Effect of injected noise on electromagnetically induced transparency and slow light

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We have examined theoretically the phenomenon of electromagnetically induced transparency (EIT) in a three-level system operating in the Λ-configuration in the presence of an externally injected noise coupling the ground level to the intermediate (metastable) level. The changes in the depth and width of the induced transparency and the slowing down of the probe light have been calculated as a function of the probe detuning and strength of the injected noise. The calculations are within the rotating-wave approximation. Our main results are the reduction and broadening of the EIT with increasing strength of the injected noise, and a reduction in the slowing down of group velocity of the probe-laser beam. Thus, the injected semi-classical noise, unlike the quantum-dynamical noise associated with the spontaneous emission, is not effectively cancelled by the EIT mechanism.

Keywords: Electromagnetically induced transparency, injected noise, slow light.

ELECTROMAGNETICALLY induced transparency (EIT) is a coherent quantum optical phenomenon in which the absorption of a weak probe beam of light vanishes, thereby opening a window of transparency narrower than the natural linewidth at the centre of the otherwise much broader resonance absorption peak[1,2]. This transparency is due to the destructive quantum interference of the transition amplitudes along the allowed alternative paths, much as in the case of the well-known Fano resonance/anti-resonance[3,4]. This phenomenon is known to give rise to several other interesting effects[1,2,5], e.g. slow light. Remarkably, this interference effectively cancels the spontaneous down-transitions (quantum noise[6]) as well, giving a sub-natural linewidth as noted above. In contrast, as the present work shows, the EIT is indeed affected/degraded by injecting a 'classical' noise into the alternative virtual paths. This causes dephasing as we will discuss later.

The present study is motivated by a recently published experimental work[7] on a closed Λ-type EIT system coupled to microwaves in a cavity, showing coherent modulation of the EIT amplitude depending on the relative phase of microwaves and the optical fields involved. Now, a microwave cavity necessarily has an associated classical noise (due to the finite Q of the cavity), that can also get injected along with the coherent microwave field. It is therefore of interest to study the effect of such an injected classical noise on the EIT and the related phenomena. The purpose of the present work is, therefore, to analyse theoretically the nature and magnitude of the effect of the injected classical noise on the depth and width of the transparency (EIT), as well as the associated change in the group velocity of the probe light as a function of the strength of noise and probe detuning. The calculations have been done using the density-matrix formalism, wherein the various natural linewidths (spontaneous decays) involved are introduced via the appropriately chosen Lindblad super-operators[6], while the injected noise $f(t)$ is introduced explicitly in the Hamiltonian. The latter is treated within the rotating wave approximation (RWA). We compute the reduced density-matrix averaged over the noise $f(t)$. This could be carried out here in a closed form by assuming the injected noise to be a Gaussian white noise (GWN), and using the well-known Novikov theorem[8]. The latter holds for averaging an arbitrary functional of the GWN. The physical quantities of interest are then calculated in terms of this noise-averaged density matrix. Our main results are a quantitative reduction and broadening of the EIT with increasing strength of the injected noise, and a reduction in the slowing down of the group velocity of the probe light.

The atomic-level scheme considered here is as in a typical EIT experiment shown schematically in Figure 1. It involves a manifold of three energy levels $\{|1\rangle, |2\rangle, |3\rangle\}$ with the respective energies $E_1 = \hbar\omega_1$, $E_2 = \hbar\omega_2$ and $E_3 = \hbar\omega_3$, connected in the Λ-configuration with $E_1 < E_2 < E_3$. Such a Λ-EIT scheme is possible, e.g. in the D2-line transitions of ^{85}Rb vapour[9,10] with $|1\rangle \equiv 5^2S_{1/2}$: F = 2, $|2\rangle \equiv 5^2S_{1/2}$: F = 3 and $|3\rangle \equiv 5^2P_{3/2}$: F′ = 3. As follows from the selection rules, in a Λ scheme, transition (here $|1\rangle \leftrightarrow |2\rangle$) between the ground and the intermediate state is necessarily an electric-dipole forbidden transition,

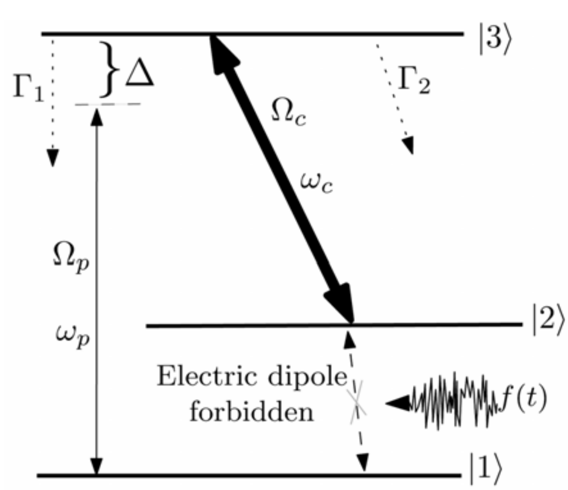

Figure 1. Electromagnetically induced transparency scheme of Λ-type three-level system perturbed by an externally injected noise $f(t)$.

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since the other two transitions involved are electric-dipole allowed. It can, however, be, say an electric-quadrupole allowed transition, much as in the case of a controlling microwave field used in the recent EIT experiment in rubidium vapour system[7]. A novel feature of the present work is a noise field $f(t)$ injected externally at the transition $|1\rangle \leftrightarrow |2\rangle$.

In a standard EIT experiment, the absorption of the probe beam is studied as function of its detuning, while the coupling laser is kept at resonance[2]. Accordingly, here the probe laser is detuned off-resonance by an amount Δ from the transition $|1\rangle \rightarrow |3\rangle$ which is being probed, i.e. $\hbar\omega_p = E_3 - E_1 - \hbar\Delta$, where ω_p and $\Omega_p(\equiv(\vec{d}_{13}\cdot\vec{\mathscr{E}}_p)/2\hbar)$ are respectively, the angular frequency and the associated Rabi frequency of the probe laser. A strong coupling laser beam is applied and kept at resonance to the transition $|2\rangle \rightarrow |3\rangle$, with $\hbar\omega_c = E_3 - E_2$, where ω_c and $\Omega_c(\equiv(\vec{d}_{23}\cdot\vec{\mathscr{E}}_c)/2\hbar)$ are respectively, the angular frequency and the associated Rabi frequency of the coupling laser. Further, Γ_1 and Γ_2 denote the rates of spontaneous decay of the excited level $|3\rangle$ to the ground level $|1\rangle$ and to the intermediate level $|2\rangle$ respectively.

The full Hamiltonian $\mathscr{H}(t)$ of the EIT system here corresponds essentially to the case of a three-level system being acted upon by two optical fields in the so-called semi-classical approximation, but now with an extra feature, namely a term $f(t)$ corresponding to the injected noise. More explicitly, we have

$$\begin{aligned}\mathscr{H}(t) = {} & \hbar\omega_1|1\rangle\langle 1| + \hbar\omega_2|2\rangle\langle 2| + \hbar\omega_3|3\rangle\langle 3| \\ & - \hbar\Omega_p(e^{-i\omega_p t} + e^{i\omega_p t})(|3\rangle\langle 1| + |1\rangle\langle 3|) \\ & - \hbar\Omega_c(e^{-i\omega_c t} + e^{i\omega_c t})(|3\rangle\langle 2| + |2\rangle\langle 3|) \\ & - \hbar f(t)(|1\rangle\langle 2| + |2\rangle\langle 1|). \end{aligned} \tag{1}$$

Let

$$\mathscr{H}(t) \equiv \mathscr{H}_0 + \mathscr{H}_1(t),$$

with

$$\mathscr{H}_0 = -\hbar\omega_p|1\rangle\langle 1| - \hbar\omega_c|2\rangle\langle 2|, \tag{2}$$

and

$$\begin{aligned}\mathscr{H}_1(t) = {} & \hbar(\omega_1 + \omega_p)|1\rangle\langle 1| + \hbar(\omega_2 + \omega_c)|2\rangle\langle 2| \\ & + \hbar\omega_3|3\rangle\langle 3| - \hbar f(t)(|1\rangle\langle 2| + |2\rangle\langle 1|) \\ & - \hbar\Omega_p(e^{-i\omega_p t} + e^{i\omega_p t})(|3\rangle\langle 1| + |1\rangle\langle 3|) \\ & - \hbar\Omega_c(e^{-i\omega_c t} + e^{i\omega_c t})(|3\rangle\langle 2| + |2\rangle\langle 3|). \end{aligned} \tag{3}$$

We now proceed to the interaction (I) picture with the corresponding Hamiltonian $\mathscr{H}_I(t)$ given by

$$\mathscr{H}_I(t) = e^{i\mathscr{H}_0 t/\hbar}\mathscr{H}_1(t)e^{-i\mathscr{H}_0 t/\hbar}. \tag{4}$$

In the RWA (i.e. neglecting the rapidly oscillating terms proportional to $e^{\pm 2i\omega_c t}$ and $e^{\pm 2i\omega_p t}$), we obtain

$$\begin{aligned}\mathscr{H}_I(t) = {} & +\hbar(\omega_1 + \omega_p)|1\rangle\langle 1| + \hbar(\omega_2 + \omega_c)|2\rangle\langle 2| \\ & + \hbar\omega_3|3\rangle\langle 3| - \hbar\Omega_p(|3\rangle\langle 1| + |1\rangle\langle 3|) \\ & - \hbar\Omega_c(|3\rangle\langle 2| + |2\rangle\langle 3|) \\ & - \hbar f(t)(e^{-i\omega_\mu t}|1\rangle\langle 2| + e^{i\omega_\mu t}|2\rangle\langle 1|), \end{aligned} \tag{5}$$

with $\omega_\mu \equiv \omega_2 - \omega_1 - \Delta$.

Hereinafter, we will use the interaction-picture Hamiltonian $\mathscr{H}_I(t)$, but drop the subscript I for convenience.

With this, the master equation of motion for the density matrix $\rho(t)$ is

$$\dot{\rho}(t) = -\frac{i}{\hbar}[\mathscr{H}(t), \rho(t)] - \hat{\Lambda}\rho(t), \tag{6}$$

where we have introduced the Linblad super-operator $\hat{\Lambda}$, given in the diagonal form as

$$\hat{\Lambda}\rho \equiv \sum_{j=1,2}\Gamma_j(L_j^\dagger L_j\rho + \rho L_j^\dagger L_j - 2L_j\rho L_j^\dagger). \tag{7}$$

Here Γ_1 and Γ_2 are the rates of spontaneous decays $|3\rangle \rightarrow |1\rangle$ and $|3\rangle \rightarrow |2\rangle$ respectively. The Lindblad operators L_1 and L_2 are chosen appropriately so as to describe the spontaneous decays as[6]:

$$L_1 = |1\rangle\langle 3|, \tag{8}$$

$$L_2 = |2\rangle\langle 3|. \tag{9}$$

As is known well, a Lindblad master equation describes the non-unitary evolution of the density matrix preserving the trace condition, without violating its complete positivity and hermiticity for all initial conditions[6].

Substituting from eqs (5) and (7) into eq. (6), we obtain

$$\begin{aligned}-i\dot{\rho}_{11}(t) = {} & f(t)(e^{-i\omega_\mu t}\rho_{21}(t) - e^{i\omega_\mu t}\rho_{12}(t)) \\ & + \Omega_p(\rho_{31}(t) - \rho_{13}(t)) - i\Gamma_1\rho_{33}(t), \end{aligned} \tag{10}$$

$$\begin{aligned}-i\dot{\rho}_{22}(t) = {} & f(t)(e^{i\omega_\mu t}\rho_{12}(t) - e^{-i\omega_\mu t}\rho_{21}(t)) \\ & + \Omega_c(\rho_{32}(t) - \rho_{23}(t)) - i\Gamma_2\rho_{33}(t), \end{aligned} \tag{11}$$

$$\begin{aligned}-i\dot{\rho}_{12}(t) = {} & \Delta\rho_{12}(t) + \Omega_p\rho_{32}(t) - \Omega_c\rho_{13}(t) \\ & + f(t)e^{-i\omega_\mu t}(\rho_{22}(t) - \rho_{11}(t)), \end{aligned} \tag{12}$$

$$\begin{aligned}-i\dot{\rho}_{13}(t) = {} & f(t)e^{-i\omega_\mu t}\rho_{23}(t) - \Omega_c\rho_{12}(t) \\ & + \Omega_c(\rho_{33}(t) - \rho_{11}(t)) + \left(\Delta + i\frac{\Gamma_1 + \Gamma_2}{2}\right)\rho_{13}(t), \end{aligned} \tag{13}$$

$$\begin{aligned}-i\dot{\rho}_{23}(t) = {} & i\frac{\Gamma_1 + \Gamma_2}{2}\rho_{23}(t) + f(t)e^{i\omega_\mu t}\rho_{13}(t) \\ & + \Omega_c(\rho_{33}(t) - \rho_{22}(t)) - \Omega_p\rho_{12}(t), \end{aligned} \tag{14}$$

along with the trace condition $Tr\,\rho = 1$ and the hermiticity condition $\rho^{\dagger} = \rho$.

Now, the density matrix $\rho(t)$ is a functional of the noise $f(t)$ occurring in eqs (10)–(14), and, therefore, it must be averaged over all the realizations of $f(t)$. For this, we have taken the noise $f(t)$ to be a GWN, i.e.

$$\langle f(t) f(t')\rangle_f = f_0^2 \delta(t-t'), \tag{15}$$

where f_0^2, having the dimension of frequency, is a measure of the strength of the noise $f(t)$ (much like the Rabi frequency Ω, which is a measure of the strength of the laser field). For this, we make use of the Novikov theorem[8] giving

$$\begin{aligned}\langle f(t)\rho[f(t)]\rangle_f &= \int_{-\infty}^{t} \mathrm{d}t' \langle f(t) f(t')\rangle_f \left\langle \frac{\delta\rho[f(t')]}{\delta f(t)} \right\rangle_f \\ &= \frac{f_0^2}{2}\left\langle \frac{\delta\rho[f(t)]}{\delta f(t)\mathrm{d}t}\right\rangle_f .\end{aligned} \tag{16}$$

Straightforward functional differentiation of eqs (10)–(14) w.r.t. $f(t)$ and using eq. (16), we obtain, within RWA (this time neglecting the rapidly oscillating terms proportional to $e^{\pm 2i\omega_\mu t}$), a closed set of equations for the noise-averaged density matrix elements $\langle\rho_{ij}(t)\rangle_f\ (\equiv \sigma_{ij}(t)$ for typographic convenience) as follows:

$$\begin{aligned}-i\dot{\sigma}_{11}(t) &= \Omega_{\mathrm{p}}(\sigma_{31}(t)-\sigma_{13}(t)) - i\Gamma_1\sigma_{33}(t) \\ &\quad - if_0^2(\sigma_{22}(t)-\sigma_{11}(t)),\end{aligned} \tag{17}$$

$$\begin{aligned}-i\dot{\sigma}_{22}(t) &= \Omega_{\mathrm{c}}(\sigma_{32}(t)-\sigma_{23}(t)) - i\Gamma_2\sigma_{33}(t) \\ &\quad + if_0^2(\sigma_{22}(t)-\sigma_{11}(t)),\end{aligned} \tag{18}$$

$$-i\dot{\sigma}_{12}(t) = (\Delta + if_0^2)\sigma_{12}(t) + \Omega_{\mathrm{p}}\sigma_{32}(t) - \Omega_{\mathrm{c}}\sigma_{13}(t), \tag{19}$$

$$\begin{aligned}-i\dot{\sigma}_{13}(t) &= -\Omega_{\mathrm{c}}\sigma_{12}(t) + \Omega_{\mathrm{p}}(\sigma_{33}(t)-\sigma_{11}(t)) \\ &\quad + \left(\Delta + i\frac{f_0^2+\Gamma_1+\Gamma_2}{2}\right)\sigma_{13}(t),\end{aligned} \tag{20}$$

$$\begin{aligned}-i\dot{\sigma}_{23}(t) &= \Omega_{\mathrm{c}}(\sigma_{33}(t)-\sigma_{22}(t)) - \Omega_{\mathrm{p}}\sigma_{12}(t) \\ &\quad + i\left(\frac{f_0^2+\Gamma_1+\Gamma_2}{2}\right)\sigma_{23}(t).\end{aligned} \tag{21}$$

These linear, first-order differential equations, along with the trace condition $Tr\,\sigma = 1$ and the hermiticity condition $\sigma^{\dagger} = \sigma$, are now solved numerically in the steady state (i.e. $\dot{\sigma}(t) = 0$), so as to calculate the physical quantities of interest, as described below.

The absorption coefficient of the probe light in a dilute gaseous medium can be expressed in terms of the density matrix as[5]:

$$\alpha(\Delta, \Omega_{\mathrm{c}}, f_0^2) = \frac{1}{\lambda_{\mathrm{p}}}\frac{N\lambda_0^3\pi}{(\Omega_{\mathrm{p}}/\Gamma)}\mathrm{Im}[\sigma_{31}(\Delta, \Omega_{\mathrm{c}}, f_0^2)], \tag{22}$$

where N is the atomic number density of the medium, λ_{p} the wavelength of the probe light at a detuning Δ, and $\lambda_0 = c/(\omega_3 - \omega_1)$ the wavelength at the corresponding resonance. Also, we have set $\Gamma_2 = 0$ (which is in fact a good approximation for most of the practical Λ-type EIT systems), and $\Gamma_1 \equiv \Gamma$.

The corresponding real part of the refractive index can be expressed in terms of the various parameters involved[5] as:

$$n_{\mathrm{R}}(\Delta, \Omega_{\mathrm{c}}, f_0^2) = 1 + \frac{N\lambda_0^3\pi}{(\Omega_{\mathrm{p}}/\Gamma)}\mathrm{Re}[\sigma_{31}(\Delta, \Omega_{\mathrm{c}}, f_0^2)]. \tag{23}$$

The associated group velocity of the probe light in the medium concerned can be conveniently expressed as[5]:

$$V_{\mathrm{g}}(\Delta, \Omega_{\mathrm{c}}, f_0^2) = \frac{c}{n_{\mathrm{R}}(\Delta, \Omega_{\mathrm{c}}, f_0^2) - \omega_{\mathrm{p}}\frac{\mathrm{d}n_{\mathrm{R}}}{\mathrm{d}\Delta}}. \tag{24}$$

In Figures 2–6, we have plotted the various physical quantities of interest (α, n_{R} and V_{g}), calculated as functions of the parameters involved (i.e. probe-detuning Δ/Γ, coupling strength $\Omega_{\mathrm{c}}/\Gamma$ and strength of noise f_0^2/Γ).

We have used the following set of parameter values appropriate to the EIT-medium, ^{85}Rb vapour[9,10]: $N = 10^{18}$ atoms per m^3, $\lambda_0 = 780$ nm, $(\Omega_{\mathrm{c}}/\Gamma) = 1$, $(\Omega_{\mathrm{p}}/\Gamma) = 0.001$, $(f_0^2/\Gamma) = \{0, 0.7, 1.6\}$ and $\Gamma = 5$ MHz.

From Figure 2, it can be readily seen, as expected for pure EIT (without noise), that absorption of the probe light beam increases with detuning (Δ/Γ), for small values of the latter.

However, with increasing strength of the noise (f_0^2/Γ), absorption as well as its spectral width are found increase. Overall, the effect of noise is more pronounced

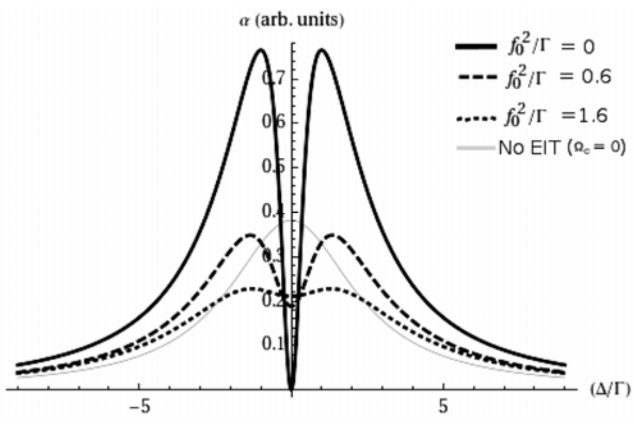

Figure 2. Absorption (arb. units) of probe light plotted against the detuning (Δ/Γ) for different values of the strength of noise (f_0^2/Γ).

within the EIT window, and much less so outside the window, as clearly seen in Figure 3.

Figure 4 gives specifically the variation of the real part of the refractive index (n_R) as a function of detuning in the anomalous regime of dispersion, within the EIT window. There is a pronounced decrease in the variation of n_R across the EIT window with the increasing strength of noise.

Now we come to the effect of noise on the group velocity of the probe light beam across the EIT window. This has an important bearing on the phenomenon of slow light associated with the EIT.

The overall effect of the noise is to reduce the slowing down of light beam, as can be seen from Figure 5. Thus, for example, $V_g = 0.52\ \mathrm{ms}^{-1}$ for zero noise and zero detuning). Figure 6 resolves the finer features of the effect of the noise on the group velocity at and near the resonance (i.e. $\Delta/\Gamma = 0, 0.25$).

Finally, a few words are in order to explain how the injected noise enters the physics of the EIT. As is known, EIT results from a coherent destructive interference of the quantum transition amplitudes along alternative (virtual)

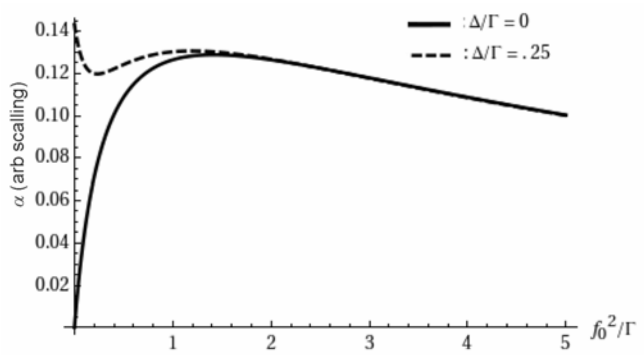

Figure 3. Absorption (arb. units) of probe light at and near resonance, plotted against the strength of noise (f_0^2/Γ).

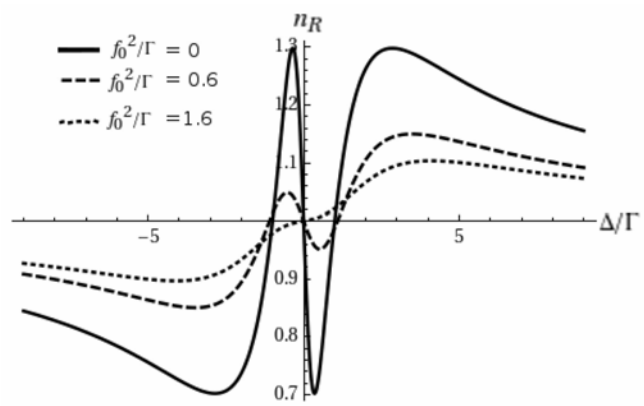

Figure 4. Real part of the refractive index (n_R) plotted against the detuning (Δ/Γ) for different values of the strength of noise (f_0^2/Γ).

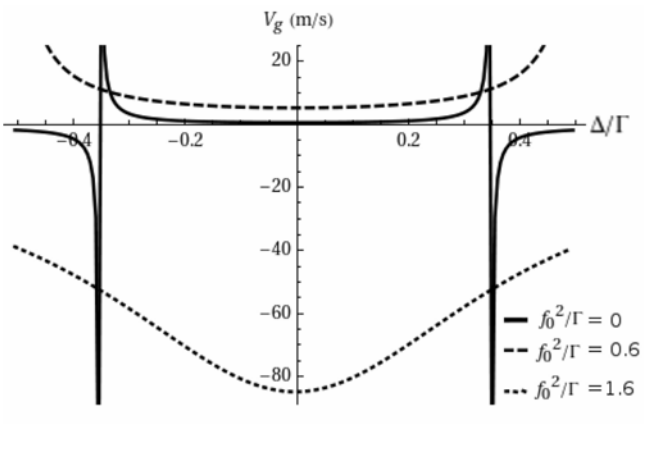

Figure 5. Group velocity (V_g m/s) of probe light in the medium plotted against the detuning (Δ/Γ) for different values of the strength of noise (f_0^2/Γ).

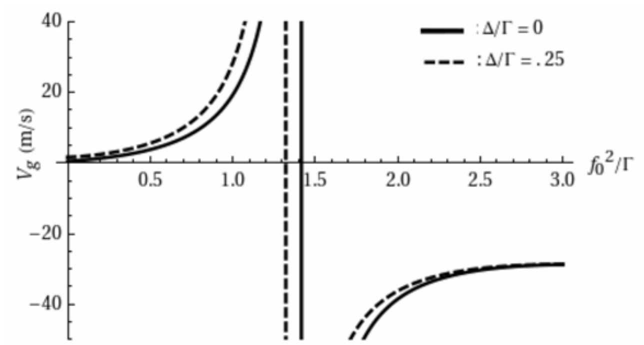

Figure 6. Group velocity (V_g m/s) of probe light in the medium at and near resonance, plotted against the strength of noise (f_0^2/Γ).

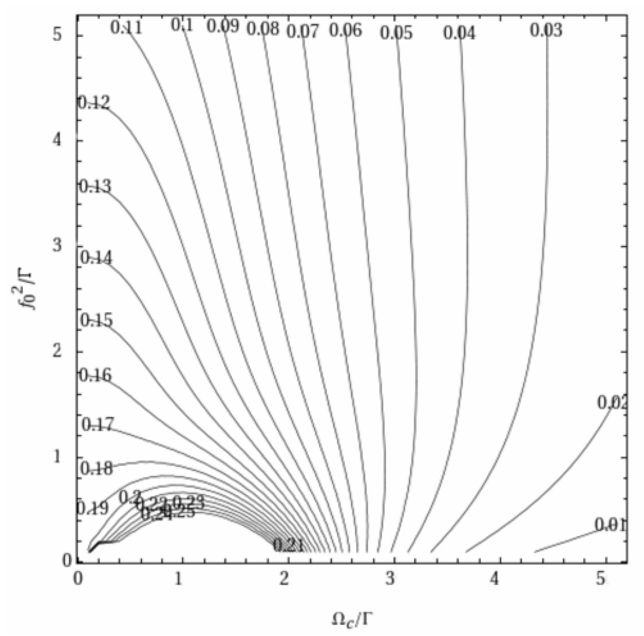

Figure 7. Contour plot of constant absorption coefficient (α) in the $(\Omega_c/\Gamma) - (f_0^2/\Gamma)$ plane. Labels on contours indicate the α values (in arbitrary units).

paths from the ground level $|1\rangle$ to the excited level $|3\rangle$ (involving, for example, the direct transition amplitude $|1\rangle \rightarrow |3\rangle$ and the alternative transition amplitudes $|1\rangle \rightarrow |3\rangle \rightarrow |2\rangle \rightarrow |3\rangle$... and so on in higher-order virtual cycles). The injected microwave noise connecting level $|2\rangle$ to level $|1\rangle$ can, however, cause the electron to make a real transition $|2\rangle \rightarrow |1\rangle$, and thus take it out of the above coherent (virtual) cycles. Such a noise-induced real transition necessarily causes dephasing of the otherwise coherent interference of the virtual alternatives, thereby degrading the EIT. This effect can be seen in Figure 7. Our calculation explicitly takes into account this effect of the injected classical noise within the Lindblad formalism, using the Novikov theorem.

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