PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 55, Nos 5 & 6
Nov. & Dec. 2000
pp. 693–698

Dromion solutions for an electron acoustic wave and its application to space observations

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Abstract. The nonlinear evolution of an electron acoustic wave is shown to obey the Davey–Stewartson I equation which admits so called *dromion* solutions. The importance of these two dimensional localized solutions for recent satellite observations of wave structures in the day side polar cap regions is discussed and the parameter regimes for their existence is delineated.

Keywords. Davey–Stewartson equation; electron acoustic wave; space plasma.

PACS No. 52.35.Mw

1. Introduction

For the last few decades exponentially localized structures like solitons in (1+1) dimensional space have been extensively studied in the context of some exactly integrable partial differential equations [1]. Its potential application in different physical fields are also well known [2]. However its higher dimensional generalization has remained an open field of investigation for a long time. It was Boiti *et al* [3] who first discovered the corresponding (2+1) dimensional analog which, unlike lump or algebraic solitons, is localized exponentially in both the spatial directions but in contrast to solitons can exchange energy during collisions. Since these solutions are 'driven by boundaries' (*dromos*), i.e., they possess time dependent boundary conditions, they are called dromions [4,5] and have drawn considerable attention in the field of nonlinear dynamics. A well known two dimensional p.d.e. that admits dromion solutions is the Davey–Stewartson I equation [6] which is often called a two dimensional generalization of the nonlinear Schrödinger equation. In recent times there have been extensive investigations into the mathematical properties of dromions with suggestions for applications in hydrodynamics, plasma physics, nonlinear optics etc. In plasma physics while a great deal of work has been done with one dimensional structures like solitons, these novel two dimensional *dromion* solutions have received limited attention [7]. Our present work is motivated by some recent satellite observations [8] of wave structures in the polar cap boundary layers where we feel dromions can help explain the experimental observations. The high time resolution data from the polar plasma wave instrument (PWI) reveal interesting low frequency electrostatic structures consisting of both large amplitude monopolar and bipolar pulses. The plasma parameters in this region typically have $T_i > T_e$ [9] so that ion acoustic waves are likely to be heavily damped. Electron acoustic waves on the other hand can exist in this region and there have been some past theoretical studies suggesting their relevance for the polar cap region [10,11]. These studies have been confined to one dimensional structures [12] and thus cannot fully account for these latest observations. Our aim is to demonstrate the possibility of forming two dimensional nonlinear structures for the electron acoustic wave in this region. For this we carry out a systematic reductive perturbation analysis in two dimensions of the model two fluid equations for the nonlinear evolution of the electron acoustic wave and show that it can be reduced to the DS-I type equations under certain conditions. We use these conditions to map out the physical parameter regimes in which dromion solutions can exist and discuss their relevance for the observed data.

2. Derivation of the DS-I equation

We start from the usual two fluid model description of the electron acoustic wave which is represented by the following set of equations [13]

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla} \cdot (n\mathbf{v}) = 0, \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \boldsymbol{\nabla})\,\mathbf{v} + 3\beta n \boldsymbol{\nabla} n - \boldsymbol{\nabla}\phi + \alpha\,(\mathbf{v} \times \mathbf{b}) = 0, \tag{2}$$

$$\nabla^2 \phi = n - \mathrm{e}^{-\phi}, \tag{3}$$

where n and v are the electron density and velocities respectively and α (= ω_{ce}/ω_{pe}) is the ratio of the electron cyclotron to the electron plasma frequency. β (= T_e/T_i) is the electron to ion temperature ratio, the magnetic field $\mathbf{b} = (0,0,1)$ is normalized by the ambient magnetic field B_0 and $\phi = e\phi/T_i$ is the normalized electrostatic potential. All the space variables are normalized by the ion Debye length (λ_i), time by the electron plasma frequency (ω_{pe}), velocities by the electron acoustic speed ($c_s = \sqrt{T_i/m_e}$) and the number densities by the ambient plasma density n_0. Note that the ion dynamics is represented by the Boltzmann distribution of the ion density which is substituted in the Poisson's equation. To carry out the reductive perturbation analysis we expand all the physical quantities as

$$q = q_0 + \sum_{n=1}^{\infty} \epsilon^n \sum_{l=-\infty}^{\infty} q_l^{(n)} \exp\left[\imath l\,(\mathbf{k} \cdot \mathbf{r} - \omega t)\right], \tag{4}$$

where $q = \{n, v_{xl}, v_{yl}, v_{zl}, \phi\}$ and $q_0 = \{1,0,0,0,0\}$, and $k = \left(0, k_\perp, k_\|\right)$ is the wave vector. We also assume the usual normalized boundary condition that $v_{x,y,z} \to 0, n \to 1$ and $\phi \to 0$ as $|x|, |y|, |z| \to \infty$. We next introduce the following stretched variables,

$$\xi = \epsilon x, \quad \eta = \epsilon\,(y - M_y t)\,, \quad \zeta = \epsilon\,(z - M_z t)\,, \quad \tau = \epsilon^2 t, \tag{5}$$

where M_y, M_z are the respective group velocities. Transforming all independent variables by eq. (5) we then carry out an order by order balance of terms. In the first order of ϵ, we recover the linear dispersion relation for the electron acoustic wave

$$\omega^4 - \omega^2 \left[\alpha^2 + |k|^2 (3\beta + K)\right] + \alpha^2 k_\parallel^2 (3\beta + K) = 0, \tag{6}$$

where $K = 1/1 + |k|^2$ and ω is the frequency of the wave. In the second order we get the expressions for the group velocities. In the third order, we get two generalized coupled equations. In the case of pure perpendicular propagation (i.e., for $k_\parallel = M_z = 0$) and restricting ourselves to two dimensions (with $\partial_\xi \to 0$) these equations have the following reduced form

$$\imath A_\tau + d_\eta A_{\eta\eta} + d_\zeta A_{\zeta\zeta} - \left(d_2 Q + d_1 |A|^2\right) A = 0, \tag{7}$$

$$-M_y^2 Q_{\eta\eta} + (3\beta + 1) Q_{\zeta\zeta} + \left[6\beta + (|k|K)^2\right] |A|^2_{\zeta\zeta} = 0, \tag{8}$$

where $A = n_1^{(1)}$ and $Q = n_0^{(2)}$. The various coefficients, namely $d_{p,j}$s are complicated algebraic functions of α, $k_\perp$ and β and are listed in the Appendix. Due to the symmetry of the system, all the cross-derivative terms in eqs (7), (8) vanish. The above two eqs (7) and (8) reduce to the DS-I type equations when the following conditions are satisfied [14],

$$d_\eta / d_\zeta > 0; \quad d_1 > 0. \tag{9}$$

In the limit of the one dimensional approximation (i.e., $\eta \to 0$), the DS-I equation further collapses to the well known nonlinear Schrödinger equation. In general, the DS-I equation admits both dromion and breather solutions which can be obtained either numerically [15] or for some specific values of the coefficients even in an analytic fashion [16,17]. For example, an idealized form of DS-I equation (under a specific transformation and for the specific values of the transformed coefficients) [14] is given by

$$\imath A_\tau + \frac{1}{2}\left(A_{\eta\eta} + A_{\zeta\zeta}\right) - \left(Q + |A|^2\right) A = 0, \tag{10}$$

$$Q_{\zeta\zeta} - \sigma Q_{\eta\eta} + 2\lambda |A|^2_{\zeta\zeta} = 0, \tag{11}$$

where σ and λ are given in the Appendix. The above equation can be solved by the Hirota bi-linear method. A simple analytic solution can be written down as [16]

$$Q' = 2p\,\mathrm{sech}^2(\zeta')\,\mathrm{sech}^2\left[p\eta' + \tanh(\zeta') - 2\ln 2p\right], \tag{12}$$

where $'$ denotes the transformed variable and p is some arbitrary real constant. Figure 1 illustrates this solution for $p = 5.5$.

For our analysis we worked with the more generalized equations (7), (8) and utilized condition (9) to determine numerically the parameter regime where they can acquire DS-I equation.

3. Results and discussion

Our numerical results are summarized in figure 2 which displays the regions in (k, α) space where condition (9) is satisfied. The region enclosed between the dashed curves

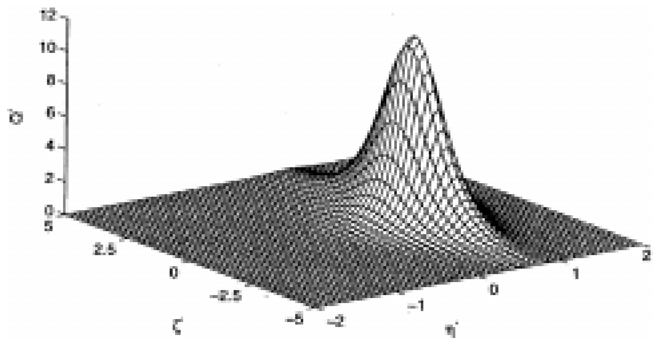

Figure 1. Analytical solution for a dromion.

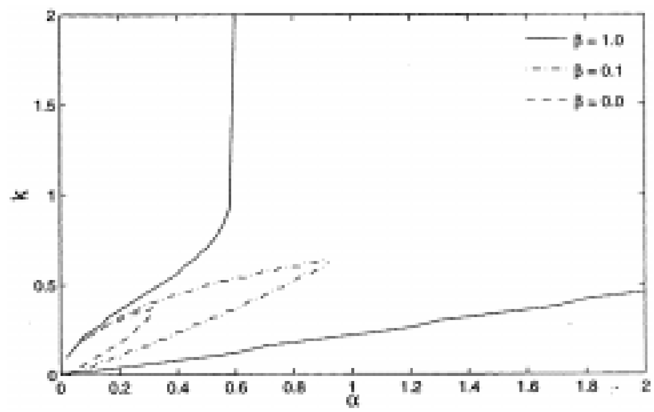

Figure 2. Existence domain of DS-I equation for different β.

is for $\beta = 0$, that within the dash–dot curves is for $\beta = 0.1$ and the largest enclosed region between solid curves is for $\beta = 1$. Thus increasing electron temperature enlarges the domain of parameter space where the evolution of the electron acoustic wave is governed by the DS-I type equation. Correspondingly the region for the existence of dromion like or breather type solutions also increases. We have next checked to see if these parametric regions include the physical parameter range corresponding to the experimental observations. The typical measured parameters in the region of the satellite observations (at a height of 5 to 7 earth radii) are $T_i = 100$ eV, $T_e = 10$ eV (i.e., $\beta = 0.1$), $n \simeq 1$ cm^{-3} ($f_{pe} \simeq 10$ kHz), $f_{ce} \simeq 5.5$ kHz (i.e. $\alpha = 0.5$) [9]. For these values figure 2 suggests the possibility of dromion-like solutions for $k \approx 0.35$ which corresponds to a wave of duration 0.15 ms ($f(\omega) \approx 6.69$ kHz). This is consistent with the observed frequency range of the wave structures. For a more quantitative understanding of the shape and finer features of the structure one needs to carry out a detailed numerical investigation of the evolution equation with appropriate initial conditions. Such an investigation is in progress and will be reported elsewhere. We are also examining the more general situation of oblique propagation (finite $k_{\|}$). Further it should also be pointed out that similar analyses can be carried out for other low frequency waves in this region, e.g. the lower hybrid waves, which can yield such two dimensional structures. Dromions thus offer a rich and new paradigm for understanding plasma wave phenomena in the ionospheric plasma and need to be investigated more intensely.

Appendix A: Coefficients for the equations

$$a = \frac{\omega}{\omega^2 - \alpha^2}\,; \quad a_\omega = \frac{2\omega}{4\omega^2 - \alpha^2}\,; \quad \kappa = \frac{1}{1 + 4|k|^2},$$

$$\frac{1}{h} = 1 + a^2 k_\perp^2 \left(1 + \frac{\alpha^2}{\omega^2}\right)(3\beta + K)\,; \quad f_\omega = a a_\omega (3\beta + K)\,,$$

$$d_\eta = a\,h\left[\left(3\beta + K^2\right) - 4k_\perp^2 \left\{K^3 + a^2 h \left(3\beta + K^2\right)^2\right\}\right],$$

$$d_\zeta = \frac{h}{a k_\perp^2}\left(1 - a^2 k_\perp^4 K^2\right),$$

$$d_1 = a_1 k_\perp^2 \left[h f_k - 2(3\beta + K)\right],$$

$$d_2 = a\,k_\perp^2 h\left[6\beta + (|k|K)^2\right],$$

where

$$f_k = f_\omega^2 k_\perp^2 \frac{\omega^2 + 2\alpha^2}{\omega a_\omega} + 2 f_\omega k_\perp^2 \left(1 + \frac{2\alpha^2}{\omega^2}\right)$$
$$\times \left[2(3\beta + \kappa) f_n + \left(3\beta - \kappa K^2\right)\right] + \left(6\beta + K - \kappa K^2\right) f_n - \kappa K^2,$$

$$f_n = \frac{\omega + a_\omega k_\perp^2 \left[\frac{a}{2\omega}\left(\alpha^2 + 2\omega^2\right)(3\beta + K) + \left(3\beta - \kappa K^2\right)\right]}{\omega - 2a_\omega k_\perp^2 (3\beta + \kappa)},$$

$$\sigma = \frac{d_\zeta}{d_\eta}\frac{M_y^2}{(3\beta + 1)}\,; \quad \lambda = \frac{d_2}{d_1}\frac{\left[6\beta + (|k|K)^2\right]}{(3\beta + 1)}.$$

Acknowledgments

One of the authors, S S Ghosh, would like to thank CSIR for its financial assistance for the work.

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