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ABSTRACT

The dielectric properties of a plasma embedded with charged dust particles is examined. The presence of dust introduces a background inhomogeneous electric field which significantly influences the dispersion properties of the plasma medium. At low frequencies we find important modifications of the ion acoustic branch as well as the existence of a new mode arising from the oscillations of the ions in the static structure of the dust distribution.

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There has been a growing interest in dusty plasmas in recent years[1-7]. Dust grains are present in space plasmas, viz., the earth's ionosphere, planetary rings, asteroid zones, cometary tails, interstellar clouds, around protostars, supernova remnants, etc. and are also encountered in many laboratory devices and plasma processing technologies. The presence of highy charged ($Z = 10^3 - 10^4$) and massive dust particles of varying sizes, distributed randomly in partially or fully ionized electron-ion plasmas can have significant influence on plasma properties. In this Letter we examine the low frequency electrostatic response of a plasma embedded with dust particles.

Our analysis is closely based on the technique used by Angelis et al.[7], who have investigated the linear damping of high frequency plasma waves in a dusty plasma. Our equilibrium system consists of charged dust particles, ions and electrons which satisfy the overall charge neutrality condition

$$\sum_{\alpha} q_\alpha n_{o\alpha} + QN_g = 0, \alpha = e, i \tag{1}$$

where $q_\alpha, n_{o\alpha}$ and Q, N_g are the charge and number density of electrons/ions and those of the dust grains, respectively. The stationary and randomly oriented dust particles give rise to a background electrostatic field which can be characterized by a potential function $\phi_o(\underline{r})$. As shown by Whipple et al.[1] the average of this potentia is given by

$$\bar{\phi}_o = \frac{1}{V}\int \phi_o(\underline{r})d^2r = \frac{3Q}{\lambda_D}(\frac{\lambda_D}{r_o})^3, 4\pi r_o^3/3N_g = 1, \tag{2}$$

where r_o and λ_D are the average separation and Debye length of dust particles, respectively. We consider the situation where $r_o >> \lambda_D$ and $e\phi_o/T_e << 1$; T_e being the temperature of electrons in units of the Boltzmann constant.

Following Angelis et al.[7] the modified equilibrium distribution functions for electrons and ions in the stationary dusty plasma represented by the potential field $\phi_o(\underline{r})$ is then given by

$$f_{\alpha o}(\underline{r}, \underline{v}) = (1 - \frac{u_\alpha}{T_\alpha} - \frac{u_\alpha^2}{T_\alpha^2})\left[F_{\alpha o} + F'_{\alpha o}u_\alpha\sigma(\underline{r}) + \frac{1}{2}F''_{\alpha o}u_\alpha^2\sigma^2(\underline{r})\right], \tag{3}$$

where the normalization factor is contained in $F_{\alpha o} = (n_{o\alpha}/\pi\sqrt{\pi}V_{th\alpha}^3)\exp(-v^2/V_{th\alpha}^2)$, $V_{th\alpha} = (2T_\alpha/m_\alpha)^{1/2}; T_\alpha, m_\alpha$ being the temperature in energy units and mass of electrons/ions, and the prime denotes derivative with respect to the kinetic energy $= \frac{1}{2}m_\alpha v^2$. In Eq.(3), $u_\alpha = q_\alpha \bar{\phi}_o$, the average potential energy of electrons/ions of charge q_α and the random statistical function $\sigma(\underline{r})$ defined from $q_\alpha \phi_o(\underline{r}) = u_\alpha \sigma(\underline{r})$ is characterized by a correlation function

$$B(\mid \underline{r} - \underline{r}' \mid) = < \sigma(\underline{r})\sigma(\underline{r}') > \tag{4}$$

where the <> denotes the ensemble average[8].

In the above representation the dust grains are assumed to be replaced by point particles having potential $\phi_o(\underline{r})$ with a finite correlation of correlation length comparable to the wavelengths of the waves of interest. The perturbed motion of the plasma electrons and ions is described by the linear Vlasov equation

$$\frac{\partial f_\alpha}{\partial t} + (\underline{v} \cdot \underline{\nabla}) f_\alpha + \frac{q_\alpha}{m_\alpha} [\underline{E}(\underline{r}, t) \cdot \underline{\nabla}_v f_{\alpha o} - \underline{\nabla} \phi_o(\underline{r}) \cdot \underline{\nabla}_v f_\alpha] = 0, \tag{5}$$

where, f_α represents the perturbed distribution function. Carrying out the usual Fourier-Laplace transformation Eq.(5) can be solved perturbatively to give

$$\begin{aligned} \tilde{f}_\alpha(\omega, \underline{k}) &= \frac{-iq_\alpha}{m_\alpha(\omega - \underline{k} \cdot \underline{v})} \int \underline{E}(\omega, \underline{k}_1) \cdot \underline{\nabla}_v f_{\alpha o}(\underline{k} - \underline{k}_1) d^3 k_1 \\ &+ \frac{iq_\alpha^2}{m_\alpha^2(\omega - \underline{k} \cdot \underline{v})} \int d^3 k_2 \phi_o(\underline{k}_2) \\ &\times (\underline{k}_2 \cdot \underline{\nabla}_v) \int \frac{\underline{E}(\omega, \underline{k}_1)}{[\omega - (\underline{k} - \underline{k}_2) \cdot \underline{v}]} \cdot \underline{\nabla}_v f_{\alpha o}(\underline{k} - \underline{k}_1 - \underline{k}_2) d^3 k_1. \end{aligned} \tag{6}$$

Substituting in Poisson's equation, the linear dielectric function for the perturbed mode $(\omega, \underline{k})$ can be obtained as

$$\epsilon(\omega, k) = 1 - \sum_{i=1}^{3} \chi_i, \tag{7}$$

where

$$\begin{aligned} \chi_1(\omega, \underline{k}) &= \sum_\alpha \frac{4\pi q_\alpha^2}{m_\alpha} [(1 - \frac{u_\alpha}{T_\alpha} - \frac{u_\alpha^2}{T_\alpha^2}) \int \frac{f_{\alpha o}(\underline{v})}{(\omega - \underline{k} \cdot \underline{v})^2} d^3 v \\ &+ u_\alpha (1 - \frac{u_\alpha}{T_\alpha}) \int \frac{f'_{\alpha o}(\underline{v})}{(\omega - \underline{k} \cdot \underline{v})^2} d^3 v + u_\alpha^2 \int \frac{f''_{\alpha o}(\underline{v})}{(\omega - \underline{k} \cdot \underline{v})^2} d^3 v], \end{aligned} \tag{8}$$

$$\chi_2(\omega,\underline{k}) = 2\sum_\alpha \frac{4\pi q_\alpha^2}{m_\alpha^2} u_\alpha^2 \int d^3q(\underline{k}\cdot\underline{q})S(\underline{q})\int \frac{f'_{\alpha o}(\underline{v})}{(\omega-\underline{k}\cdot\underline{v})^3[\omega-(\underline{k}-\underline{q})\cdot\underline{v}]}d^3v, \tag{9}$$

$$\begin{aligned}\chi_3(\omega,\underline{k}) &= -\sum_\alpha \frac{4\pi q_\alpha^2}{m_\alpha^2} u_\alpha^2 \int d^3q \frac{(\underline{k}\cdot\underline{q})[\underline{k}\cdot(\underline{k}-\underline{q})]}{k^2} S(\underline{q}) \\ &\times \int \frac{f'(\underline{v})}{(\omega-\underline{k}\cdot\underline{v})^2[\omega-(\underline{k}-\underline{q})\cdot\underline{v}]^2}d^3v,\end{aligned} \tag{10}$$

where the spectral density of the correlation is given by

$$S(\underline{q})\delta(\underline{q}+\underline{q}') = \langle\sigma(\underline{q})\sigma(\underline{q}')\rangle. \tag{11}$$

We now solve Eq.(7) in the low frequency range and consider two limits (i) $kV_{thi} \ll \omega \ll kV_{the}$ and (ii) $\omega \ll kV_{thi}$. In both cases we evaluate the integrals in Eqs.(8-10) by assuming a model Gaussian distribution for the correlation function

$$S(q) = (1/\pi\sqrt{\pi}q_o^3)\exp(-q^2/q_o^2), \tag{12}$$

where q_o is the correlation length for the static dust grains in the plasma.

(i) In the first case, the linear dielectric function for the low frequency mode reduces to

$$\begin{aligned}\epsilon(\omega,k) &= 1+\frac{1}{k^2\lambda_{De}^2}\left[1-2\mu_e+\mu_e^2\{\frac{2}{3}-\frac{k}{3q_o}Z(\frac{k}{q_o})\}\right] \\ &- \frac{\omega_{pi}^2}{\omega^2}\left[1-2\mu_i+\mu_i^2\{1-\frac{q_o^2V_{thi}^2}{4\omega^2}-i(\frac{kV_{thi}}{\omega}I_1-\frac{\omega}{q_oV_{thi}}I_2)\}\right] \\ &+ \frac{2i\sqrt{\pi}\omega_{pi}^2\omega}{k^3V_{thi}^3}(1-2\mu_i+\mu_i^2)\exp(-\omega^2/k^2V_{thi}^2),\end{aligned} \tag{13}$$

where $Z(\frac{k}{q_o})$ is the usual plasma dispersion function, $\mu_\alpha = q_\alpha\bar{\phi}_o/T_\alpha$, $\omega_{p\alpha}^2 = 4\pi q_\alpha^2 n_{o\alpha}/m_\alpha, \lambda_{D\alpha}^2 = T_\alpha/4\pi q_\alpha^2 n_{o\alpha}$, and

$$\begin{aligned}I_1 &= \frac{1}{q_o}\int_0^\infty dq_\parallel \frac{q_\parallel}{k+q_\parallel}\exp\left[-\frac{q_\parallel^2}{q_o^2}-\frac{\omega^2}{(k+q_\parallel)^2V_{thi}^2}\right], \\ I_2 &= \int_0^\infty dq_\parallel \frac{q_\parallel}{(k+q_\parallel)^2}\exp\left[-\frac{q_\parallel^2}{q_o^2}-\frac{\omega^2}{(k+q_\parallel)^2V_{thi}^2}\right].\end{aligned} \tag{14}$$

Neglecting μ_e^2 and μ_i^2 compared to 1, and solving $\epsilon_r = 0$, where ϵ_r is the real part of the dielectric function, we obtain the linear dispersion relation of the low frequency electrostatic mode as

$$\omega^2 = \frac{k^2 C_s^2 (n_{oi}/n_{oe})}{1 + k^2\lambda_{De}^2 - 2\mu_e\{\frac{2}{3} - \frac{k}{3q_o} ReZ(\frac{k}{q_o})\}}, \tag{15}$$

where ReZ represents the real part of Z, $C_s = (2T_e/m_i)^{1/2}$ is the ion sound speed in the plasma, and $n_{oi} > n_{oe}$ for a dusty plasma[9,10]. This is the usual ion acoustic branch of the low frequency mode modified by the dust electrostaic potential, and the nonneutrality of the dusty plasma.

The linear damping rate of this ion acoustic mode in the dusty plasma is given by

$$\begin{aligned} \gamma_L &= -\epsilon_i/(\partial\epsilon_r/\partial\omega) \\ &\simeq \left[\frac{\mu_e^2}{3k^2\lambda_{De}^2}\frac{k}{q_o}\sqrt{\pi}\exp(-k^2/q_o^2) - \frac{\mu_i^2\omega_{pi}^2}{\omega^2}(\frac{kV_{thi}}{\omega}I_1 - \frac{\omega}{q_oV_{thi}}I_2) - \frac{2\sqrt{\pi}\omega_{pi}^2\omega}{k^3V_{thi}^3}\exp(-\omega^2/k^2V_{thi}^2)\right] \\ &\div \left[\frac{2\omega_{pi}^2}{\omega^3}\{1 - 2\mu_i + \mu_i^2(1 - \frac{q_o^2V_{thi}^2}{2\omega^2})\}\right], \end{aligned} \tag{16}$$

where I_1 and I_2 are given by Eq.(14).

(ii) For the second case, $\omega \ll kV_{thi}$ the linear dielectric function of the dusty plasma becomes

$$\epsilon(\omega, k) = \epsilon_R + i\epsilon_I, \tag{17}$$

where

$$\begin{aligned} \epsilon_R &= 1 + \frac{1}{k^2\lambda_{De}^2}\left[1 - 2\mu_e + \mu_e^2\{1 - \frac{1}{3}ReZ(\frac{k}{q_o})\}\right] \\ &+ \frac{1}{k^2\lambda_{Di}^2}(1 - 2\mu_i + \mu_i^2) - \frac{3\mu_i^2\omega_{pi}^2q_o^2}{2\omega^2k^2}, \end{aligned} \tag{18}$$

$$\begin{aligned} \epsilon_I &= -\frac{\mu_e^2}{3k^2\lambda_{De}^2}ImZ(\frac{k}{q_o}) + \frac{\sqrt{\pi}\omega}{k^3\lambda_{Di}^2V_{thi}}(1 - 2\mu_i + \mu_i^2)\exp(-\omega^2/k^2V_{thi}^2) \\ &+ \frac{\mu_i^2\omega_{pi}^2V_{thi}}{k^2\omega^3q_o}\int_{-\infty}^{\infty} dq_\parallel q_\parallel (k - q_\parallel)^2 \exp\left[-\frac{q_\parallel^2}{q_o^2} - \frac{\omega^2}{(k - q_\parallel)^2V_{thi}^2}\right] \\ &- \frac{\mu_i^2\omega_{pi}^2\omega}{2k^2q_oV_{thi}^3}\int_{-\infty}^{\infty} dq_\parallel \frac{q_\parallel}{(k - q_\parallel)^2}\exp\left[-\frac{q_\parallel^2}{q_o^2} - \frac{\omega^2}{(k - q_\parallel)^2V_{thi}^2}\right]. \end{aligned} \tag{19}$$

It is noted that the resonant contribution to the dielectric function comes from the ion terms in χ_2^{ion} and χ_3^{ion} where $\omega \approx (k - q_{\|})v_{\|}$. Solving $\epsilon_R = 0$ we get

$$\omega^2 = \frac{3\mu_i^2 q_o^2 \omega_{pi}^2}{2k^2[1 + \frac{1}{k^2\lambda_{De}^2}\{1 - 2\mu_e + \mu_e^2(1 - \frac{1}{3}ReZ(\frac{k}{q_o}))\} + \frac{1}{k^2\lambda_{Di}^2}(1 - 2\mu_i + \mu_i^2)]}. \tag{20}$$

Since $\mu_e, \mu_i \ll 1$ the above dispersion relation reduces to

$$\omega^2 = \frac{3\mu_i^2 q_o^2 \omega_{pi}^2}{2k^2[1 + \frac{1}{k^2\lambda_{De}^2} + \frac{1}{k^2\lambda_{Di}^2}]} \simeq (3/4)\mu_i^2 q_o^2 V_{thi}^2. \tag{21}$$

This is a new mode in the dusty plasma having frequency much less that the usual ion acoustic branch. The linear damping rate of this low frequency mode is given by

$$\gamma_L = -\epsilon_I/(\partial\epsilon_R/\partial\omega), \tag{22}$$

where ϵ_I is given by Eq.(19) and $\partial\epsilon_R/\partial\omega = 3\mu_i^2\omega_{pi}^2 q_o^2/k^2\omega^3$.

We have investigated the low frequency electrostatic response of a plasma embedded with a stationary distribution of massive dust particles. The background inhomogeneous electric field created by the dust has significant effects on the low frequency modes. For the limit $q_\alpha\bar{\phi}_o/T_\alpha \ll 1, \alpha = e, i$ we have examined two low frequency modes. The usual ion acoustic branch is found to be only slightly modified by the electric field contribution. The major modification to the ion acoustic mode comes from the equilibrium charge imblance condition between ions and electrons as is commonly found in multispecies plasmas as well[9,10]. We also find a new mode below the ion thermal velocity which arises from the presence of the static dust distribution. It involves the ion thermal motion and the correlation length of the dust distribution, and arises from the oscillation of the ions in the background electric potential. It is analogous to a quisi-mode found in external pump driven parametric instability phenomena[11] - the dust potential can be viewed as a zero frequency finite wavenumber pump wave. The presence of this new ultralow frequency mode could have interesting consequences for wave scattering experiments off dusty plasmas.

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