THEORY OF AN ANASTIGMATIC MONOCHROMATOR FOR VACUUM ULTRA-VIOLET REGION

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It is proposed in the present paper to use a concave grating along with a concave mirror to obtain a monochromator in which there is no beam displacement, and the astigmatism is very small. To change the wavelength the concave grating is rotated about an axis passing through itself. The angle between the incident and the diffracted beams are kept small so that the astigmatism is negligible. An analysis of the focussing properties of such a monochromator is made, and it is shown that it can be profitably used in the range 500 Å to 4000 Å. The addition of an extra reflection may not be in itself a disadvantage in view of the fact that reflection properties of coatings are constantly being improved.

INTRODUCTION

In the vacuum ultra-violet region, due to low reflection properties of many surfaces, concave gratings are used as the only single dispersing and focussing element. There are mainly two types of monochromators for this region which are manufactured and used. One type (Fujioka and Ito 1951; Tousey *et al.* 1951) is called the radius mounting in which the concave grating is rotated about an axis passing through the centre of the Rowland Circle. This mounting has very little astigmatism, but it suffers from the displacement of the incident and exit beams as the grating is rotated to obtain different wavelengths. The second type of monochromator is called Seya-Namioka type (Seya 1952; Namioka 1954, 1959*b*; Greiner and Schaffer 1957) and uses a very large angle between the incident and the exit beams (about 70°). To change the wavelength the grating is rotated about an axis passing through itself. Hence the incident and the exit beams are not displaced during scanning. The major disadvantage of this mounting is the presence of large amount of astigmatism. We have discussed a new type of monochromator, in which the dispersing element is a concave grating and the focussing element is a concave mirror. For scanning various wavelengths the grating is rotated about an axis passing through itself. The angle between the incident and the emergent beam is kept very small so that astigmatism is negligible.

THEORY

Fig. 1 shows the diagram of the monochromator. G and M are the concave grating and the concave mirror respectively. S_1 and S_2 are the entrance and the exit slits. O_1Z and O_2Z' are the axes of the concave grating and the concave mirror respectively which are taken parallel to each other. O_1 and O_2 are the central points on the grating and mirror surface respectively. S'_1 is the position of the tangential

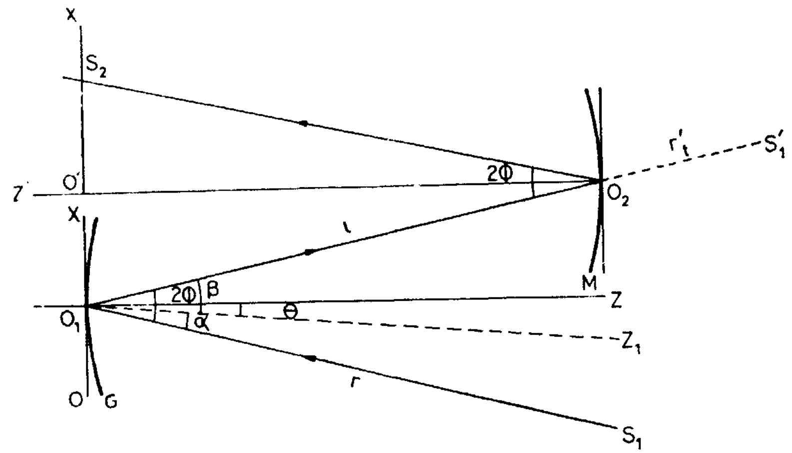

FIG. 1. Schematic arrangement of the concave grating and the concave mirror indicating the various parameters.

focus in the absence of the mirror M. The line OX is tangent to the grating surface at O_1 and $O'\ X'$ is the line passing through S_2 and perpendicular to the mirror axis O_2Z'. For focussing various wavelengths the grating is rotated about an axis which passes through O_1 and perpendicular to the plane of the paper. Thus, S_1O_1 is the principal incident ray, O_1S_1 and O_2S_2 are the corresponding diffracted and reflected rays. O_1Z_1 is the position of the axis O_1Z when the grating is rotated for focussing various wavelengths. Let

$S_1\,O_1$	$= r,$	distance from the entrance slit S_1 to the pole of the grating,
$O_1\,O_2$	$= l,$	distance between the poles of the grating and the mirror along the principal ray,
$O_1\,S_1$	$= r'_t$	distance between the pole of the grating and the focus S'_1,
S'_T	$=$	distance of the tangential focus from O_2 along the principal ray $O_2\,S_2$,
S'_S	$=$	distance of the sagittal focus from O_2 along the principal ray $O_2\,S_2$,
$Z_1\,\hat{O}_1\,Z$	$= \theta,$	angle of rotation of the grating,
$S_1\hat{O}_1O_2$	$= 2\phi,$	angle between the incident ray and the diffracted ray at the pole of the grating and since the mirror and the grating axis are parallel to each other, angle subtended by the incident ray $O_1\,O_2$ and the reflected ray $O_2\,S_2$ at the pole of the mirror is also 2ϕ,
$S_1\,\hat{O}_1\,Z_1$	$= \alpha,$	angle of incidence for any wavelength,
$Z_1\hat{O}_1O_2$	$= \beta,$	angle of diffraction for any wavelength,
$O_1\,X$	$= X,$	measures the aperture of the grating,
$O'\,X'$	$= X',$	distance of the point of intersection from O', of any ray reflected from M, with the line $O'\,X'$,

r'_s = distance of the sagittal focus from the pole O_1 along the principal ray $O_1 O_2$ when the mirror M is absent, and

R = radius of curvature of the grating and the mirror.

Following Beutler (1945) and Namioka (1959*a*) the focussing equation of a concave grating is given by

$$(cos^2\alpha/\Upsilon - cos\alpha/R) + (cos^2\beta/\gamma_t' - cos\beta/R) = 0 \quad .. \quad (1)$$

$$(1/\gamma - cos\alpha/R) + (1/\gamma'_s - cos\,\beta/R) = 0 \quad .. \quad (2)$$

Eqn. (1) is for the tangential focus and Eqn. (2) is for sagittal focus. The equation of diffraction is given by

$$sin\alpha + sin\beta - \lambda/\beta \quad .. \quad (3)$$

where λ is the wavelength and d is the grating constant. When the grating is rotated by an angle θ to obtain any wavelength λ, angles of incidence and diffraction are given by

$$\alpha = \phi - \theta \quad .. \quad (4)$$

$$\beta = \phi + \theta \quad .. \quad (5)$$

Substituting the value of α and β from (4) and (5) in (1) and (2) we have

$$\rho_t' = [2\,cos\theta\,cos\phi - \rho\,cos^2(\theta-\phi)]/\,cos^2(\theta+\phi) \quad .. \quad (6)$$

$$\rho_s' = cos\,(\phi-\theta) + cos\,(\phi+\theta) - \rho \quad .. \quad (7)$$

where

$$\rho_t' = R/\gamma_t',\ \rho_s' = R/\gamma_s',\ \rho = R/\Upsilon \quad .. \quad (8)$$

For any angle of rotation θ, the corresponding wave-length λ can be computed using (3), (4) and (5) and the focussing position for the tangential and sagittal rays can be computed by (6), (7) and (8). Assuming several values of ρ and θ, ρ_t' was computed for various values of θ and the results were plotted graphically. It has been found that for values of ρ in the region of 1.2 and 2.0, ρ_t' remains more or less constant for long range of values of θ. This has been shown in Figs. 2 and 3. In Fig. 2, for ϕ=35°, ρ_t' is fairly constant upto large values of θ. This is the case of Seya-Namioka monochromator (1959*b*). In Fig. 3, it is seen that for smaller values of ϕ, ρ_t' is more or less constant upto certain values of θ. Assuming a suitable small value of ϕ, the value of ρ in the region of 2.0 for constant values of ρ_t' is computed in the following way.

Differentiating (1) with respect to θ and noting that $\partial\beta/\partial\theta$=1.0 and $\partial\alpha/\partial\theta$= —1.0 from (4) and (5) we have

$$\partial\rho_t'/\partial\theta = sec^2\beta\,[sin\,\beta - sin\,\beta + \rho_t'\,sin\,2\beta - \rho\,sin\,2\alpha] \quad .. \quad (9)$$

From (1) and (8) we have

$$\rho_t' = sec^2\beta\,[cos\,\alpha + cos\,\beta - \rho\,cos^2\alpha] \quad .. \quad (10)$$

For constant value of ρ_t', $\partial\rho_t'/\partial\theta = 0$. So from (9) we have

$$sin\,\alpha - sin\,\beta + \rho_t'\,sin\,2\beta - \rho\,sin\,2\alpha = 0 \quad .. \quad (11)$$

substituting the value of ρ_t' from (10) in (11) we have

$$\rho = (sin\,\alpha - sin\,\beta + 2\,tan\,\beta\,cos\,\alpha + 2\,sin\,\beta)/(2tan\,\beta\,cos^2\alpha + sin\,2\alpha) \quad .. \quad (12)$$

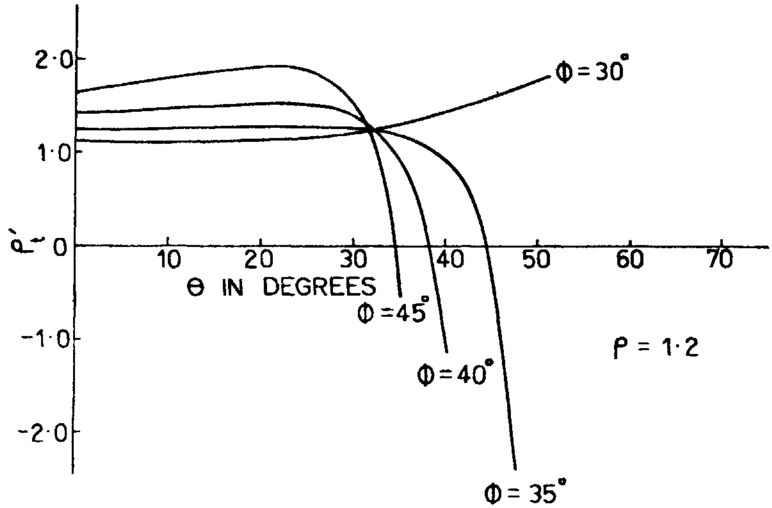

FIG. 2. Plot of the tangential focal distance ρ'_t against angle of rotation θ for various angles ϕ between the incident and the diffracted directions. Note that for the value $\phi = 35°$, ρ'_t is constant even at an angle of rotation of 35°, for $\rho = 1.2$.

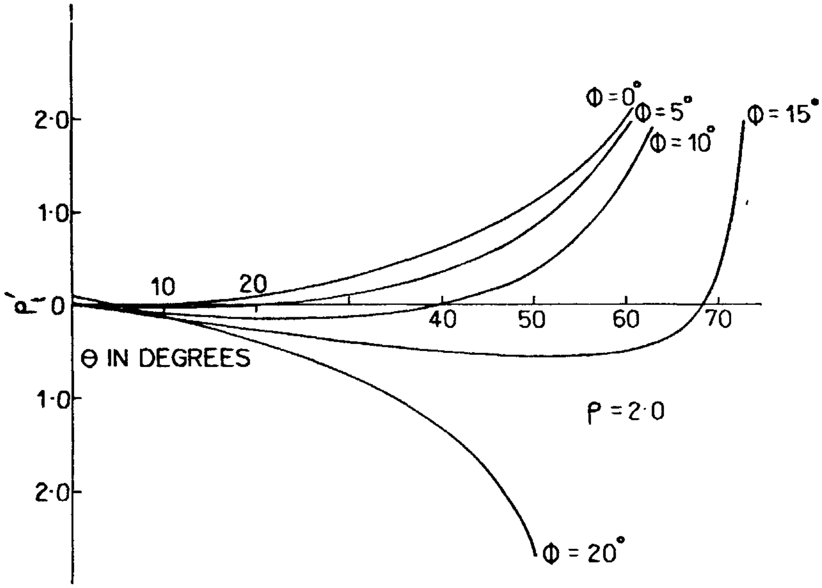

FIG. 3. Same as Figure 2 except that the values of the angle ϕ are near zero and $\rho = 2.0$.

Eqn. (12) gives the value of ρ for which ρ_t' will be constant for values of θ. From Eqn. (3) for a grating of 600 lines/mm it has been found that $\lambda = 0$ to 5780Å corresponds to rotation of the grating by 10° from zero order position. So for computation purposes θ was taken as 5°. Taking $\phi = 3°$, using (4), (5) and (12) it is found that $\rho = 1.8260$. Thus, when the angle between the incident and diffracted rays is 6° and $\rho=1.8260$ focussing positions of the tangential rays will be more or less constant for different wavelengths.

From Fig. 3, it is seen that when $\rho=2.0$ and ϕ is small, ρ_t' is also small upto large values of θ. From (8) it is seen that small value of ρ_t' means a large value of γ_t'. Thus, when we select $\rho=1.8260$ and $\phi=3°$, the tangential rays are diffracted nearly parallel and in order to focus them at a convenient position, a concave mirror is used. The focussing position when the grating and the mirrors are used together are found in the following way.

Referring to Fig. 1, S'_1, the focussing point of the grating G becomes the off-axis object point of the mirror M and S_2 is the final image point. The object distance along the principal ray with respect to the mirror is given by

$$S_T = l - \gamma_t' \quad .. \quad (13)$$

$$S_S = l - \gamma_s' \quad .. \quad (14)$$

where S_T and S_S are for the tangential and sagittal rays respectively. The Coddington equation (Jenkins and White 1957), for reflection at a concave mirror is given by

$$1/S_T + 1/S'_T = -2/(R \cos \phi) \quad .. \quad (15)$$

$$1/S_S + 1/S'_S = -2 \cos \phi/R \quad .. \quad (16)$$

where S'_T and S'_S are distances of the tangential focus and sagittal focus from the pole of the mirror along the principal ray. Thus, using Eqns. (1) to (8) and (13) to (16) one can compute S'_T and S'_S for various values of λ and hence we can find the positions of the tangential and the sagittal foci on the principal ray. For the purpose of computation the following values were taken.

$R=1000$ mm, $l=500$ mm, grating constant=600 lines/mm, ruling area 50×50 mm, $\phi=3°$ and $\rho=1.825984$. It has been found that for $\lambda=580.9$Å, $S'_T=455.92$ mm. The shift of the focus $\triangle S'_T$ from the focus of $\lambda=580.9$Å along the principal ray was computed for other wavelengths. Fig. 4 shows the plot of $\triangle S'_T$ against λ. It is seen that maximum defocussing between 0 to 5000Å is about 1.2 mm. It seems that this amount of defocussing is large compared to that in Seya-Namioka monochromator, but actual ray tracing will show that the blur of the image due to this defocussing is well within usable limits.

Astigmatism

For computation of astigmatism let us assume that the sources S_1 (Fig. 1) is a point source. As shown in Fig. 5, AB is the ruling length of the grating. OS is the direction of the principal ray and AS and BS are two rays from the upper and lower end of the grating ruling. Thus S is the position of the sagittal focus. Let T be the position of the tangential focus. The line CD which is perpendicular to OS represents the spread due to astigmatism. Let LA'_{ast} be the astigmatic focal difference. Thus,

$$LA'_{ast} = S'_S - S'_T \quad .. \quad (17)$$

In the present mirror-grating system, since the diffracted rays are more or less parallel we can assume that the distance between the points of intersections of the rays coming from the upper and lower ends of the grating ruling, with the mirror

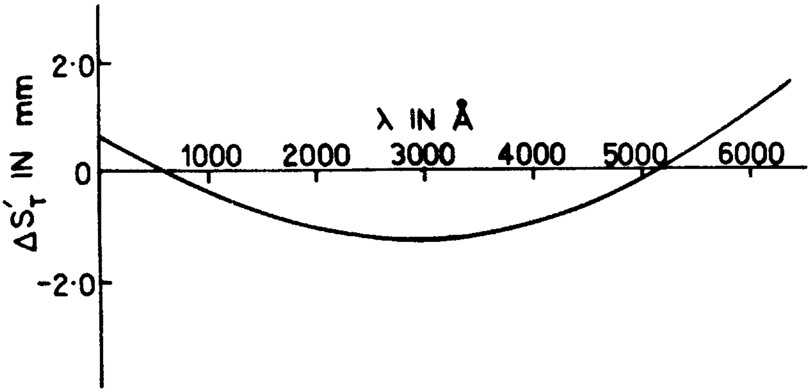

FIG. 4. Plot of the defocussing parameter $\triangle S'_t$ for various wavelengths.

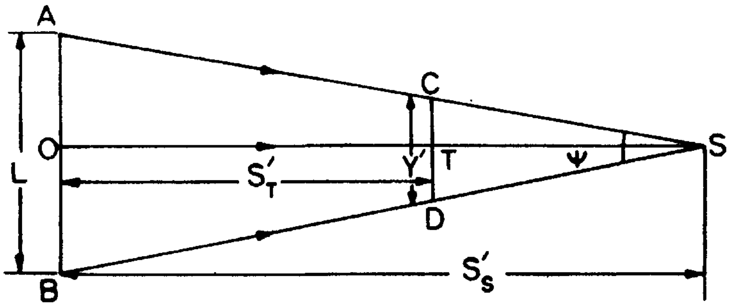

FIG. 5. Diagram used for the culculation of astigmatic length of the image.

surface is also $AB=L$. Let $A\hat{S}B=\Psi$ be the angle between the upper and the lower rays. From $\triangle$ AOS

$$\tan(\Psi/2) = L/(2S'_s) \simeq L/(2f) \qquad .. \quad (18)$$

where f is the focal length of the mirror. Let γ' be the astigmatic length of the image. Again from $\triangle$ CTS

$$\tan(\Psi/2) = CT/TS = \Upsilon'/(2LA'_{ast}) \qquad .. \quad (19)$$

Equating (18) and (19)

$$\gamma'/L = LA'_{ast}/f \qquad .. \quad (20)$$

Eqn. (20) gives the spread of the image of a point source due to astigmatism, as a fraction of the ruling length. Using (17) and (20) the fractional image spread was computed for various wavelengths and the results were plotted graphically as represented by curve (a) in Fig. 6. The ordinate scale in the left gives the various values of Y'/L for the curve (a). For comparing the results with that of the Seya-Namioka monochromator (1959a), ρ was taken to be 1.222470 and $\phi=35°\ 7.5'$. Other values remaining the same, r'_t and r_s' for different λ were computed using Eqns. (1) to (8). In Fig. 5 S_T' and S_S' were replaced by r'_t and r'_s respectively Thus,

$$TS = LA'_{ast} = \gamma'_s - \gamma'_t \qquad .. \quad (21)$$

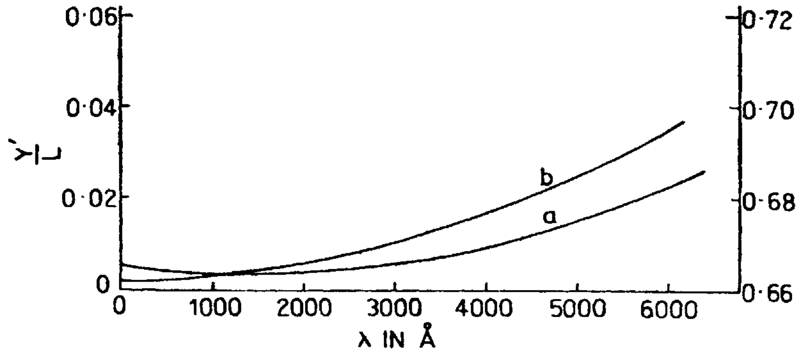

FIG. 6. Plot of the astigmatism parameter Y′/L Vs. wavelength. Curve (a) whose ordinate scale is in the left is for the Mirror-Grating system. Curve (b) whose oradinate scale is in the right is for the Seya-Namioka monochromator.

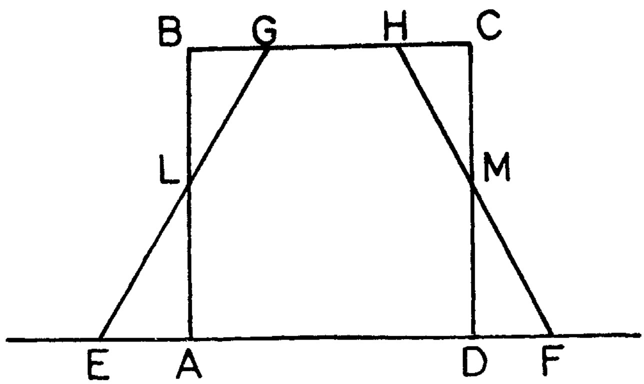

FIG. 7. Diagram for calculating the efficiency due to presence of astigmatism.

From the similar triangles AOS and CTS, $AO/CT=OS/TS$, noting that $AO=L/2$ and $CT=Y'/2$, we have

$$Y'/L = LA'_{ast}/\gamma'_s \qquad .. \quad (22)$$

using (21) and (22) fractional image spread due to astigmatism was computed for various wavelengths and the results were plotted graphically as represented by curve (*b*) in Fig. 6. The ordinate scale on the right gives the various values of Y'/L for the curve (*b*). From the curves (*a*) and (*b*) in Fig. 6 it is seen that in the mirror-grating system, astigmatism is negligible compared to that in the Seya-Namioka monochromator.

EFFICIENCY DUE TO REDUCED ASTIGMATISM

Normally, the photocathode of the detector may not be big enough to collect all the light coming out of the exit slit of the monochromator. This is specially true for a monochromator of the Seya-Namioka type having large astigmatism. Thus, in such a monochromator, a long exit slit has to be used to collect all the light flux and also a large photocathode is required. If the astigmatism is zero, the exit slit need not be longer than the entrance slit. Neglecting the effect of diffraction, if there is no spread of the image due to astigmatism, the intensity distribution along the length of the exit slit will be a rectangular function. If there is spread due to astigmatism, the intensity distribution will be modified and for all practical purposes one can assume the intensity curve to be trapezium shaped at the exit slit portion. The situation is illustrated in Fig. 7. In the ideal situation $A\,B\,C\,D$ is the intensity curve whereas in the actual case the curve will be represented by the trapezium *ELGHMF*. The light flux is simply represented by the area under the curves. If the exit slit of the monochromator is of same length as the entrance slit, the light flux emerging from it is equal to the area $G\,L\,A\,D\,M\,H$. Therefore, efficiency of the monochromator can be defined by

$$\text{Percentage Efficiency} = \frac{\text{Area } GLADMH}{\text{Area } GLEFMH} \times 100 \qquad \text{.. (23)}$$

Depending on the size of the image-blur and the length of the slit, the shape of the trapezium will be changed. For the present case length of the entrance and the exit slits was taken as 10 mm. Using an analysis similar to one given by Murty (1970), the areas required by equation (23) were computed and percentage efficiency was computed for the mirror-grating system as well as for the Seya-Namioka monochromator, for various wavelengths. The results are plotted as shown in Fig. 8 in which curve (*a*) and (*b*) are the efficiency curves for the mirror-grating system and Seya-Namioka monochromator, respectively. Scale of percentage efficiency is represented on the left and the right ordinates for the curves (*a*) and (*b*), respectively. From the curves (*a*) and (*b*) it is seen that the efficiency in the mirror-grating system is much more higher than that in the Seya-Namioka monochromator. Thus, the addition of one more optical element is not a disadvantage and the system may have an equal or slightly higher flux gathering power.

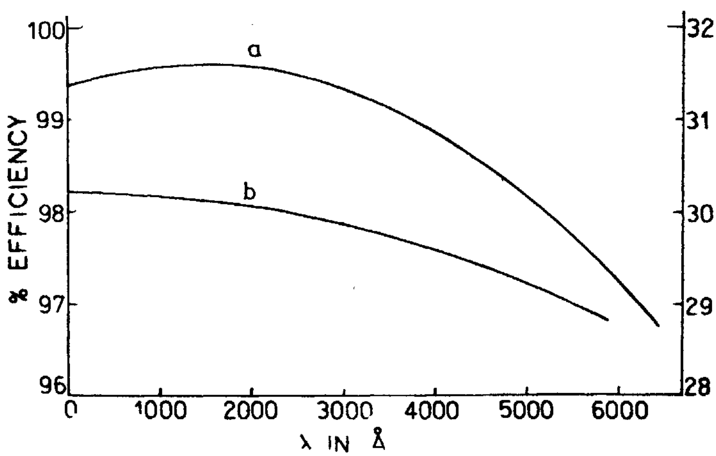

FIG. 8. Plot of efficiency *Vs.* wavelength. Curve (a) whose efficiency scale is in the left is for the Mirror-Grating System. Curve (b) whose efficiency scale is in the right is for the Seya-Namioka monochromator

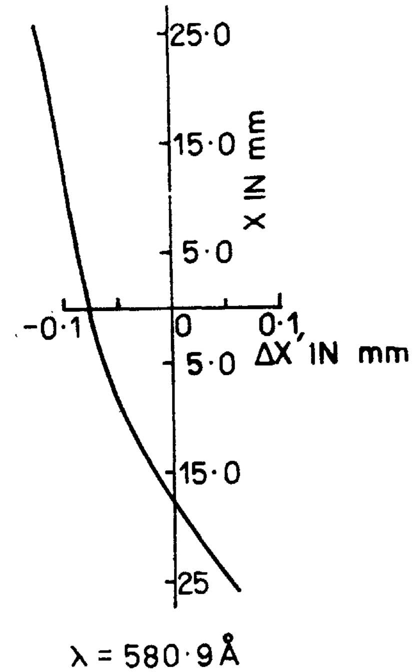

FIG. 9. Ray trace data curve. Plot of $\triangle X'$ (deviation of the ray intersection point from the image point) *Vs*. Semi-aperture X of the grating for $\lambda = 580.9$ A.

RAY TRACE ANALYSIS

The system was analysed using Spencer and Murty (1962) method. Referring to Fig. 1, S_1 is considered as a point source. Aperture variable X of the grating was measured along the stationary line O_1 X. For a particular configuration of the grating tangential rays coming from S_1 were traced and their intersection height X' along the line O' X' from the base point O' were computed. The distance of the image plane O' X' from the pole O_2 of the mirror was taken as 456.5 mm. The entrance slit S_2 was situated at a distance of 24 mm from the point O'. $\triangle X'$, the shift of the intersection point of various rays from the point S_2 were computed. Finally X and $\triangle X'$ were plotted for various wavelengths. Figs. 9 and 10 are the ray trace data curves for the wavelengths 580.9 Å and 4623.7 Å, respectively. From the ray trace data curves image blur for several wavelengths between 581 to 4633 were evaluated and they are represented in Table I. It is seen from Table I that the maximum blur from 581 Å to 4633 Å is only about 0.2 mm which is fairly small compared to the slit width at S_2. The dispersion for the instrument is about 30Å/*mm*. Thus, the blur 0.2 mm corrresponds to about 6Å. This Figure may be taken as the limit of resolution for this instrument. For better resolution the exit slit may be moved as the grating is rotated.

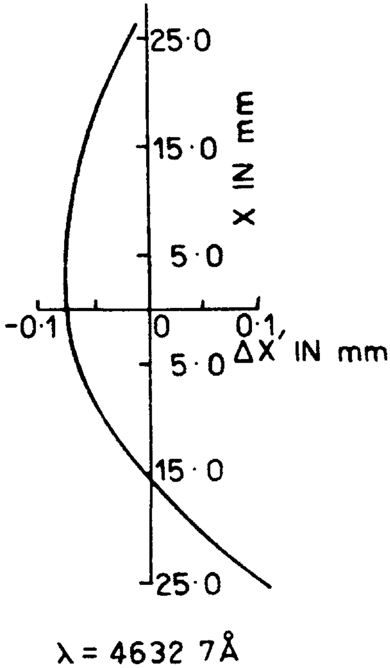

FIG. 10. Ray trace data curve. Plot of $\triangle X'$ (deviation of the ray intersection point from the image point) *Vs*. Semi-aperture X of the grating for $\lambda=4632.7$ Å.

TABLE 1

λ in Å	Blur in mm
581	0·18
1162	0.13
1742	0.11
2322	0.11
2901	0.12
3480	0.13
4057	0.15
4633	0.18

CONCLUSION

The mirror-grating system as proposed here seems to be more efficient in collecting the light flux even though at the expense of some resolution. Where high resolution is also to be maintained in addition to low astigmatism, the radius mounting (Tousey *et al.* 1951) or the system proposed earlier by Murty (1961, 1966) may be preferable.

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