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STOCHASTIC VECTORIAL FLOW GRAPH ANALYSIS OF METEOROLOGICAL CAUSE-EFFECT RELATIONSHIPS FOR DROUGHT PREDICTION

N. SESHAGIRI
National Informatics Centre
E-Wing, Pushpa Bhavan
Chiragdelhi , Madangir Road
New Delhi - 110 062, India

ABSTRACT

Advocating the view that the drought prediction models which neglect the complex feedback mechanisms in the cause effect relationships between variables of meteorological, hydrological and topographic origin may give faulty predictions, a system theoretic method is proposed for taking into consideration certain dominant cause-effect relationships. The method is based on the concept of stochastic vectorial flow graph developed earlier by the author which is used in conjunction with a characterization of the interacting stochastic variables by cross-autocorrelation. A comprehensive feedback system of important interacting meteorological variables is proposed for drought prediction along with an approach for computer aided analysis of such a system.

1. INTRODUCTION

Drought prediction methodology is predominantly based on the study of trends of effects from isolated causes. Invariably the methods tend to neglect the interactive feedback mechanisms in the cause-effect relationships between variables of meteorological, hydrological and topographic origin. To this extent such methods do not faithfully reflect the phenomena causing drought. Whereas such approaches are important in understanding the effect of variations of individual causes and trends on the precipitation or hydrologic characteristics of drought on a region of interest, they must be imbedded in a larger model treating the climatological system including all important feedback mechanisms in order to be able to predict drought accurately.

Any index of drought defined necessarily requires the analysis of a highly complex network of dependences between the set of meteorological, hydrological and topographic factors. Even granting that we can enumerate the more important factors with our present state of knowledge, the interaction network constructed with these factors as the nodes will be quite complex because there are not only open loop and feed forward interactions between the factors but also feedback interactions. From the theory of feedback systems, it is well known that such a system can exhibit complex patterns of spacially and temporally varying stability and instability which can make any effort at the prediction of the system behaviour by neglecting these patterns somewhat unrealistic. Recognizing the importance of this requirements in any drought prediction model, a beginning is sought to be made in this paper for representing the precipitation pattern over a region in terms of cause-effect graph as a special case of the concept of stochastic Vectorial flow-graphs developed earlier by the author (Seshagiri) 1971). This approach can give an implementation modality for the diagnosis made by Klemes (1978) regarding the essentiality of taking cause-effect relationships in drought prediction models.

In the development of the Stochastic Vectorial Flow Graph of climatic variables that is of relevance to drought prediction, analysis made for example, in the Report of the Study of Man's Impact on Climate (SMIC, 1971), Manabe and Bryan (1969); Manabde and Wetherald (1967), Budyko (1971) and Mason (1976) have been utilized. In particular; the feedback mechanisms in the ocean-atmosphere coupling, water vapour-greenhouse coupling, snow cover-albedo-temperature coupling, cloud cover —radiation balance coupling are sought to be characterized in a single graph.

The Stochastic Vectorial Flow Graph (SVF) incorporating such couplings is constructed by regarding various locations in the region of interest, along with telelink factors, as nodes of the network, at which is defined a stochastic variable composed of time functions of factors like temperature, albedo, cloud cover, precipitation, relative humidity, horizontal wind and vertical wind. The flow network connecting these nodes is formed by related branches between the nodes with weightages on them making use of the concepts of 'Cross-autocorrelation' of the version developed by Makridakis et al. It appears from the predominance of interrelated causes and effects that autocorrelation approaches are inadequate and it would be necessary to combine the concept of autocorrelation with that of cross correlation. The resulting concept of cross-autos has been found to be a powerful one in prediction exercises. In the present study, the concept is found to be the most natural characterization of the branch weight in the flow graph. A method is outlined for analysing such a graph so as to determine the effect, on precipitation, of changes in the values of other variables in the graph.

2. GENERAL CONCEPT OF THE SVF GRAPH

A brief summary of the general concept of the SVF graph and mathematical rules for analysing them developed in the earlier paper by Seshagiri (1971), is given in this section.

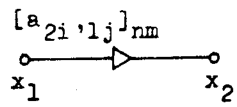
A general feedback system can be characterized by inter-actions among n sets of variables

$$V_i = \left\{ v_{ij} \,/\, j = 1,2,\ldots,\ldots, m_i \right\} \; ; \; i = 1,2\ldots,n.$$

If all m_i are assumed to be unity, a scalar flow graph may be described as in Fig.1(a). If the m_i are greater than unity, a flow graph with similar feedback interactions between sets of nodes may be described as in Fig.1(b). This describes a case in which the nodal variable v_{11} and v_{12}, v_{21}, v_{31} and v_{32} form the vector V_1, V_2 and V_3

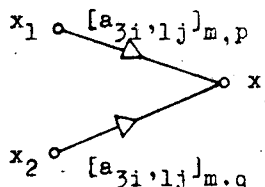
respectively. It is possible to represent the flow graph if Fig.1 (b) as a flow graph in which each node stands for a vector and each branch for a matrix as shown in Fig.1(c). For notational convenience, scalar variables and transmittances (of disturbances) are denoted by small letters and the vector variables and transmittance matrices by captial letters. Similarly, the nodes and branches for the two cases are distinguished as shown in Fig.1(d)- The Fig.1(a) is a 'scalar flow graph' and Fig.1(b) a 'vectorial flow graph'. If the nodal variables are stochastic, we have a 'stochastic vectorial flow graph'.

Two vector variables $X_1 = x_{11}, x_{12}, \ldots, x_{1m}$ and $X_2 = x_{21}, x_{22}, \ldots, x_{2n}$ interact in all possible combinations of their constituents resulting in a maximum of mn branches. The corresponding mn branch transmittances can be represented as a rectangular matrix with m rows and n columns, i.e., $[a_{2i,1j}]_{nm}$. This is referred to as the 'transmatrix'. We can then define a vector correspondance rule by



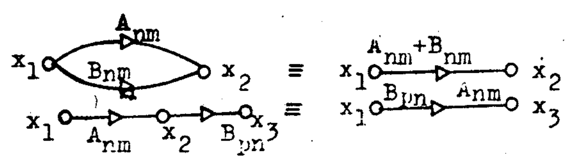
or, $x_2 = [a_{2i,1j}]_{nm}\, x_1$

For several incoming branches a correspondence rule can be derived as



or, $x_3 = [a_{3i,1j}]_{mp} x_1 + [a_{3i,2j}]_{mq} x_2$

Generalization of this to n in-coming branches is straight-forward. The addition and multiplication rules are given by

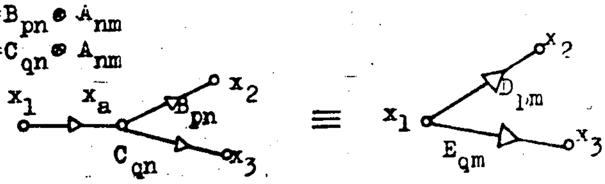


It is important to note that the rectangular matrices during addition and multiplication should be conformable in dimensions.

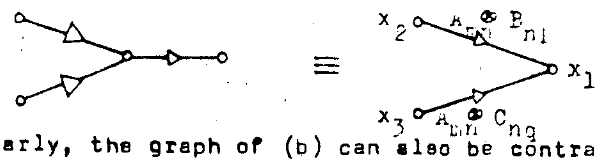
Contraction of the vectorial flow graph can be defined as follows: For the graph shown at (a) below, the transmatrix between X_1, X_2, is given by $(B_{pq} \otimes A_{nm})$ and that tween X_1 and X_3 by $(C^2_{qn} \otimes A_{nm})$. Hence the three branches can be contracted into two branches with transmatrices D_{pm} and E_{qm}, where

$$D_{pm} = B_{pn} \otimes A_{nm}$$
$$E_{qm} = C_{qn} \otimes A_{nm}$$

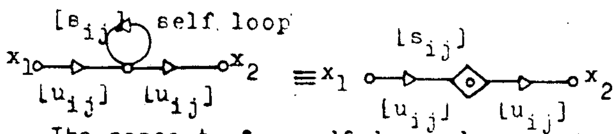
(a)



(b)



Similarly, the graph of (b) can also be contracted.



The concept of a self-loop plays an important role in the reduction of vectorial flow graphs with feed back loops. The vectorial self loop is shown above.

Here $[u_{ij}]$ is the identify matrix and $[s_{ij}]$ is a square matrix of dimension equal to the number of elements in the nodal vector V. From the correspondance rule, we can derive the infinite matrix series,

$$x_2 = [[u_{ij}] + [s_{ij}] + [s_{ij}]^2 + \dots + \text{to } \infty]x_1$$

From matrix methods it can be shown that, granting convergence, the infinite matrix series can be reduced to

$$x_2 = [[u_{ij}] - [s_{ij}]]^{-1} x_1$$

Also, the basic feedback loop with a forward transmatrix A_{mn} and a feedback transmatrix B_{nm} can be reduced to a branch with a transmatrix

$$A_{mn} = A_{mn} [U_{nn} - B_{nm} \quad A_{mn}]^{-1}$$

With the above set of rules of operation, it is possible to analyze highly complex interactions between numerous stochestic vector variables. These rules help to bridge any two variables in the interactive network of variables by collapsing all other variables into a transmatrix composed of elements which are functions of the other variables. By this, the effect of a disturbance or change in one variable on another variable anywhere in the graph can be computed.

3. THE BASIC METEOROLOGICAL SVF GRAPH

Based on the definitions outlined in Section 2, a basic meteorological stochastic vetorial flow graph is proposed which represents the cause effect relationships between the more important meteorological variables at the macro level. In the construction of this SVF graph, the following more important couplings have been accounted for.

In the ocean-atmosphere coupling, heat is exchanged through radiative fluxes and turbulent transport of sensible and latent heat. Water is exchanged through precipitation and evaporation. Momentum is exchanged through surface wind stress at the oceanic surface. For taking into consideration the water vapour -greenhouse feedback coupling the chain process considered is: warmer temperature → more water vapour in the atmosphere → more greenhouse effect → warmer temperature. The snow cover - albedo - temperature coupling is dependent on the positive feedback loops wider snow cover → more reflection of insolation → colder surface temperature → wider snow cover. Since the depth of the snow cover decreases during the summer and increases during winter, seasonal vafiation of solar radiation has strong controlling effects over the net change of snow depth from year to year. It is therefore necessary to incorporate the effects of the seasonal variation of solar radiation in order to discuss the stability of the snow or ice cover or the sensitivity of climate. Cloud cover has important effect upon the radiation balance of the earth - atmosphere system. It is therefore determined as a function of the large scale distribution of quantities such as relative humidity, temperature and vertical velocity.

The above feedback mechanisms are typical examples of important meteorological interactions that should go into a basic SVF graph characterization. A comprehensive SVF graph incorporating all such feedback and feed forward mechanisms in a consolidated manner is given in Fig. 2. Here all variables are vectors whose n elements give the stochastic variables quantifying the factors denoted in the legend for Fig. 2. These n elements of the vector variable refer to a set of n regions. This enables the model to be implemented hierarchically from the global to the local level. In the first instance, the n elements may refer to n regions of the globe. After determining the average trends for these n regions, the model can be applied to n_1 subregions within a region of interest. The model can be further applied to n_2 areas within a subregion of interest. By the hierarchic application of the model from the global to the local, it is possible to modulate the local trends with the global trends. The topology of the SVF graph through successive levels of the hierarchy will be identical to what is given in Fig. 2.

4. NODAL NORMALIZATION AND BRANCH WEIGHT INDEX

The stochastic variables denoting the nodes of the graph have to be made dimensionless through a normalization procedure. One of the simplest criterion is to define the variables such that the average value over a sufficiently long past is taken to be zero and the extrema of the modulus over the same duration as unity. All other values are linearly interpolated. This normalizes all the nodal variables in Fig. 2. to values between +1 and -1 with their average at zero. Such a normalization will be termed 'linear normalization'. However, for each variable, it is also possible to introduce a non-linear interpolation scheme to reflect certain non-linear trends in their cause-effect relations with other variables. In this paper, we confine ourselves to linear normalization as the method of non-linear normalization which has distinct theoretical basis of its own, will be discussed in a separate paper.

The normalized variables being themselves stochastic, interaction between pairs of them can be characterized by indices based on correlation. Such an index should not only reflect autocorrelations of the individual variables butalso the cross-correlation between the pair. The cross- autocorrelation coefficient or 'cross-auto' is found to be an ideal index for describinn the degree of association between the two variables for various time lags. They can be visualized as combining the characteristics of correlation coefficients and auto-correlation coefficients into a single measure of association. As with correlations and autocorrelations the cross-auto correlations are relative measure of association that vary from -1 to +1. A value close to -1 indicates a strong negative relationship between x_t and y_{t+k}, i.e. k time lags apart; a value close to +1 indicates a strong positive relationship, while a value of zero indicates the absence of an association between x_t and y_{t+k}. Estimated cross-autos based on sample data are usually expressed as

$$r_{xy}(k) \; ; \; k = 0, \pm 1, \pm 2, \pm 3, \ldots \pm m$$

where x is the independent variable

y is the dependent variable

k is the number of time lags separating X_t and Y_{t+k}

m is the number of time lags computed

The formula for computing the cross-autos for positive k is:

$$r_{XY}(k) = \frac{\sum_{t=1}^{n-k}(X_t - \bar{X})(Y_{t+k} - \bar{Y})}{\sqrt{\left[\sum_{t=1}^{n}(X_t - \bar{X})^2\right]\left[\sum_{t=1}^{n}(Y_t - \bar{Y})^2\right]}}$$

$$r_{YX}(k) = r_{XY}(-k)$$

In the SVF graph of Fig.2, the branches will have a weightage equal to the cross-auto between the two nodal variables on either side of the branch. Along with this weightage, the time delay between the cause and the effect, either, in particular, as the mean value or, in general, as a distribution is also specified as a distinct branch weight. Together, the cross-auto and delay, constitute a binary branch weight. As the nodal variables are vectors, the binary branch weight will be composed of a cross-auto transmatrix and a delay transmatrix. For predicting the trend of any variable, e.g., precipitation P, the time lags at all other nodes have to be determined along with the corresponding cross-autos. Each nodal variable will be computed accounting for the corresponding time lag with respect to P.

When the time profile of P, T, A_i, A_s, M_s, E, etc. are determined from the graph over an area servicing the stream flow, the run estimate can be made using known techniques.

5. ANALYSIS OF THE BASIC SVF GRAPH

In the basic graph of Fig. 2, the variable of interest is P, the vector index of precipitation over the n regions. Disturbances or trends in the other variables will eventually transmit their effects to P. For example, a variation in the index of solar radiation R_s, at a time accounted for by the delay weightages over the branches connecting R_s and P, will have its effect on P at the present time. On the other hand, if the value of R_s at a later time is considered, a prediction of P will be realized, provided all other variables are computed at a time before the present time. The latter situation will arise in the simulation of the system iteratively with unit time steps. P at a future time in terms of the other nodal values at different times decided by the delay weightages and the cross-auto branch weightage can be computed basically by the judicious use of the rules of analysis described in Section -2 like the addition rule, multiplication rule, contraction rule, self-loop, node splitting and branch splitting. All these rules lend themselves for being incorporated in a computer simulation. If the delay branch weights are included as the mean value, P will result as a vector whose values are sharply estimated. On the other hand, if the delay branch weight is a statistical distribution, P will also be a statistical distribution.

Computer simulation efforts on the above lines is underway for the prediction of droughts.

6. CONCLUSION

By developing a Stochastic Vector Flow Graph model of the cause-effect relationships in the meteorological variables related to the drought phenomenon, it is shown that highly complex feedback mechanisms exist which have to be fully incorporated in any model proposing to give a prediction of drought. It is also shown that the concept of SVF graph developed earlier by the author is a national representation of such feedback mechanisms if a binary branch weight comprising of the cross-auto transmatrix and delay transmatrix is adopted.

The SVF graph description given in this paper has also been investigated by the author in terms of equivalent State Space Concepts which will be the subject matter of a separate communication.

The theory proposed in this paper is still in its formative stages. Much more development along these lines is required to be carried out to enable realistic computer-aided prediction of drought.

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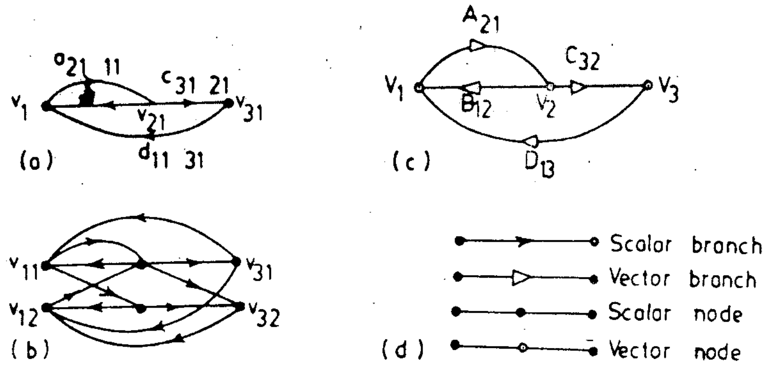

FIG. 1

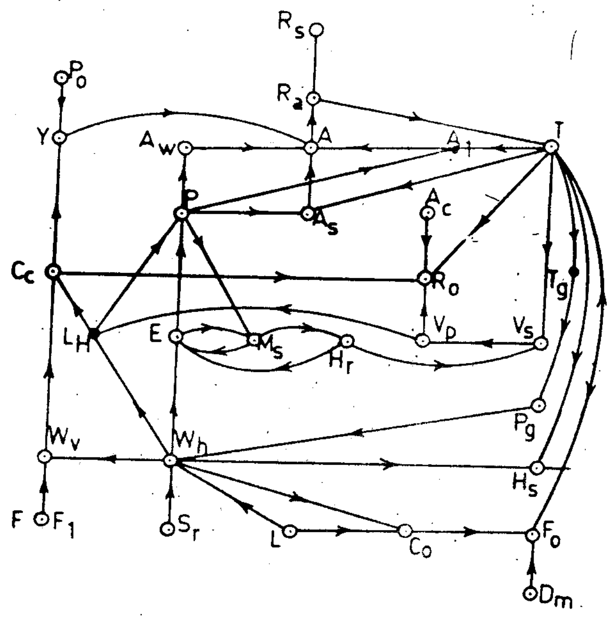

FIG. 2. Cause-effect relationships expressed as a stochastic vectorial flow graph.

LIST OF SYMBOLS IN FIGURE 2

P	-	Precipitation
R_s	-	Solar radiation
R_a	-	Absorbed sunlight
A	-	Albedo
A_w	-	Land & Water albedo
A_1	-	Ice area, A_s - Snow Area
T	-	Temperature
P_o	-	Optical index of atmosphere
Y	-	Transmitivity & reflectivity
A_c	-	Atmospheric composition affecting R_o
R_o	-	Outgoing IR radiation
C_c	-	Cloud cover
V_s	-	Surface vapour per square unit
V_p	-	Precipitable water vapour
H_r	-	Relative humidity
M_s	-	Soil Moisture
E	-	Evaporation
L_H	-	Latent hea t flux
W_h	-	Horizontal wind; W_v - Vertical wind
F_1	-	Local factors affecting W_v
S_r	-	Surface roughness
L	-	Latitude
C_o	-	Ocean current, F_o - Ocean flux
D_m	-	Mixing depth
H_s	-	Sensible heat + Potential energy flux.
P_g	-	Pressure gradient
T_g	-	Temperature gradient.