ was taken first, so that the length L_h of the throat became much smaller than the wavelength ω^{-1} , and the nature of the wave equation and the boundary condition in the inner region became different from what we had above. Nevertheless, the cross section computed from this alternative set of limits agrees with what was computed above (where the limit of extremality is taken first, and $\omega \rightarrow 0$ is taken later).

7. Comparison of the D-brane and Classical result

According to the standard semiclassical derivation of Hawking radiation [22] the total luminosity of particles of a given bosonic species radiated from a given black hole in an energy interval $(\omega, \omega + d\omega)$

$$\left[\frac{dE(\omega)}{dt}\right]_{sc} = \sum_i \frac{d\omega}{2\pi} \frac{\omega \Gamma_{\omega i}}{e^{\beta_H \omega} - 1} \quad (7.1)$$

where i collectively denotes the various angular momentum quantum numbers and Γ_i denotes the absorption probability of a wave with energy ω and angular quantum numbers i . As mentioned earlier, for scalars the dominant decay at low ω is into S-waves. In this case $\Gamma_{\omega} = |A|^2(\omega)$ computed in the previous section.

Assuming S-wave dominance and using (6.27) in (7.1) we thus get for our case

$$\left[\frac{dE(\omega)}{dt}\right]_{sc} = \frac{A_H}{8\pi^2} \frac{\omega^4 d\omega}{e^{\beta_H \omega} - 1} \quad (7.2)$$

This is in *exact* agreement with the D-brane calculation (5.7).

8. Discussion

One interesting feature of the D-brane picture for this five dimensional black hole is that the low energy excitations on the brane are scalars or spin-1/2 fermions from the five dimensional point of view. As a result, in the lowest order process considered in this paper, photons or gravitons are not produced. Presumably these may be produced from annihilation of pairs of fermionic open strings. As mentioned above, in the semiclassical calculation of Hawking radiation, production of particles with spin one and spin two are suppressed as well⁴. For four dimensional Schwarzschild holes, the absorption probability for spin-1 particles behaves as $\sim A_H^2 \omega^4$ for spin-2 particles it behaves as $\sim A_H^3 \omega^6$, while it is $\sim A_H \omega^2$ for scalars [17]. It is reasonable to expect that a similar situation holds for the five dimensional extremal hole e.g. the absorption probability for spin-1 should be $\sim A_H^2 \omega^6$ and $\sim A_H^3 \omega^9$ for spin-2. These powers of A_H should follow from the various thermal distribution functions. It is important to check whether the D-brane results and the semiclassical calculations for these cases are in agreement as well. We expect to report

⁴ We would like to thank J. Maldacena, G. Mandal and L. Susskind for discussions on this point

on this in the near future. Another interesting computation is a comparison of the emission of charged particles (i.e. particles which have a nonzero momentum along the D-string direction).

In this paper we have computed the process of emission of scalars from the collision of open strings on the D-string. It is also possible to compute the reverse process [18], where an incoming scalar interacts with the D-string to create a pair of oppositely moving open strings on the D-string. This computation leads directly to an absorption cross section equal to the area of the horizon of the 5-dimensional hole, as expected from the result presented here.

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