SOILS AND FOUNDATIONS Vol. 30, No. 4, 53-62, Dec. 1990
Japanese Society of Soil Mechanics and Foundation Engineering

ANALOG SOLUTIONS FOR DESIGN OF MACHINE FOUNDATIONS

A. SRIDHARAN[i], D. K. BAIDYA[ii] and D. M. RAJU[iii]

ABSTRACT

Design of foundations subjected to dynamic load can be carried out either by elastic half-space theory or by mass-spring-dashpot analogy. Another simplified method, called analog method has also been proposed. Using this method resonance frequency and resonant amplitude which are the major criteria in design of machine foundations, can be expressed in terms of modified mass ratio knowing appropriate damping factor. In this paper analog solutions in the form of equations (non-dimensional frequency factor vs. modified mass ratio, non-dimensional magnification factor vs. modified mass ratio) are presented for all modes of vibration, viz., vertical, horizontal, rocking and torsional. Finally results obtained by this method are compared with elastic half-space theory which shows good agreement between the two.

Key words: damping, design, dynamic, machine foundations, vibration (**IGC:** H 1)

INTRODUCTION

Machine foundation design can be carried out either by elastic half space theory or theory based on mass-spring dash pot model or empirical methods (e. g., Richart et al, 1970, Sridharan and Nagendra, 1981). Out of these, elastic half-space theory is widely used and most popular. Using this theory many investigators [Sung (1953), Bycroft (1956), Hsieh (1962), Richart and Whitman (1967); Sridharan and Nagendra (1981, 1982 and 1984), to name a few] have carried out research on vertical and horizontal modes of vibration taking into consideration the three different contact pressure distributions, viz. Rigid, Uniform and Parabolic and with three displacement conditions, viz. Central, Average and Weighted average. Using the elastic half-space theory, Arnold, Bycroft and Warburton (1955), Bycroft (1956) and Hall (1967) carried out research on the rocking mode of vibration taking into consideration only rigid base pressure distribution and weighted average displacement condition. Sreekantaiah (1978) studied the rocking mode of vibration for other two pressure distributions, i. e. Uniform and Parabolic, considering weighted average displacement condition only. Reissner and Sagoci (1944) and Arnold, Bycroft and Warburton (1955) obtained solutions for torsional mode of vibration of a rigid circular

i) Professor, Dept. of Civil Engg., Indian Institute of Science, Bangalore 560 012, India.
ii) Research Scholar, Dept. of Civil Engg., Indian Institute of Science, Bangalore 560 012, India.
iii) Lecturer in Civil Engg., University Visveswaraya College of Engg., Jnana Bharathi, Bangalore, India.
Manuscript was received for review on June 22, 1989.
Written discussions on this paper should be submitted before July 1, 1991, to the Japanese Society of Soil Mechanics and Foundation Engineering, Sugayama Bldg. 4 F, Kanda Awaji-cho 2-23, Chiyoda-ku, Tokyo 101, Japan. Upon request the closing date may be extended one month.

footing resting on the elastic half space assuming rigid base pressure distribution that is linear variation of displacement from center to circumference of the footing.

Based on the solution of elastic half space theory, Lysmer and Richart (1966) presented the analog solutions for the weighted average displacement of a circular foundation with rigid base contact pressure distribution for vertical mode.

The mass of the equivalent lumped system can, with an accuracy which is adequate for engineering purposes, simply be taken as the mass of the foundation block plus machinery. The "effective mass" of the soil, which has caused so much controversy and confusion in the past, is so small as to be of little consequence (Whitman and Richart, 1967).

In order to satisfy the static state, the stiffness parameter was taken same as the static stiffness coefficient. Whitman and Richart (1967) discuss at great length about the choice of spring constant using theoretical equation and measured stress-strain relations. They could be summarised as.

(a) use formulas for spring constants derived from the theory of elasticity and evaluate the elastic constants either from insitu shear wave velocity measurements or from laboratory tests.

(b) Determine spring consstants from small scale plate bearing tests using static repeated loadings.

(c) Deduce spring constants from the results of small-scale vibrator tests.

(d) Use the concept of an elastic subgrade modulus together with tables or charts correlating subgrade modulus to soil type. None of these methods is necessarily better than the others because each involves approximations and assumptions, and considerable engineering judgement is required. Based on the theory of elasticity formulas for stiffness coefficients are listed in this paper elsewhere for different modes of vibrations. They are merely functions of shear modulus, G, Poisson's ratio, μ and equivalent radius, r_0 of the foundation contact area. For simplicity the above method is suggested to obtain stiffness coefficients.

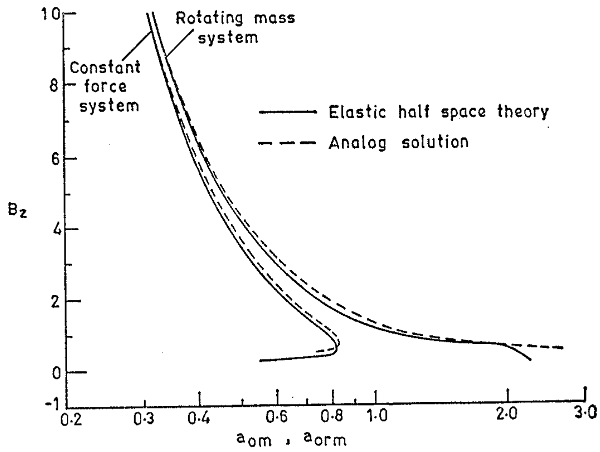

Fig. 1. Modified mass ratio, B_z vs. nontimensional frequency factor a_{om} and a_{orm} for constant and rotating mass system respectively, for vertical mode of vibration

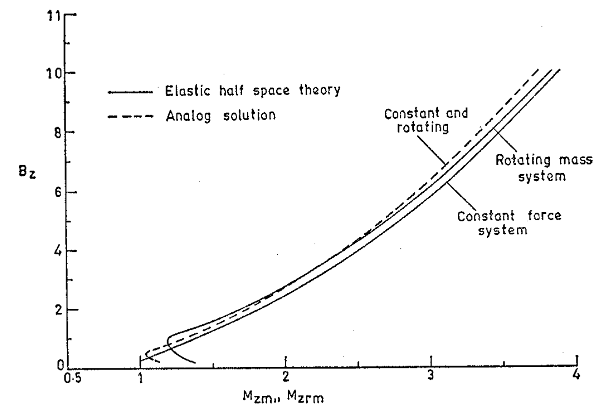

Fig. 2. Modified mass ratio, B_z vs. nondimensional magnification factor, M_{zm} and M_{zrm} for constant and rotating mass system respectively, for vertical mode of vibration

The analog parameter for damping was obtained by averaging the value of damping coefficient in the range $0 \leq a_0 \leq 0.8$, where a_0 is nondimensional frequency factor. Using modified displacement functions F_1 and F_2 in the expression for damping coefficient, C and average the value of C in the range $0.3 \leq a_0 \leq 0.8$, they obtained the solutions. Based on this, Nagendra and Sridharan (1984) obtained the analog parameter for vertical and horizontal vibrations considering three types of pressure distributions and displacement condition.

Hall (1967) using the displacement functions of Bycroft (1956) obtained the analog solutions for the horizontal displacement of a rigid circular footing. Hall (1967) developed analog model for rocking mode of vibration for rigid base pressure distribution and weighted average displacement condition. In this paper, using analog coefficients for all modes of vibrations, analog solutions i. e. relationship between modified mass ratio vs. nondimensional frequency factor and modified mass ratio vs. magnification factor have been developed and presented in a tabular form.

Analog solutions have the distinct advantages of their simplicity, yet their engineering accuracy adds to the versatility of applying them to machine foundation problems.

Finally, results obtained by the two methods (i. e. elastic half space theory and analog solutions) are presented in the form of figures. By analysing the result, it can be concluded that analog solutions can be used in the design and avoid laborious calculations using elastic half space theory.

ANALYSIS

(*a*) *Vertical Mode:*

Nagendra and Sridharan (1982) based on elastic half space theory obtained expressions for stiffness coefficients for three different pressure distributions and displacement conditions and their results are summarised in Table 1. In Table 1, the stiffness coefficients, k_z are given in terms of shear modulus, G; Poisson's ratio, μ and equivalent radius, r_0.

Defining damping factor for vertical mode as

$$D_z=\frac{C_z}{C_{zc}}$$

Table 1. Stiffness coefficients, k_z for vertical mode (Nagendra and Sridharan, 1982)

Displacement condition	Contact pressure distribution		
	Rigid	Uniform	Parabolic
Central	$\frac{4Gr_0}{(1-\mu)}$	$\frac{\pi Gr_0}{(1-\mu)}$	$\frac{3\pi Gr_0}{4(1-\mu)}$
Average	$\frac{4Gr_0}{(1-\mu)}$	$\frac{3\pi^2 Gr_0}{8(1-\mu)}$	$\frac{45\pi^2 Gr_0}{128(1-\mu)}$
Weighted average	$\frac{4Gr_0}{(1-\mu)}$	$\frac{3\pi^2 Gr_0}{8(1-\mu)}$	$\frac{945\pi^2 Gr_0}{3072(1-\mu)}$

where, C_z=damping coefficients for vertical mode, and

C_{zc}=critical damping coefficients for vertical mode.

Lysmer and Richart (1966) obtained expressions relating D_z with the modified mass-ratio, B_z (Eq. (1))

$$D_z=\frac{0.425}{\sqrt{B_z}} \qquad (1)$$

for rigid base pressure distribution and weighted average displacement condition, where

$$B_z=\frac{m}{\rho r_0^3}\ \frac{(1-\mu)}{4}$$

m=mass of the foundation block

ρ=mass density of the media, and

r_0=equivalent radius of the foundation contact area.

For different modes expressions ror r_0 are given in App. II-under notations.

In general, it can be written that

$$D_z=\frac{D_{kz}}{\sqrt{B_z}} \qquad (1a)$$

where, D_{kz}=constant depending on type of pressure distribution and displacement condition.

$$B_z=\frac{b_z G r_0}{k_z}$$

k_z=static stiffness coefficient mode as given in table-1, and

b_z=mass ratio=$m/\rho r_0^3$.

Following similar approach as Lysmer and Richart (1966), Nagendra (1982) obtained the values of D_{kz} for various pressure distributions and displacement conditions as given in Table 2 under vertical mode.

Frequency factor: It is well known [Richart et al (1970)] that a mass spring dashpot

Table 2. Values of D_{kz} for various pressure distribution and displacement condition, $D_z=D_{kz}\ B_z^{-1/2}$

Displacement condition	Contact pressure distribution		
	Rigin	Uniform	Parabolic
Central	0.402	0.320	0.241
Average	0.412	0.380	0.355
Weighted average	0.425	0.380	0.311

model subjected to constant dynamic force which is independent of frequency, the resonance frequency, f_m is given by

$$f_m=\frac{1}{2\pi}\sqrt{\frac{k_z}{m}}\sqrt{1-2D_z^2} \qquad (2)$$

Substituting for k_z and D_z from Tables (1) and (2) for weighted average displacement condition and rigid base pressure distribution

$$f_m=\frac{1}{2\pi}\sqrt{\frac{4Gr_0}{m(1-\mu)}}\sqrt{1-\frac{2(0.425)^2}{B_z}}$$

$$f_m=\frac{\sqrt{G/\rho}}{2\pi r_0}\frac{\sqrt{B_z-0.36}}{B_z} \qquad (3)$$

Defining $a_{0m}=\omega_m r_0\sqrt{\rho/G}=2\pi f_m r_0\sqrt{\rho/G}$ (4)

where ω_m = circular resonance frequency
a_{0m} = nondimensional frequency factor at resonance

One can get,

$$a_{0m}=\frac{\sqrt{B_z-0.36}}{B_z} \qquad (5)$$

Similarly for rotating mass we have (Richart, (1970))

$$f_{mr}=\frac{1}{2\pi}\sqrt{\frac{k_z}{m}}\frac{1}{\sqrt{1-2D_z^2}} \qquad (6)$$

and Substituting for k_z and D_z from Tables 1 and 2 for weighted average displacement condition and rigid base pressure distribution and simplifying,

$$a_{0rm}=\frac{1}{\sqrt{B_z-0.36}} \qquad (7)$$

Magnification factor:

For constant force system (Richart et al. 1970) amplitude is given by

$$A_{zm}=\frac{Q_0}{k_z}\frac{1}{2D_z\sqrt{1-D_z^2}} \qquad (8)$$

where, Q_0 = amplitude of dynamic load.

Substituting for k_z and D_z from Tables 1 and 2 for weighted average displacement con dition and rigid base pressure distribution.

One can obtain,

$$A_{zm}=\frac{Q_0(1-\mu)}{4Gr_0}\frac{B_z}{0.85\sqrt{B_z-0.18}} \qquad (9)$$

$$\text{Defining, } A_{zm}=\frac{Q_0(1-\mu)}{4Gr_0}M_{zm} \qquad (10)$$

where, M_{zm} is the nondimensional magnification factor

$$M_{zm}=\frac{B_z}{0.85\sqrt{B_z-0.18}} \qquad (11)$$

Similarly for rotating mass system,

$$A_{zrm}=\frac{m_e e}{m}\frac{1}{2D_z\sqrt{1-D_z^2}} \qquad (12)$$

where, Q_0 = amplitude of dynamic load $=m_e e\omega^2$
m_e = eccentric rotating mass
e = eccentricity
ω = circular frequency

Substituting for D_z from Table 2

$$A_{zrm}=\frac{m_e e}{m}\frac{B_z}{0.85\sqrt{B_z-0.18}}$$

$$\text{Defining, } A_{zrm}=\frac{m_e e}{m}M_{zrm}$$

where, M_{zrm} = nondimensional maginification factor at resonance,

$$M_{zrm}=\frac{B_z}{0.85\sqrt{B_z-0.18}} \qquad (13)$$

It can be thus seen from Eqs. (11) and (13) that the nondimensional parameter M_{zm} and M_{zrm} are one and the same.

$$\text{i.e., } M_{zm}=M_{zrm}=\frac{B_z}{0.85\sqrt{B_z-0.18}} \qquad (14)$$

Eqs. (5) and (7) relates the nondimensional frequency factor for constant and rotating mass system respectively with modified mass ratio, B_z.

Eq. (14) relates the nondimensional amplitude factor (M_{zrm} and M_{zrm}) and the modified mass ratio, B_z.

Now, expressing the relationships of Eqs. (5), (7) and (14) in generalised form:

$$a_{0m}=\frac{\sqrt{B_z-\alpha_z}}{B_z} \qquad (15)$$

$$a_{0rm}=\frac{1}{\sqrt{B_z-\alpha_z}} \qquad (16)$$

$$M_{zm}=M_{zrm}=\frac{B_z}{\beta_z\sqrt{B_z-\alpha_z/2}} \qquad (17)$$

where, α_z and β_z are constants depending on contact pressure distribution and displacement conditions. Adopting similar procedure for other contact pressure distributions and displacement conditions, constant α_z and β_z are

Table 3. Constants α_z and β_z for vertical mode for use in Eqs. 15, 16 and 17

Displacement condition	Constant α_z and β_z	Contact pressure distribution		
		Rigid	Uniform	Parabolic
Central	α_z	0.324	0.205	0.116
	β_z	0.805	0.640	0.482
Average	α_z	0.340	0.289	0.252
	β_z	0.825	0.760	0.710
Weighted	α_z	0.360	0.289	0.192
Average	β_z	0.850	0.760	0.620

obtained by using appropriate k_z and D_z values from Tables 1 and 2.

Table 3 gives the values of α_z and β_z for three pressure distributions and displacement conditions.

Using the Eqs. (15) to (17) and Table 3 one can easily obtain the nondimensional amplitude and frequency factors for different modified mass ratios, B_z.

Results are presented in Figs. 1 and 2 for rigid base pressure distribution and weighted average displacement condition as example. Comparing the elastic half-space theory (Richart et al, 1970) with the analog solutions it can be seen that the agreement is very good. Similar comparison for other pressure distributions and displacement conditions also showed good agreement between the two procedures.

(*b*) *Horizontal Mode of Vibration*

Analysis has been carried out for horizontal mode of vibrations on the similar terms of the vertical mode. Nagendra et al (1982) based on elastic half-space theory obtained expressions for stiffness coefficients assuming the vertical displacement of the footing due to horizontal force as zero for three different contact shear stress distributions and displacement conditions. Solutions are also available (Nagendra and Sridharan, 1984) for $\sigma_{zz}=0$ at the interface. This condition is more appropriate for purely horizontal mode of vibration. Hence in this paper the analog solutions are developed for the condition of zero vertical stress due to horizontal force. The stiffness coefficients k_x for vertical stress, σ_{zz} due to horizontal force is zero, obtained by Nagendra and Sridharan (1984) are reproduced in Table 4.

Table 4. Stiffness coefficients, k_x for horizontal mode of vibration (vertical stress σ_{zz} due to horizontal load is zero) (Nagendra and Sridharan, 1984)

Displacement condition	Contact pressure distribution		
	Rigid	Uniform	Parabolic
Central	$\frac{8Gr_0}{(2-\mu)}$	$\frac{2\pi Gr_0}{(2-\mu)}$	$\frac{3\pi Gr_0}{2(2-\mu)}$
Average	$\frac{8Gr_0}{(2-\mu)}$	$\frac{2\pi^2 Gr_0}{4(2-\mu)}$	$\frac{45\pi^2 Gr_0}{64(2-\mu)}$
Weighted average	$\frac{8Gr_0}{(2-\mu)}$	$\frac{3\pi^2 Gr_0}{4(2-\mu)}$	$\frac{315\pi^2 Gr_0}{512(2-\mu)}$

Table 5. Analog coefficients, C_{kx} for horizontal mode of vibrations ($\sigma_{zz}=0$) $D_x=D_{kx}B_x^{-2/1}$

Displacement condition	Contact pressure distribution		
	Rigid	Uniform	Parabolic
Central	0.249	0.194	0.144
Average	0.252	0.231	0.216
Weighted average	0.252	0.231	0.188

Defining damping factor,

$$D_x=\frac{C_x}{C_{xc}}=\frac{D_{kx}}{\sqrt{B_x}} \tag{18}$$

where, C_x=damping coefficients in the horizontal mode

C_{xc}=critical damping coefficients

D_{kx}=analog coefficients.

B_x=modiffied mass ratio for horizontal mode$=(b_xGr_0)/(k_x)$

k_x=static stiffness coefficient in horizontal mode as given in table 4; and

b_x=mass ratio$=(m)/(\rho r_0^3)$.

Table 5 gives the values of D_{kx} for different shear stress distributions and displcement conditions.

Frequency factor:

Resonance frequency for constant force system is given as

$$f_m=\frac{1}{2\pi}\sqrt{\frac{k_x}{m}}\sqrt{1-2D^2_x} \tag{19}$$

Knowing, $k_x=\frac{8Gr_0}{2-\mu}$

and $D_x = \frac{0.249}{\sqrt{B_x}}$ from Tables 4 and 5 respectively for rigid base contact shear stress distribution and weighted average displacement condition

and, $a_{0m} = 2\pi f_m r_0 \sqrt{\rho/G}$

One can obtain, $a_{0m} = \frac{\sqrt{B_x - 0.124}}{B_x}$ (20)

Resonance frequency for rotating mass system is

$$f_{mr} = \frac{1}{2\pi}\sqrt{\frac{k_x}{m}} \frac{1}{\sqrt{1-2D_x^2}}$$

Substituting for k_x, D_x and simplifying

$$a_{0rm} = \frac{1}{\sqrt{B_x - 0.124}} \quad (21)$$

Magnification factor:

For constant force system (Richart et al. 1970) amplitude is given by

$$A_{xm} = \frac{Q_0}{k_x} \frac{1}{2D_x\sqrt{1-D_x^2}} \quad (22)$$

Substituting for D_x from Teble 5 and defining

$$A_{xm} = \frac{Q_0}{k_x} M_{xm}$$

where, M_{xm} = magnification factor for constant force sysem

$$= \frac{B_x}{0.498\sqrt{B_x - 0.062}} \quad (23)$$

Similarly for rotating mass system,

$$A_{xrm} = \frac{m_e e}{m} \frac{1}{2D_x\sqrt{1-D_x^2}}$$

Substituting for D_x and defining,

$$A_{xrm} = \frac{m_e e}{m} M_{xrm}$$

where, M_{xrm} = magnification factor for rotating mass system

$$= \frac{B_x}{0.498\sqrt{B_x - 0.062}} \quad (24)$$

Thus, $M_{xm} = M_{xrm} = \frac{B_x}{0.498\sqrt{B_x - 0.062}}$ (25)

Now, expressing Eqs. (20), (21) and (25) in the following form:

$$a_{0m} = \frac{\sqrt{B_x - \alpha_x}}{B_x} \quad (26)$$

Table 6. Constants, α_x and β_x for use in equations (26, 27 and 28)

Displacement condition	α_x and β_x	Contact shear stress distribution		
		Rigid	Uniform	Parabolic
Central	α_x	0.124	0.075	0.041
	β_x	0.498	0.388	0.288
Average	α_x	0.127	0.107	0.093
	β_x	0.504	0.462	0.432
Weighted average	α_x	0.127	0.107	0.071
	β_x	0.504	0.462	0.376

$$a_{0rm} = \frac{1}{\sqrt{B_x - \alpha_x}} \quad (27)$$

$$M_{xm} = M_{xrm} = \frac{B_x}{\beta_x\sqrt{B_x - \alpha_x/2}} \quad (28)$$

where, α_x and β_x are constants which depends on shear stress distributions and displacement conditions. Table 6 gives the values of α_x and β_x for three different shear stress distributions and displacement conditions.

Using Eqs. (26) to (28) and Table 6 one can easily obtain the nondimensional frequency factor and magnification factors for different contact shear stress distributions and displacement conditions for any value of modified mass ratio, B_x.

Finally, results are presented in Figs. 3 and 4 for rigid base pressure distribution and

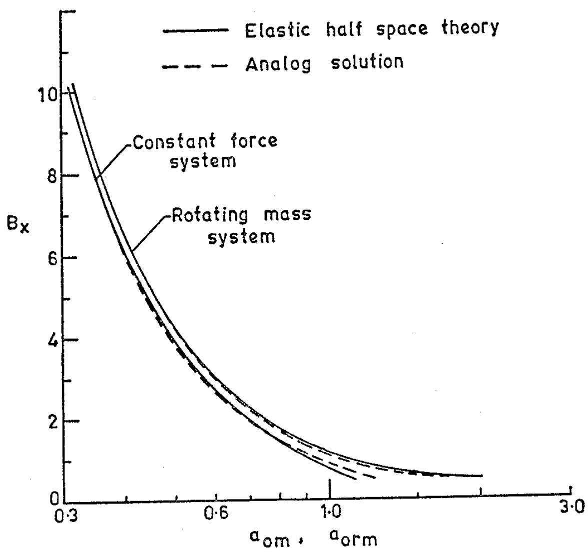

Fig. 3. Modified mass ratio, B_x vs. nondimensional frequency factor, a_{om} and a_{orm} for constant and rotating mass system respectively for horizontal mode of vibration

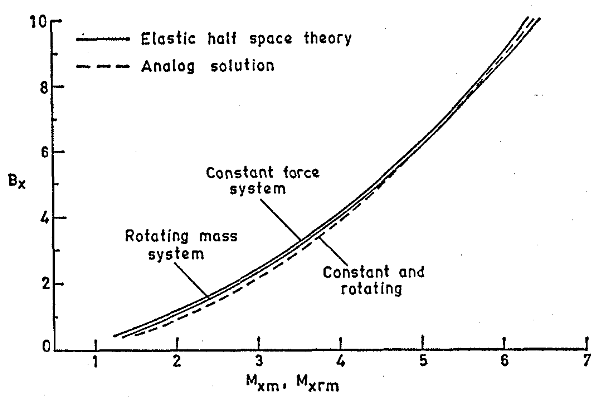

Fig. 4. Modified mass ratio, B_x vs. nondimensional magnification factor, M_{xm} and M_{xrm} for constant and rotating mass system respectively for horizontal mode of vibration

weighted average displacement condition for both elastic half space theory (Nagendra and Sridharan 1984) and analog model. It can be seen that agreement between two is very good.

(*c*) *Rocking Mode of Vibrations:*

Similar to what has been carried out for vertical and horizontal mode, analog solutions have been obtained for rocking mode for three types of pressure distribution namely rigid, uniform and parabolic and for weighted average displacement condition only. For other displacement conditions stiffness coefficient k_ϕ, damping factor, D_ϕ are not available. Tables 7 and 8 presents the stiffness coefficient, k_ϕ and damping factor, D_ϕ respectively as obtained from literature.

Table 7. Stiffness coefficients, k_ϕ, for rocking mode

Contact pressure distribution	Rigid	Uniform	Parabolic
Values of k_ϕ	$\frac{8Gr^3_0}{3(1-\mu)}$ (By Croft, 1956)	$\frac{15\pi^2Gr^3_0}{64(1-\mu)}$	$\frac{67.5\pi^2Gr^3_0}{1024(1-\mu)}$
		(Sridharan, Baidya and Raju, 1989)	

Table 8. Values of damping factor, D_ϕ

Contact pressure distribution	Rigid	Uniform	Parabolic
Values of D_ϕ	$\frac{0.11}{(1+B_\phi)B^{0.4}}$	$\frac{0.094}{(1+B_\phi)B^{0.4}}$	$\frac{0.073}{(1+B_\phi)B^{0.4}}$
	(Sridharan, Baidya and Raju, 1989)		

Substituting for k_ϕ (table 7) and D_ϕ (table 8) in Eqs. (2), (6) and (8) one can obtain the expression for frequency factor and magnification factor for rocking mode as follows:

$$a_{0m}=\frac{\sqrt{(1+B_\phi)^2B_\phi^{0.8}-\alpha_\phi}}{(1+B_\phi)B_x^{0.9}} \quad (29)$$

$$a_{0rm}=\frac{(1+B_\phi)}{B_\phi^{0.1}\sqrt{(1+B_\phi)^2B_\phi^{0.8}-\alpha_\phi}} \quad (30)$$

$$M_{\phi m}=M_{\phi rm}=\frac{(1+B_\phi)^2B_\phi^{0.8}}{B_\phi\sqrt{(1+B_\phi)^2B_\phi^{0.8}-\alpha_\phi/2}} \quad (31)$$

where B_ϕ=modified inertia ratio for rocking mode $=\frac{b_\phi G r_0^3}{k_\phi}$

b_ϕ=inertia ratio$=I/\rho r_0^5$

and I_ϕ=mass momet of inertia about the axis of rotation.

whera α_ϕ and β_ϕ are the constants depends on the contact pressure distribution and displacement conditions and are tabulated in Table 9.

Finally, results are presented in Figs. 5 and 6 for rigid base pressure distribution and weighted average displacement condition for both elastic half space theory (Richart et al. 1970) and analog model which shows good

Table 9. Values of α_ϕ and β_ϕ

Contact pressure distribution	Rigid	Uniform	Parabolic
α_ϕ	0.024	0.0176	0.011
β_ϕ	0.022	0.1880	0.146

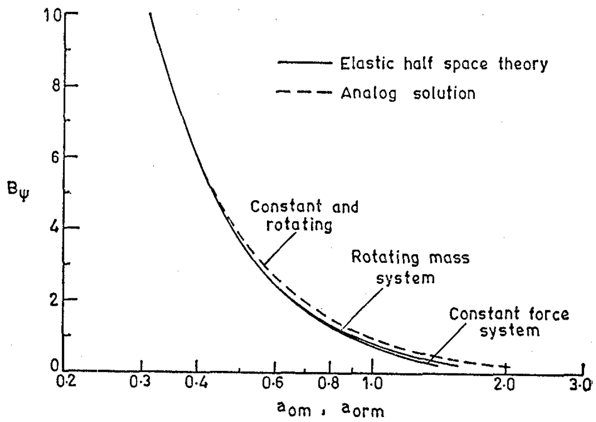

Fig. 5. Modified inertia ratio, B_ϕ vs. nondimensional frequency factor, a_{om} and a_{orm} for constant and rotating mass system respectively for rocking mode of vibration

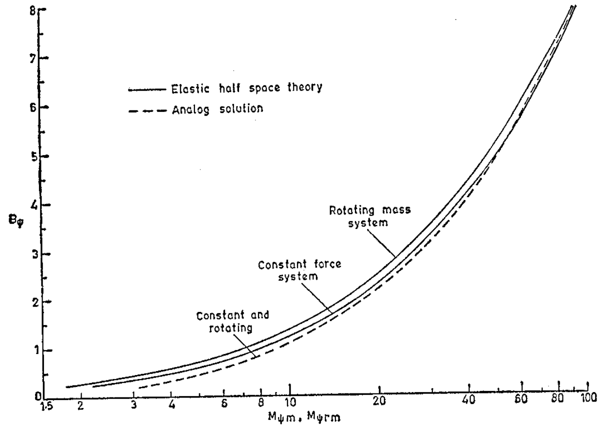

Fig. 6. Modified inertia ratio, B_ϕ vs. nondimensional magnification factor, $M_{\phi m}$ and $M_{\phi rm}$ for constant and rotating mass system respectively for rocking mode of vidration

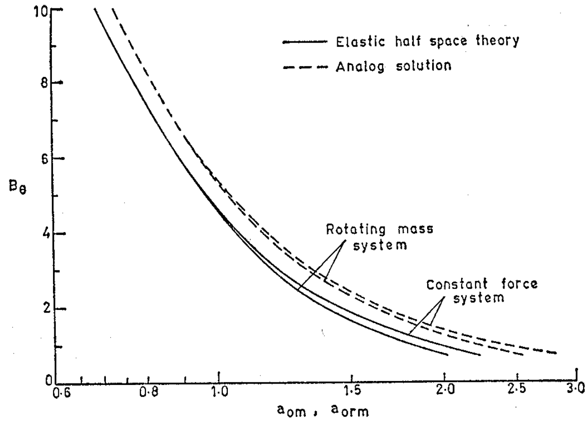

Fig. 7. Modified inertia ratio, B_θ vs. nondimensional frequency factor, a_{om} and a_{orm} for constant and rotating mass system respectively for torsional mode of vibrations

agreement between the two approaches.

(d) Torsional Mode of Vibrations:

Sufficient literature far torsional mode of vibrations are not available like all other mode of vibrations. From available literature (Reissner and Sagoci, 1944) damping ratio, D_θ and stiffness coefficient, k_θ for torsional mode of vibration for rigid base pressure distribution and weighted average displacement condition, are:

$$D_\theta=\frac{0.5}{1+2B_\theta},\quad k_\theta=\frac{16}{3}Gr_0^3 \qquad (32)$$

Substituting it in Eqs. (2), (6) and (8) one can get analog equations for torsional mode of vibrations for B_θ vs. nondimensional frequency factor and amplitude factor as follows:

$$a_{0m}=\frac{4}{\sqrt{3}}\ \frac{\sqrt{(1+2B_\theta)^2-0.5}}{\sqrt{B_\theta}(1+2B_\theta)} \qquad (33)$$

$$a_{0rm}=\frac{4}{\sqrt{3}}B_\theta\ \frac{(1+2B_\theta)}{\sqrt{(1+2B_\theta)^2-0.5}} \qquad (34)$$

$$M_{\theta m}=M_{\theta rm}=\frac{(1+2B_\theta)^2}{\sqrt{(1+2B_\theta)^2-0.25}} \qquad (35)$$

where, B_θ = modified inertia ratio for torsional mode $=\dfrac{I_\theta}{\rho r_0^5}$

I_θ = mass moment of inertia about axis of rotation.

Finally, results obtained by elastic half-space theory (Richart et al. 1970) and analog

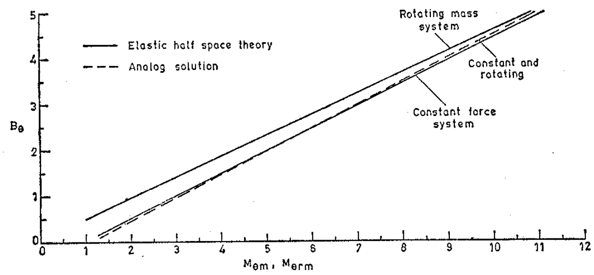

Fig. 8. Modified inertia ratio, B_θ vs. nondimensional magnification factor, $M_{\theta m}$ and $M_{\theta rm}$ for constant and rotating mass system respectively for torsional mode of vibration

model are presented in Figs. 7 and 8 and are compared. It can be seen that the agreement between them is very good.

CONCLUSIONS

Solution are available for predicting the resonance frequency and resonance amplitude for foundations subjected to different modes of vibrations based on elastic half-space theory. However, they are not in simple form to be used in practice. Analog solutions have been proposed as a simple solutions. In this paper coefficients are obtained using analog solutions which can be readily used in the equations relating nondimensional mass ratio vs. frequency factor; and nondimensional mass ratio vs. magnification factor. Results have been obtained for vertical and horizontal mode of vibrations for three types of pressure distributions (Rigid, Uniform, Parabolic) and three displacement conditions (Central, Average, Weighted average). For rocking mode, results are presented for weighted average displacement condition and three types of pressure distribution. For torsional mode analog equations have been presented only for rigid base pressure distribution and weighted average displacement condition.

The results obtained from analog solutions have been compared with those obtained from elastic half-space theory and their agreement is found to be very good.

NOTATIONS

A_{zm}, A_{xm} = displacement amplitude for vertical and horizontal mode respectively due to vibrations

a_0 = nondimensional frequency factor

a_{0m}, a_{0rm} = nondimensional frequency factor at resonance for constant and rotating mass system respectvely

B_z, B_x = modified mass ratio for vertical, horizontal mode respectively

B_ψ, B_θ = modified inertia ratio for rocking and torsional mode respectively

C_z, C_x = damping coefficients for vertical and horizontal mode respectively

C_{zc}, C_{xc} = critical damping coefficients for vertical and horizontal mode respectively

$D_z, D_x, D_\psi, D_\theta$ = damping factor for vertical, horizontal rocking and torsional mode respectively

D_{kz}, D_{kx} = analog parameter for vertical and horizontal mode respectively

e = eccentricity of rotating mass

f_m, f_{mr} = resonance frequency for constant and rotating mass system respectively

G = shear modulus of the mediun

I_ψ = mass moment of inertia of machine and foundation about axis of rotation, i. e. about base

I_θ = mass moment of inertia of machine and foundation about axis of rotation, i. e. about vertical axis

$k_z, k_x, k_\psi, k_\theta$ = static stiffness coefficients for vertical, horizontal rocking and torsional mode respectively

m = mass of the vibrating foundations

$M_{zm}, M_{xm}, M_{\psi m}, M_{\theta m}$ = nondimensional magnification factor for constant force system for vertical, horizontal, rocking and torsional mode respectively

r_0 = equivalent radius of the foundation block corresponding to each mode

For a rectangular footing having dimensions $2c$ by $2d$, the equivalent radius, r_0 is given by:

$$r_0=\left[\frac{4cd}{\pi}\right]^{1/2} \text{ for vertical and horizontal}$$

$$r_0=\left[\frac{16cd^3}{3\pi}\right]^{1/4} \text{ for rocking}$$

$$r_0=\left[\frac{16cd(c^2+d^2)}{6\pi}\right]^{1/4} \text{ for torsion}$$

α_z, α_x and α_ψ = constants in analog equations for vertical, horizontal and rocking mode respectively

β_z, β_x and β_ψ = constants in analog equations for vertical, horizontal and rocking mode respectively

μ = poisson's ratio

ρ = mass density of soil

ω = circular frequency

ω_{rm}, ω_m = circular resonance frequency for rotating mass system and constant force respectively.

REFERENCES

1) Arnold, R. N., Bycroft, G. N. and Warburton, G.B. (1955): "Forced vibrations of a body of an infinite elastic solid," J. Appl. Mech., Trans. ASME, Vol. 77, pp. 391-401.

2) Bycroft, G.N. (1956): "Forced vibrations of a rigid circular plate on a semi-infinite elastic space and on an elastic stratum," Philosophical Trans., Royal Society, London, Ser. A., Vol. 248, pp. 327-368.

3) Hall, J. R. Jr. (1967): "Coupled rocking and sliding oscillations of rigid circular footing," Proc. Int. Symp. on Wave Propagation and Dynamic Properties of Earth Materials, Al bu querque. New Mexico. Aug.

4) Hsieh, T.K. (1962): "Foundation vibrations," Proc. Institute of Civil Engineers, Vol. 22

5) Lysmer, J. (1965): "Vertical motion of rigid footings," Dept. of Civil Engg., Univ. of Michigan, Report to WES Contract report No. 3-115 Under Contract No. DA-22-0. 79-eng-340, also a Ph.D dissertation, Univ. of Michigan, Aug.

6) Lysmer, J. and Richart, F.E. Jr. (1966): "Dynamic response of a footings to vertical loading," Jl. of Soil Mech. and Found. Divn., Proc. ASCE, Vol. 92, No. SM1, Jan.

7) Nagendra, M.V. and Sridharan, A. (1981): "Re-

sponse of circular footing to vertical vibrations," Proc. Jl. of Geotech. Div. ASCE, Vol. 107, pp. 989-995.
8) Nagendra, M. V. and Sridharan, A. (1982): "Stiffness coefficients of elastic medium," Proc. Jl. of Geotech. Div. ASCE, Vol. 108, pp. 661-668.
9) Nagendra, M.V. and Sridharan, A.(1984): "Footing response to horizontal vibration," Proc. ASCE Engineering Mech. Div. Vol. 110, No. 4, 648-654.
10) Nagendra, M.V., Sridharan, A. and Seenivasan, M. (1982): "Footing response to horizontal vibration," Indian Geotech. Journal, Vol. 12, pp. 132-151.
11) Nagendra, M.V. (1982): "Studies on foundation soil system subjected to vertical and horizontal vibrations," Ph. D Dissertation, I.I. Sc., Bangalore.
12) Richart, F.E., Jr., Hall, J.R., Jr. and Woods, R.D. (1970): "Vibrations of soils and foundations," Prentice Hall, Inc., Englewood, Cliffs, New Jersey.
13) Reissner, E. and Sagoci, H.F. (1944): "Forced torsional oscillations of an elastic half space," J. of Appl. Phys. Vol. 15, pp. 652-662.
14) Sridharan, A., Baidya, D. K. and Raju, D. M. (1989): "Foundations response to rocking vibrations," Communicated for Publications.
15) Sung, T.Y. (1953): "Vibrations in semi-infinite solids dne to periodic surface loading", Symp. on Dynamic Testing of Soils, Special Technical Publications, No. 156, ASTM, Philadelphia.
16) Whitman, R.V. and Richart, F.E. Jr. (1967): "Design procedure for dynamically loaded foundations", J. of Soil Mech. & Foundation Divns. Proc. ASCE, Vol. 93, SM6, pp. 169-193.