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Electric dipolarizability of ^{7}Li

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Abstract. We calculate the electric dipolarizability of ^{7}Li nucleus within the cluster model and estimate a value of about 0.0188 fm^3. We also discuss the possibility of observing this in the scattering of ^{7}Li from a ^{208}Pb target at energies about 30 MeV.

Keywords. Polarizability; cluster models.

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There has been a long tradition of cluster models [1] of light nuclei – primarily introduced to simplify the studies of these strongly interacting few-body systems. In a series of papers [2–4], through variational calculations of energy levels of lithium isotopes, cluster models of these nuclei were established. Using the cluster model, the reaction $^7\text{Li}(p, pt)^4\text{He}$ was successfully analysed [5,6]. The isotopes ^{6}Li and ^{7}Li are best described respectively as (α-d) and (α-t) clusters. In this paper, based on cluster models, we calculate electric polarizabilities employing the Green function approach and obtain a value of 0.0188 fm^3. Calculations based on sum rules give 0.05 fm^3 for the cluster contribution and 0.082 fm^3 for the single-particle contribution to the dipolarizability of ^{7}Li [7]. Measurements of dipolarizability of ^{7}Li have been carried out and comparison between theory and experiment has been made [7] but there are large experimental errors. Thus, it is useful to present an estimate based on a different method to have a greater confidence on theoretical values obtained in [7].

The particles that constitute these isotopes may or may not have the same charge to mass ratio; in case the ratio is not the same, the nucleus will re-orient and stretch under an external electric field. This change results in a polarization potential. For the case of deuteron, ground-state polarizability was calculated by Ramsey *et al* [8], and the value he found was about 0.56 fm^3 for deuteron in S-state with much smaller corrections from the D-state. Clearly, the result crucially depends on the correctness of the ground state description. Almost thirty years later, measurement of polarizability [9] gave 0.70 fm^3. There is no theoretical model that obtains this value, several calculations lead to a value of about 0.64 fm^3.

For the ^{6}Li isotope modelled as an α-d cluster, because the centre of charge coincides with the centre of mass, the dipole polarizability is zero. The polarizability may arise if ^{6}Li is described as a cluster of ^{3}H and ^{3}He. However, the results in that case will depend on polarizabilities of ^{3}H and ^{3}He besides that of their intercluster polarizability. It may be argued however that strong binding of ^{3}H and ^{3}He will lead to a very small polarizability. Besides, the separation energies of ^{3}H and ^{3}He in ^{6}Li will be comparatively large. This will again lead to very small contribution to the polarizability of ^{6}Li.

For the ^{7}Li isotope, the polarizability α will be non-zero, as seen by the calculation of polarization potential. Let Z_{T} be the atomic number of the target nucleus, which we assume to be the origin of the coordinate system. With this, the position vectors of t and α in the isotope are $\mathbf{r}_{\mathrm{t}}$ and $\mathbf{r}_\alpha$. Let the centre of mass be denoted by $\mathbf{R}$ and the vector from α to t in the cluster be denoted by $\mathbf{r}$. The total Hamiltonian is

$$\begin{aligned} H &= T + V_{\mathrm{t}\alpha}(\mathbf{r}) + \frac{Z_{\mathrm{T}}e^2}{|\mathbf{r}_{\mathrm{t}}|} + \frac{2Z_{\mathrm{T}}e^2}{|\mathbf{r}_\alpha|} \\ &= H_0(\mathbf{R}) + H_1(\mathbf{r}, \mathbf{R}), \end{aligned} \tag{1}$$

where T corresponds to kinetic energy. In (1),

$$\begin{aligned} H_0(\mathbf{R}) &= T_{\mathbf{R}} + \frac{3Z_{\mathrm{T}}e^2}{R} \quad \text{(centre of mass)} \\ H_1(\mathbf{r}, \mathbf{R}) &= T_{\mathbf{r}} + V_{\mathrm{t}\alpha}(r) + \frac{Z_{\mathrm{T}}e^2}{r_{\mathrm{t}}} + \frac{2Z_{\mathrm{T}}e^2}{r_\alpha} - \frac{3Z_{\mathrm{T}}e^2}{R} \\ &= T_{\mathbf{r}} + V_{\mathrm{t}\alpha}(r) + V_1(\mathbf{r}, \mathbf{R}). \end{aligned} \tag{2}$$

Writing $\mathbf{r}_{\mathrm{t}} = \mathbf{R} + \frac{4}{7}\mathbf{r}$ and $\mathbf{r}_\alpha = \mathbf{R} - \frac{3}{7}\mathbf{r}$,

$$\begin{aligned} V_1(\mathbf{r}, \mathbf{R}) &= \frac{Z_{\mathrm{T}}e^2}{|\mathbf{R} + \frac{4}{7}\mathbf{r}|} + \frac{2Z_{\mathrm{T}}e^2}{|\mathbf{R} - \frac{3}{7}\mathbf{r}|} - \frac{3Z_{\mathrm{T}}e^2}{R} \\ &= \frac{Z_{\mathrm{T}}e^2}{\sqrt{R^2 + \frac{16}{49}r^2 + \frac{8}{7}\mathbf{r}\cdot\mathbf{R}}} + \frac{2Z_{\mathrm{T}}e^2}{\sqrt{R^2 + \frac{9}{49}r^2 - \frac{6}{7}\mathbf{r}\cdot\mathbf{R}}} - \frac{3Z_{\mathrm{T}}e^2}{R} \\ &= \frac{2}{7}\frac{Z_{\mathrm{T}}e^2}{R^3}\mathbf{r}\cdot\mathbf{R} + O\left(\frac{r^2}{R^2}\right) \end{aligned} \tag{3}$$

after Taylor expansion and retaining terms up to $O(r/R)$. With $\mathbf{r}\cdot\mathbf{R} = rR\cos\theta = zR$, we have the polarization energy given by the second-order Stark effect in the presence of electric field $\mathcal{E}$:

$$W_{\mathrm{p}} = -\frac{4}{49}e^2\mathcal{E}^2 \sum_{n\neq 0} \frac{\langle 0|z|n\rangle\langle n|z|0\rangle}{E_n - E_0}, \tag{4}$$

where the sum is over all the states except the ground state, including continuum. Employing the definition of polarizability, we have

$$\alpha = -\frac{2W_{\rm p}}{\mathcal{E}^2}$$
$$= \frac{8}{49}e^2\left(\frac{2m_{{\rm t}\alpha}c^2}{\hbar^2c^2}\right)\sum_{n\neq 0}\frac{\langle 0|z|n\rangle\langle n|z|0\rangle}{k_n^2+\gamma^2}, \tag{5}$$

where $m_{{\rm t}\alpha}$ is the reduced mass of the cluster, $k_n^2 = 2m_{{\rm t}\alpha}E_n/\hbar^2$, $\gamma^2 = 2m_{{\rm t}\alpha}\epsilon/\hbar^2$, and $\epsilon = -E_{{\rm t}\alpha} = 2.47$ MeV denotes the binding energy. The ground state of ^{7}Li has a definite parity, hence $\langle 0|z|0\rangle = 0$. We can extend the summation over all the complete set of wave functions if we consider ϵ to be slightly different from the binding energy. Writing the wave functions explicitly,

$$\alpha = \frac{8}{49}e^2\left(\frac{2m_{{\rm t}\alpha}c^2}{\hbar^2c^2}\right)\int {\rm d}\mathbf{r}\int {\rm d}\mathbf{r}'\psi_0^*(\mathbf{r})zG_{{\rm t}\alpha}(\mathbf{r},\mathbf{r}')z'\psi_0(\mathbf{r}'), \tag{6}$$

where

$$G_{{\rm t}\alpha}(\mathbf{r},\mathbf{r}') = \sum_n \frac{\psi_n(\mathbf{r})\psi_n^*(\mathbf{r}')}{k_n^2+\gamma^2}. \tag{7}$$

This Green function satisfies the Schrödinger equation for ^{7}Li, viz.,

$$\left[-\nabla^2+\gamma^2+\frac{2m_{{\rm t}\alpha}}{\hbar^2}V_{{\rm t}\alpha}(\mathbf{r})\right]G_{{\rm t}\alpha}(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'), \tag{8}$$

where $V_{{\rm t}\alpha}(\mathbf{r})$ is the relative potential between t and α that binds them together. The Green function $G_{{\rm t}\alpha}(\mathbf{r},\mathbf{r}')$ is related to the free Green function $G(\mathbf{r},\mathbf{r}')$ by

$$G_{{\rm t}\alpha}(\mathbf{r},\mathbf{r}') = G(\mathbf{r},\mathbf{r}') - \frac{2m_{{\rm t}\alpha}}{\hbar^2}\int {\rm d}\mathbf{r}''G(\mathbf{r},\mathbf{r}'')V_{{\rm t}\alpha}(\mathbf{r}'')G_{{\rm t}\alpha}(\mathbf{r}'',\mathbf{r}'). \tag{9}$$

In the following, we ignore the second term for it is quite small for the following reasons. Firstly, note that the excited state of ^{7}Li is $^2P_{1/2}$ which can only be excited by a spin-orbit interaction between t and α. As seen in figure 5 of [4], the spin-orbit interaction energy in the permissible range of separation parameter is below 0.3 MeV. Therefore, the second term of (9) will contribute marginally due to the excited states because their separation is $\sim$0.5 MeV. Secondly, $V_{{\rm t}\alpha}$ is peaked at $r = 0$ at short t-α separation while the intercluster wave function is peaked at $\sim$2.5 fm (figure 5 of [5]). Thus the contribution of the second term must be small, particularly because the z-matrices vanish for the states of even parity.

Thus, we have the polarizability given by

$$\alpha = \frac{8}{49}e^2\left(\frac{2m_{{\rm t}\alpha}c^2}{\hbar^2c^2}\right)\int {\rm d}\mathbf{r}\int {\rm d}\mathbf{r}'\psi_0^*(\mathbf{r})zG(\mathbf{r},\mathbf{r}')z'\psi_0(\mathbf{r}'), \tag{10}$$

with the free Green function

$$G(\mathbf{r},\mathbf{r}') = \frac{1}{(2\pi)^3}\int {\rm d}\mathbf{k}\frac{\exp[i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')]}{k^2+\gamma^2}. \tag{11}$$

The ground state eigenfunction [5] is given by ($\beta = 0.288$ fm^{-2})

$$\psi_0(r) = r^3 \mathrm{e}^{-\frac{6}{7}\beta r^2}, \quad r < 3.53 \text{ fm (we call this part as } \psi_1)$$
$$= 9.84336 \left[\frac{1}{\gamma r} + \frac{1}{(\gamma r)^2}\right] \mathrm{e}^{-\gamma r}, \quad r > 3.53 \text{ fm}$$
$$\text{(we call this part as } \psi_2). \tag{12}$$

This wave function corresponds to the intercluster t-α wave function obtained by detailed, fully antisymmetrized, microscopic variational calculations using a Serber force which fits two-nucleon data upto about 40 MeV. The logarithmic derivative of the intercluster wave function is then matched at 3.53 fm with the logarithmic derivative of an exponentially decaying ($l = 1$) wave function corresponding to the t-α separation energy in ^{7}Li ground state. This combination of functions thus satisfies the binding energy of ^{7}Li by its ψ_1-part, and, fits the cluster knock out data due to its exponential tail.

The total wave function will be written as a sum over different spin-angular harmonics weighted with the above wave function. With $\mathcal{A}$ as a constant to be determined from normalization, we write

$$\psi_{\mathrm{t}\alpha}(\mathbf{r}) = \mathcal{A} \sum_{m=0,\pm 1} \psi_1(r) Y_{1m}(\theta,\phi), \quad r < 3.53 \text{ fm}$$
$$= \mathcal{A} \sum_{m=0,\pm 1} \psi_2(r) Y_{1m}(\theta,\phi), \quad r > 3.53 \text{ fm}, \tag{13}$$

where the sum corresponds to an unpolarized nucleus. Normalization condition on $\psi_{\mathrm{t}\alpha}(\mathbf{r})$ implies that $\mathcal{A}$ is given by

$$\frac{1}{3\mathcal{A}^2} = \int_0^{3.53} \mathrm{d}r r^2 \psi_1^2(r) + \int_{3.53}^{\infty} \mathrm{d}r r^2 \psi_2^2(r). \tag{14}$$

The polarizability is then given by $\alpha = \frac{8}{49}\frac{e^2}{\hbar c}\frac{2m_{\mathrm{t}\alpha}c^2}{\hbar c}\mathcal{J}$ where

$$\mathcal{J} = \frac{1}{(2\pi)^3}\int \mathrm{d}\mathbf{k} \int \mathrm{d}\mathbf{r}\mathrm{d}\mathbf{r}' \psi_{\mathrm{t}\alpha}(\mathbf{r})\psi_{\mathrm{t}\alpha}(\mathbf{r}') r\cos\theta \frac{\mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}} \cdot \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{r}'}}{k^2+\gamma^2} r'\cos\theta'$$
$$= \frac{1}{(2\pi)^3}\int \mathrm{d}\mathbf{k} |\mathcal{R}|^2. \tag{15}$$

$\mathcal{R}$ is the integral over $\mathbf{r}$ wherein we can now insert the explicit form for the eigenfunctions and the polar representation of plane wave in terms of the unit vectors $\hat{r}$ and $\hat{k}$:

$$\mathcal{R} = \int \mathrm{d}\mathbf{r} \psi_{\mathrm{t}\alpha}(\mathbf{r}) r\cos\theta \frac{\mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{k^2+\gamma^2}}$$
$$= \frac{\mathcal{A}}{\sqrt{k^2+\gamma^2}} \int \mathrm{d}r r^3 \mathrm{d}\Omega \sqrt{\frac{4\pi}{3}} Y_{10}(\theta,\phi)$$
$$\times \sum_{m'} \psi_{1,2}(r) Y_{1m'}(\theta,\phi) \sum_{l,m} 4\pi i^l Y_{lm}^*(\hat{r}) Y_{lm}(\hat{k}) j_l(kr). \tag{16}$$

We know that the product of spherical harmonics appearing above is

$$Y_{10}(\hat{r})Y_{1m'}(\hat{r}) = \sqrt{\frac{9}{4\pi}} \begin{bmatrix} 1 & 1 & 0 \\ 0 & m' & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} Y_{00}(\hat{r}) + \sqrt{\frac{45}{4\pi}} \begin{bmatrix} 1 & 1 & 2 \\ 0 & m' & m' \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} Y_{2m'}(\hat{r}). \tag{17}$$

The values of the symbols are

$$\begin{aligned} \begin{bmatrix} 1 & 1 & 0 \\ 0 & m' & 0 \end{bmatrix} &= -\sqrt{\frac{1}{3}}\delta_{m'0}, \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} &= -\sqrt{\frac{1}{3}}, \\ \begin{bmatrix} 1 & 1 & 2 \\ 0 & m' & m' \end{bmatrix} &= \frac{(-1)^{-m'}}{\sqrt{5}}\sqrt{\frac{(2-m')(2+m')}{6}}, \\ \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} &= \sqrt{\frac{2}{15}}. \end{aligned} \tag{18}$$

The product in (17) becomes

$$Y_{10}(\hat{r})Y_{1m'}(\hat{r}) = \sqrt{\frac{1}{4\pi}} Y_{00}\delta_{m'0} + \frac{(-1)^{-m'}}{\sqrt{20\pi}}\sqrt{4-m'^2}Y_{2m'}. \tag{19}$$

After some simple manipulations, $\mathcal{R}$ can be written as follows:

$$\begin{aligned} \mathcal{R} = &-\frac{\mathcal{A}\sqrt{3}}{\sqrt{k^2+\gamma^2}} \int \mathrm{d}r r^3 \psi_{1,2}(r) j_0(kr) \\ &+ i\frac{3\mathcal{A}\sqrt{2}}{\sqrt{k^2+\gamma^2}} \sin\theta_k \cos\theta_k \sin\phi_k \int \mathrm{d}r r^3 \psi_{1,2}(r) j_2(kr) \\ &+ \frac{\mathcal{A}}{\sqrt{k^2+\gamma^2}}(3\cos^2\theta_k - 1) \int \mathrm{d}r r^3 \psi_{1,2}(r) j_2(kr). \end{aligned} \tag{20}$$

The dipolarizability of ^{7}Li in the ground state, within the cluster model, is found to be 0.0188 fm^3. This result is reliable because the cluster model wave function of ^{7}Li not only reproduces the binding energy of ^{7}Li from microscopic cluster model calculations but also reproduces the cluster knock-out data which is highly sensitive to the surface region of the nucleus.

We now evaluate the observability of the polarizability in scattering experiment with ^{7}Li as projectile (P) and a heavy nucleus (e.g. ^{208}Pb or ^{238}U) as target (T). It is well-known that the effect shows up in the deviation of the cross-section, $\sigma(\theta)$ with the Rutherford cross-section, $\sigma_{\mathrm{R}}(\theta)$:

$$\Delta(\theta) = \frac{\sigma(\theta) - \sigma_{\mathrm{R}}(\theta)}{\sigma_{\mathrm{R}}(\theta)}. \tag{21}$$

Classical first-order calculation gives [10]

$$\Delta(\theta) = -g(\theta)\left(\frac{\nu}{R_{\rm int}}\right)^3 \left(\frac{Z_{\rm T}}{Z_{\rm P}}\alpha_{\rm P} + \frac{Z_{\rm P}}{Z_{\rm T}}\alpha_{\rm T}\right). \tag{22}$$

Here, $g(\theta)$ is a universal function, $E_{\rm P}$ is the projectile energy, $E_{\rm CB}$ is the energy at the Coulomb barrier, $\nu = E_{\rm P}/E_{\rm CB}$, $R_{\rm int} = r_0(A_{\rm P}^{1/3} + A_{\rm T}^{1/3})$ (we take $r_0 = 1.44$).

To obtain the projectile energy at which there could be observable deviation in cross-section, we employ (22) for the two systems: ^{2}H–^{208}Pb and ^{7}Li–^{208}Pb. To have the same Δ at a backward angle, a simple calculation gives us

$$E_{\rm Li} \sim 10E_{\rm d}, \tag{23}$$

where $E_{\rm d}$ is the energy of the deuteron beam used in the experiment to observe the deviation [9]. We have used the values of the Coulomb barriers in d–Pb and Li–Pb systems as 13.67 MeV and 37.6 MeV respectively. The polarizability of Pb is calculated using the relation based on the sum rule, valid for mass numbers greater than 40, $\alpha \approx 3.5 \times 10^{-3} A^{5/3}$ fm^3 (for ^{208}Pb, it is 25.5 fm^3) [11].

Thus, we may observe the effect of the polarizability of ^{7}Li at an energy of about 30 MeV. Since this is close to the Coulomb barrier, due to possible presence of nuclear effects, the experiment will have to be rather sensitive. The observation could be relatively easier if the target is ^{238}U. The value of energy estimated here is consistent with those given in [7]. Moreover, there is a recent measurement from which the extracted value of α is about 0.02–0.03 fm^3 [12]. The reasonable agreement with experimental value makes the theoretical estimation interesting as it shows that the model and assumptions are consistent with the physical picture.

The polarizability of ^{7}Li turns out to be about 20 times smaller than that of deuteron. Accordingly, the energy at which one can observe significant effect of ^{7}Li-stretching will be higher. In addition to this, the energy will be pushed up further due to somewhat larger α-t separation energy. Therefore, as estimated above, the deviation in cross-section could be seen only at relatively higher energy, giving rise to further complication due to nuclear effects by being closer to the energy of the Coulomb barrier. The contribution from triton polarizability (which is expected to enhance the value of α) is assumed to be small due to its tight binding as well as its averaging over t-α intercluster distribution.

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