A COMPARISON OF TWO IMAGE ROTATING PRISMS

by M. V. R. K. MURTY, *Spectroscopy Division, Bhabha Atomic Research Centre, Bombay*

(*Received 15 February 1973*)

The dove prism is widely used in periscopes and other optical instruments to rotate the image azimuth in the field of view. The dove prism is compared to another image rotating prism. The comparison is in respect of the base length of the prism and the area required to pass a given beam width. Both the prisms have three polished sides, which are made of a single piece of glass, and have to be used in collimated light and hence the comparison.

INTRODUCTION

Image rotating prisms are generally used in periscopes and other optical instruments to orient the azimuth of the image in the field of view. A typical prism used for this purpose is a dove prism placed in parallel light. Several prisms are described by Hopkins (1965). In this reference, prisms 1, 3, 5, 7 and 9 give image rotation and a left-handed image because the number of reflections is odd. Prisms 5, 7 and 9 can be used in convergent light because the optical axis is normal to the entrance and exit faces. But these prisms have to be fabricated by cementing two or more individual prisms. Prisms 1 and 3 are made of single piece of glass and are, therefore, comparable. However, the optical axis enters and emerges obliquely through the faces of these prisms and hence they should be used in parallel light. Prism 1 is known as dove prism and prism 3 is called delta prism. The path of optical axis in these two prisms is shown in Figs. 1 and 3. The object of the present study is to compare the physical dimensions of these two types of prisms to pass a beam of light of given width.

THE DOVE PRISM

The shape of the dove prism shown in Fig. 1 (*a*) is generally trapezoidal. The angle of incidence of the optical axis on the bottom reflecting surface is always greater than the critical angle and hence it need not be coated with any reflecting material unless done for some other reasons. The angle of incidence of the optical axis on the entrance face may be chosen at will. The following notation is used in relation to the dove prism shown in Fig. 1.

I = angle of Incidence on the entrance face
R = angle of Refraction on the entrance face
W = width of the parallel beam passed by the dove prism
B = base length of the dove prism
A = area of the dove prism cross section in the plane of the paper.
L = length of the optical axis as shown in figure.

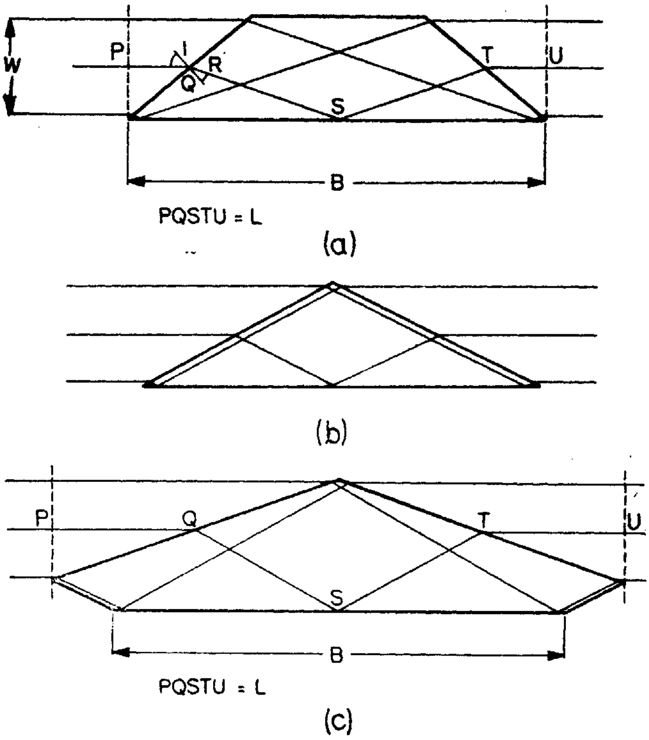

FIG. 1. Ray diagram for a parallel bundle of rays through a Dove prism : (*a*), (*b*) and (*c*) are three shapes for the Dove prism as the angle of incidence increases. The length *PQSTU* represents the total length of the light path between the planes *P* and *U*.

It can be shown that the base length, area of the prism and the length of the optical axis are given by :

$$B = \frac{W \cos R}{\cos I.\ \sin(I - R)} \tag{1}$$

$$A = W^2 \left(\frac{\cos R}{\cos I.\ \sin(L - R)} - \tan I \right) \tag{2}$$

$$L = W \left(\frac{1}{\sin(I - R)} + \tan I \right) \tag{3}$$

The above formulae are valid for the dove prism for values of I satisfying the inequality given below :

$$(90° - I) \geqslant (I - R) \tag{4}$$

In Eqn. (4), the equality sign corresponds to the shape of dove prism which is a triangle as shown in Fig. 1*b*. For the situation

$$(90° - I) \leqslant (I - R) \tag{5}$$

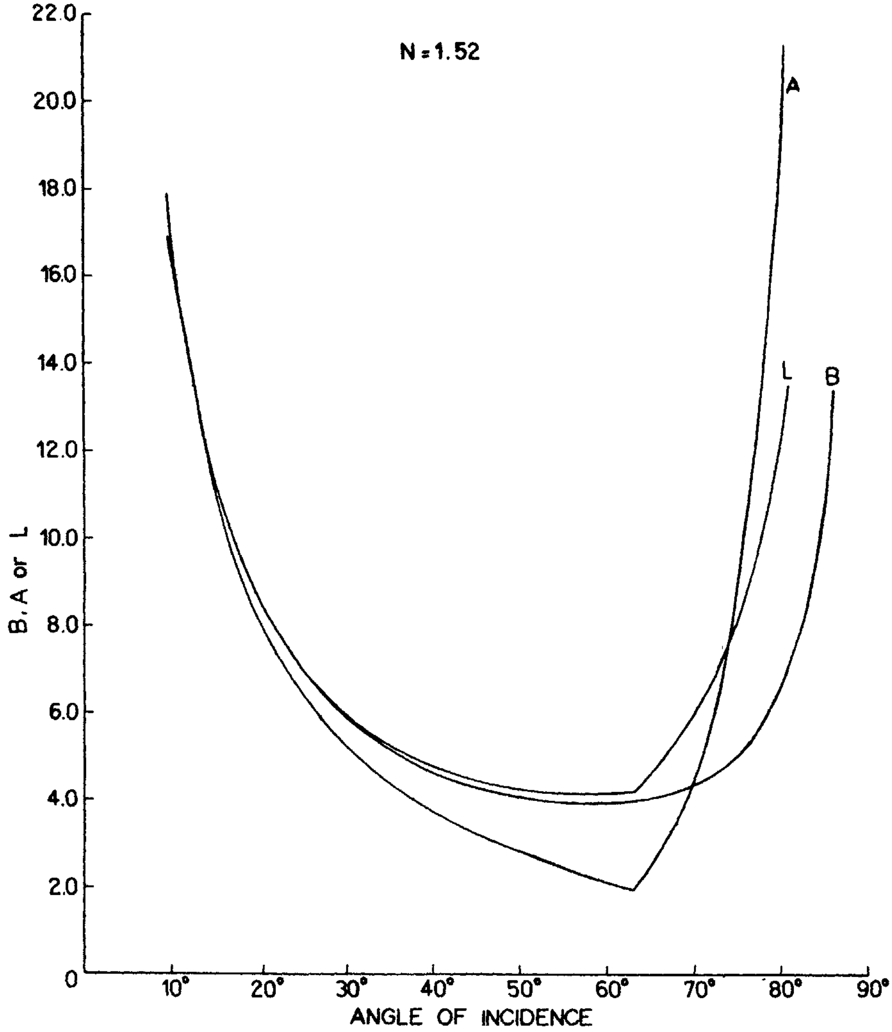

FIG. 2. The plots of B, A and L for various angles of incidence for the Dove prism made of glass of refractive index 1.52. The width of the beam is taken as unity. It may be seen from the curves that the optimum angle of incidence is between 50° and 60°.

formula (1) remains unchanged and formulae (1) and (3) are modified as follows :

$$A = W^2 \frac{\cos^2 R}{\cos^2 I . \sin^2 (I - R)} \left[\frac{\sin(2\,I - R - 90°)}{\cos R} + \tfrac{1}{2} \right] \tag{2a}$$

$$L = W \left[\frac{1}{\sin\,(I - R)} + \tan\,I + \frac{2 \sin(2I - R - 90°)}{\cos I . \sin(2I - R)} \right] \tag{3a}$$

The shape of the dove prism corresponding to the inequality (5) is shown in Fig. 1 (c) its bottom corners are truncated. The length of the optical axis L is measured between two planes which just touch the bottom corners at both ends. This length

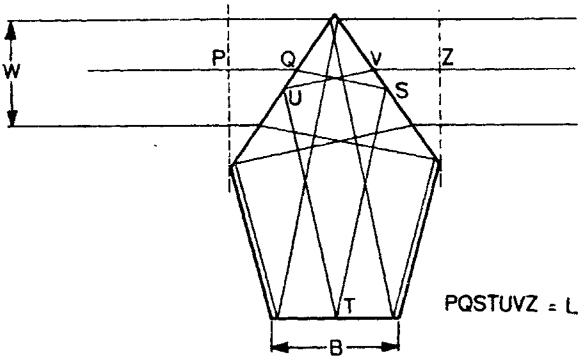

FIG. 3. Ray diagram for the Delta prism.

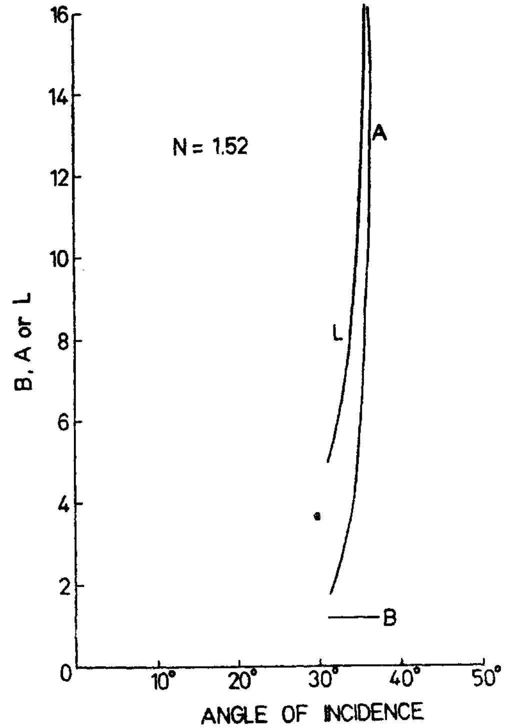

FIG. 4. Plots of B, A and L for various angles of incidence for a Delta prism made of glass of refractive index 1.52. The width of the beam is unity. There is really no optimum angle of incidence for this prism as can be seen from the steep curves for A and L.

is an indirect measure of the field handling capacity of the prism. The longer the path L, the smaller the field that can be handled. Thus, two prisms can be compared regarding their field handling capacity by means of this quantity L. The parameters B, A and L are plotted as functions of the angle of incidence I in Fig. 2 for refractive index $n=1.52$.

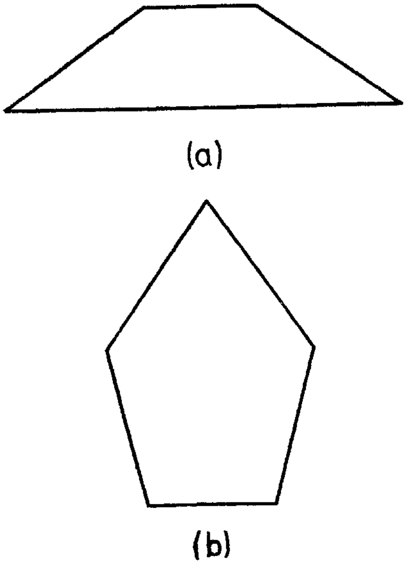

FIG. 5. Comparative shapes of the Dove and Delta prisms. For the Dove prism the angle of incidence has been chosen to be about 56° and for the Delta prism the angle of incidence is chosen to be about 33°.

THE DELTA PRISM

This prism is shown in Fig. 3 and there are three reflections and two transmissions. The entrance and exit faces also act as reflecting surfaces and hence it is necessary that total internal reflection occurs at these two faces. The bottom face also acts as a reflecting surface but, this surface has to be coated with reflecting material as the angles of incidence on this face are small. The same notation may be used here also as in the case of dove prism. Therefore, the various parameters are given by :

$$B = \frac{W \cos R}{\cos I . \sin(3I - R)} \tag{6}$$

$$A = \frac{W^2 \cos^2 R}{\cos^2 I . \sin^2(3I - R)} \left[\frac{\sin(4I - R - 90^\circ)}{\cos(2I - R)} + \tfrac{1}{2} \right] \tag{7}$$

$$L = W \tan I \left[\frac{2(\cos I + \cos R)}{\cos(2I - R)} + \frac{\cos R}{\cos(3I - R) . \cos(2I - R)} - 1 \right] \tag{8}$$

In Fig. 4, the various parameters are plotted as a function of the angle of incidence for a glass of refractive index $n = 1.52$.

It can be seen that the angle of incidence in the case of delta prism has a very narrow range of values due to the restriction that the reflection at the entrance and exit faces should be at more than the critical angle.

This means that we should have :

$$(2I - R) \geqslant \sin^{-1}(1/n) \tag{9}$$

Another condition is that the angle of incidence on the bottom surface should be positive (then only the ray will be cyclic) which means that

$$(3I - R) < 90^\circ \tag{10}$$

Under these restrictions (9) and (10), the range of angles of incidence is limited in most cases to about 29° to 37° the lower limit being governed by (9) and the upper limit by (10).

DISCUSSION

From an examination of the plots in Figs. 2 and 4, it is seen that the dove prism is superior in all respects to the delta prism when a finite field is to be handled. On the other hand, when we are interested in a narrow field, the delta prism may be advantageous because its base length is about the same as the width of the beam to be transmitted. For the dove prism, the base length is usually not less than four times the width of the beam to be passed. But, the area of both prisms is the same. Hence, the weight of the two prisms will be same. Thus, while the dove prism is long, the delta prism is high. Fig. 5 shows the two prisms admitting a beam of 25 mm wide and the relative size and shape may be judged.

REFERENCES

Hopkins, R. E. (1965). Mirror and Prism Systems in Applied Optics and Optical Engineering. R. Ed. Academic Press, Kingslake, New York, Vol. 3, p. 303.